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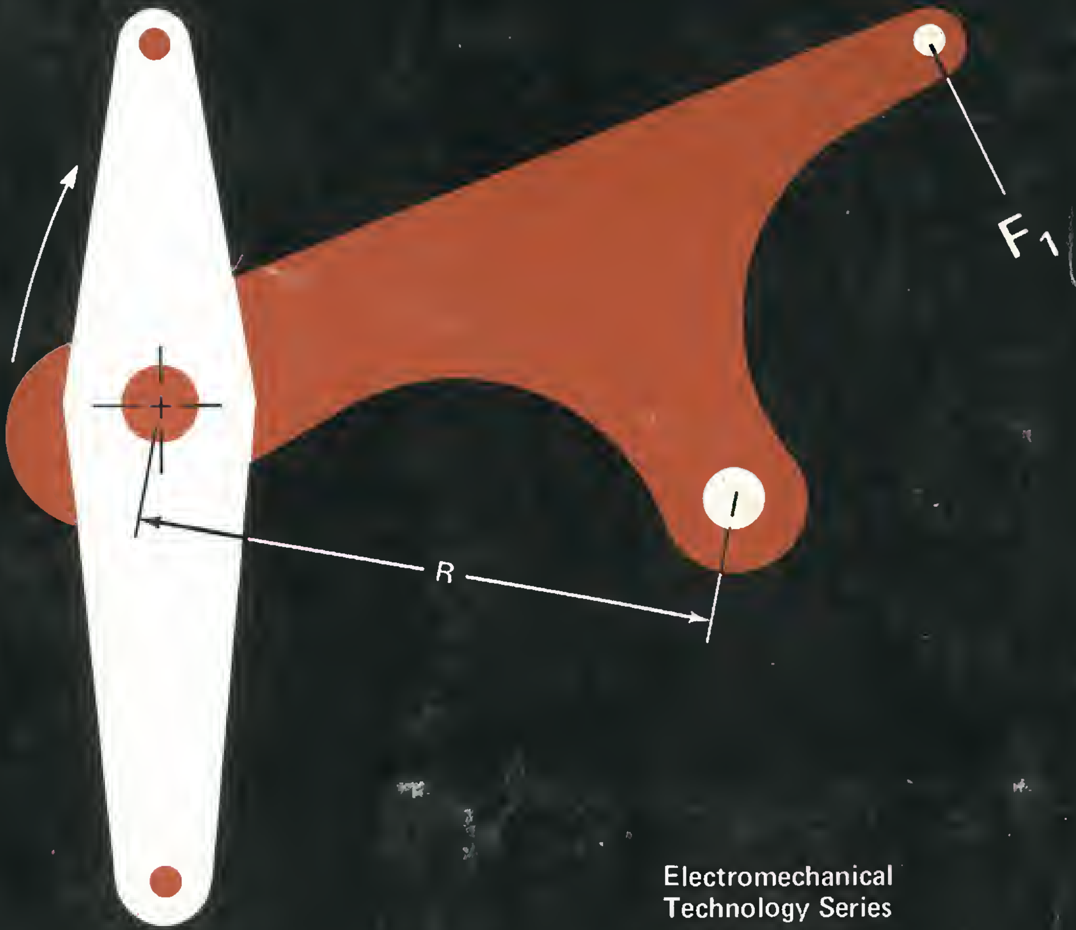

Electromechanical Technology Series

TERC EMT STAFF

DELMAR PUBLISHERS, MOUNTAINVIEW AVENUE, ALBANY, NEW YORK 12205

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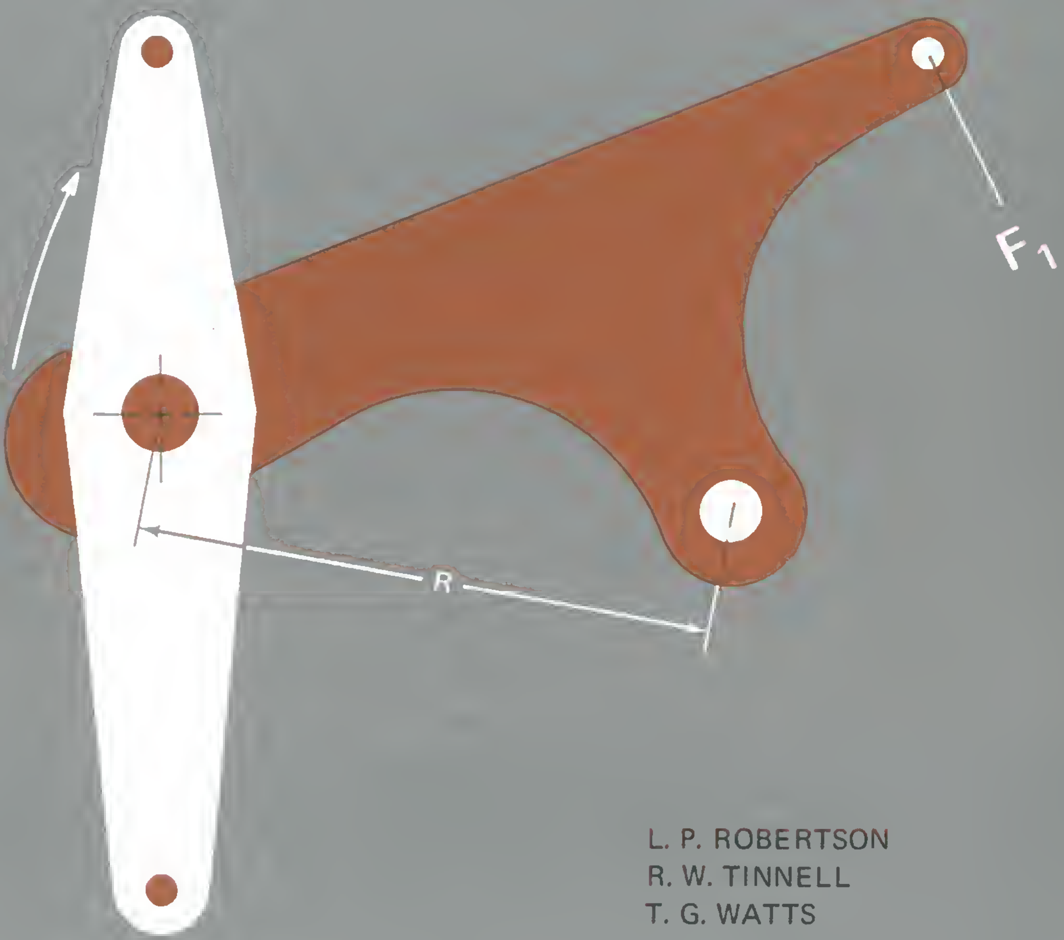

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Foreword

The marriage of electronics and technology is creating new demands for technical personnel in today's industries. New occupations have emerged with combination skill requirements well beyond the capability of many technical specialists. Increasingly, technicians who work with systems and devices of many kinds — mechanical, hydraulic, pneumatic, thermal, and optical — must be competent also in electronics. This need for combination skills is especially significant for the youngster who is preparing for a career in industrial technology.

This manual is one of a series of closely related publications designed for students who want the broadest possible introduction to technical occupations. The most effective use of these manuals is as combination textbook-laboratory guides for a full-time, post-secondary school study program that provides parallel and concurrent courses in electronics, mechanics, physics, mathematics, technical writing, and electromechanical applications.

A unique feature of the manuals in this series is the close correlation of technical laboratory study with mathematics and physics concepts. Each topic is studied by use of practical examples using modern industrial applications. The reinforcement obtained from multiple applications of the concepts has been shown to be extremely effective, especially for students with widely diverse educational backgrounds. Experience has shown that typical junior college or technical school students can make satisfactory progress in a well-coordinated program using these manuals as the primary instructional material.

School administrators will be interested in the potential of these manuals to support a common first-year core of studies for two-year programs in such fields as: instrumentation, automation, mechanical design, or quality assurance. This form of *technical core* program has the advantage of reducing instructional costs without the corresponding decrease in holding power so frequently found in general core programs.

This manual, along with the others in the series, is the result of six years of research and development by the *Technical Education Research Centers, Inc.,* (TERC), a national nonprofit, public service corporation with headquarters in Cambridge, Massachusetts. It has undergone a number of revisions as a direct result of experience gained with students in technical schools and community colleges throughout the country.

Maurice W. Roney

The Electromechanical Series

TERC is engaged in an on-going educational program in *Electromechanical Technology.* The following titles have been developed for this program:

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Preface

The study of mechanisms is one of the oldest of the applied sciences. The early Greeks and Romans used simple levers and linkages in a wide variety of application; and the American Industrial Revolution can truly be said to have been based on mechanical components. The advent of space exploration has demanded a rebirth of interest in mechanics and mechanisms. In the past we have thought primarily of applications in the automotive, machine tool, and watchmaking fields. Today, it is more common to think of aerospace, defense weaponry, computer and precision instrument applications. These changes in emphasis have created subtle but important new demands upon training programs in mechanisms.

This material is an introductory treatment of modern Mechanical Linkages, combining the elements of mechanical theory with those of practicality. The topics treated include: various levers and four-bar configurations, and some selected special topics. The materials are intended for use by technology students who have had little or no previous exposure to practical applied mechanics. Consequently, no attempt has been made to cover the material in the fine detail that would be appropriate for the experienced specialist in mechanical linkages. An attempt *has* been made to expose the student to the practical skills of mechanical assembly and to the principles of operation of a variety of mechanisms.

The sequence of presentation chosen is by no means inflexible. It is expected that individual instructors may choose to use the materials in other than the given sequence.

The particular topics chosen for inclusion in this volume were selected primarily for convenience and economy of materials. Some instructors may wish to omit some of the exercises or to supplement some of them to better meet their local needs.

The materials are presented in an action-oriented format combining many of the features normally found in a textbook with those usually associated with a laboratory manual. Each experiment contains:

1. An INTRODUCTION which identifies the topic to be examined and often includes a rationale for doing the exercise.
2. A DISCUSSION which presents the background, theory, or techniques needed to carry out the exercise.

3. A MATERIALS list which identifies all of the items needed in the laboratory experiment. (Items usually supplied by the student such as pencil and paper are not included in the lists.)

4. A PROCEDURE which presents step-by-step instructions for performing the experiment. In most instances the measurements are done before calculations so that all of the students can at least finish making the measurements before the laboratory period ends.

5. An ANALYSIS GUIDE which offers suggestions as to how the student might approach interpretation of the data in order to draw conclusions from it.

6. PROBLEMS are included for the purpose of reviewing and reinforcing the points covered in the exercise. The problems may be of the numerical solution type or simply questions about the exercise.

Students should be encouraged to study the text material, perform the experiment, work the review problems, and submit a technical report on each topic. Following this pattern, the student can acquire an understanding of, and skill with, modern mechanisms that will be very valuable on the job. For best results, these students should be concurrently enrolled in a course in technical mathematics (introductory calculus).

This material on Mechanical Linkages comprises one of a series of volumes prepared for technical students by the TERC EMT staff at Oklahoma State University, under the direction of D.S. Phillips and R.W. Tinnell. The principal authors of this material were L.P. Robertson, R.W. Tinnell, T.G. Watts, and D.A. Yeager.

An *Instructor's Data Book* is available for use with this volume. Mr. Harlan Cook was responsible for testing the materials and compiling the instructor's data book for them. Other members of the TERC staff made valuable contributions in the form or criticisms, corrections and suggestions.

It is sincerely hoped that this volume as well as the other volumes in the series, the instructor's data books, and the other supplementary materials will make the study of technology interesting and rewarding for both students and teachers.

THE TERC EMT STAFF

TO THE STUDENT

Duplicate data sheets for each experiment are provided in the back of the book. These are perforated to be removed and completed while performing each experiment. They may then be submitted with the experiment analysis for your instructor's examination.

Contents

experiment 1 CLASS–ONE LEVERS

INTRODUCTION. Machines and mechanisms often appear to be quite complicated; however, closer examination shows them to be composed of various combinations of simple machine elements. One of the elements found in most machines is the lever. In this experiment we will examine the most basic of the machine elements — the class-one lever.

DISCUSSION. Complex machines are only combinations of two or more simple machine elements. Many persons classify the basic machine elements as being the lever, the pulley, the wheel and axle, the inclined plane, the screw, and the gear. However, most scientists and engineers recognize that there are only two basic principles in machines: namely the lever and the inclined plane. The wheel and axle, the pulley, and gears may be considered levers. The wedge and the screw use the principle of the inclined plane. By becoming familiar with the principles of these simple machines, you can more easily understand the operation of complex machinery.

Machines have many purposes. They may be used to *transform* energy. For example, a generator transforms mechanical energy into electrical energy. Or, machines may be used to *transfer* energy from one place to another. For example, the connecting rods, crankshaft, drive shaft, and rear axle transfer energy from the automobile engine to the rear wheels.

Another use of machines is to *multiply force*. We can use a system of pulleys to raise a large weight with a much smaller force exerted. But we must exert this force over a greater distance than the height through which the weight is raised: thus, the load moves more slowly than the pulley chain on which we pull. A machine lets us gain force, but only at the expense of speed or distance.

Machines may also be used to *multiply speed*. A good example of this is the bicycle; we gain speed by exerting a greater force.

Finally, machines are used to *change the direction of a force*. For example, a flag is raised to the top of a flagpole by exerting a downward force on the hoisting rope which causes an upward force on the attached flag.

Probably the simplest and most often used type of machine is the lever. A lever consists of a rigid bar that is free to rotate about a bearing known as the *fulcrum*. The bar may be either straight or curved. Levers have been grouped into three different classes, depending upon the location of the fulcrum with respect to the weight and effort application. The class-one lever shown in figure 1-1 has the fulcrum located between the *effort* or force application and the *weight* or resistance. A seesaw or a crowbar are good examples of the class-one lever. Other examples with which you may be familiar are

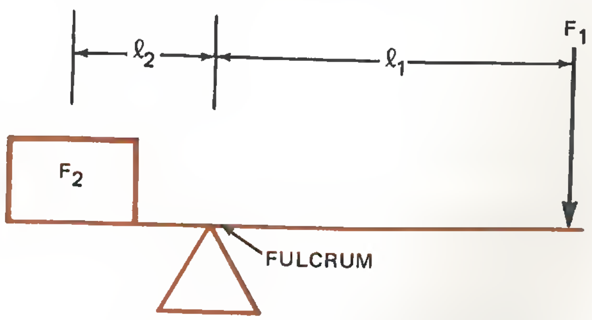


Fig. 1-1 Class-One Lever

shears, pliers, and oars. A factor involved in many problems in mechanics is the *moment of force.* This factor is well illustrated in the class-one lever. The moment of force about a point is the product of the force and the *perpendicular* distance from that point to the line of action of the force. This distance is the *lever arm* of the force and the fixed point is the *center of moment.* For example, in figure 1-1, the center of moment can be the fulcrum point. The moment of force caused by the weight about the fulcrum would be equal to $F_2 \times \ell_2$. Assuming that F_2 is measured in pounds and ℓ_2 is measured in feet, the units would be in pound-feet.

To have *equilibrium* (or balance), the moments about the center of moment must be equal. In other words, in figure 1-1,

$$\boxed{F_2 \times \ell_2 = F_1 \times \ell_1} \qquad (1.1)$$

To illustrate this fact, assume that a force of 80 pounds is applied as indicated by F_1 and that distance ℓ_1 is 3 feet. If distance ℓ_2 is 1 foot, how much weight would we be able to balance? Using the moment equation 1.1 and inserting the given values gives:

$$F_2 \times 1 = 80 \text{ lb} \times 3 \text{ ft}$$

Solving for F_2,

$$F_2 = \frac{80 \times 3}{1} = 240 \text{ lbs.}$$

In this example, 80 pounds will balance 240 pounds. In this case we have a positive *mechanical advantage* in that our effort has been magnified 3 times. The mechanical advantage of a machine is defined as the ratio of the output force to the input force.

$$\boxed{MA = \frac{F_2}{F_1}} \qquad (1.2)$$

In our example, $MA = \frac{F_2}{F_1} = \frac{240}{80} = 3.$

Another frequently used ratio in mechanics is the *velocity* or *displacement* ratio. This ratio is defined as the distance through which the *input* force moves divided by the distance through which the *output* force moves. Using S_1 for the input distance and S_2 for the output distance,

$$\boxed{\text{Velocity (Displacement) ratio} = \frac{S_1}{S_2}} \qquad (1.3)$$

In figure 1-2, the lever rotates about the ful-

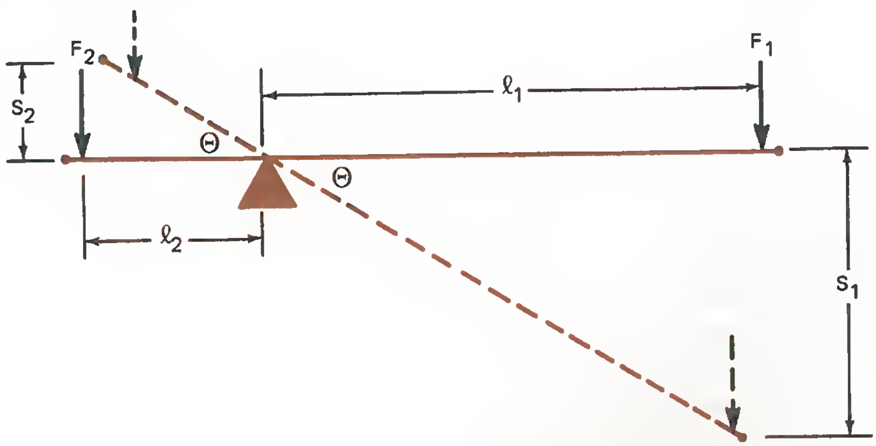


Fig. 1-2 Class-One Lever Velocity Ratio

crum and moves the weight, F_2, a distance, S_2. At the same time, the input force, F_1, is moving through the distance S_1. From similar triangles, it can be seen that

$$\frac{S_1}{S_2} = \frac{\ell_1 \sin \Theta}{\ell_2 \sin \Theta} \tag{1.4}$$

Since the friction is quite small, it can be neglected for this discussion. The work performed (force X distance in direction of the force) at the input will equal the work at the output; that is,

$$F_2 \times S_2 = F_1 \times S_1$$

This equation is equal to:

$$\frac{F_2}{F_1} = \frac{S_1}{S_2}$$

In short, if friction and the weight of the lever are neglected, the displacement ratio (S_1/S_2) is equal to the mechanical advantage (F_2/F_1). In practice, it takes a little more effort (F_1 in our illustrations) to overcome whatever friction may be in the system. Therefore, the actual mechanical advantage is always somewhat *less* than the velocity or displacement ratio.

From equation 1.1, it can be seen that $F_2 = F_1 \times \frac{\ell_1}{\ell_2}$. This indicates that the effort expended by force F_1 will be magnified or increased by an amount equal to length ℓ_1 divided by length ℓ_2. That is, the mechanical advantage is equal to the effort arm divided by the resistance arm. For example, a class-one lever having 8 inches on one side of the fulcrum and 2 inches on the other would possess a mechanical advantage of 8/2 or 4. If you applied 50 lbs. of force to the 8-inch arm, you would expect 4 times that force to be exerted by the other arm. Your 50 lbs. would be increased to 200 lbs.

To this point, the levers we have considered have had straight arms, and the direction in which the weight acts has been parallel to the direction in which the force was exerted. However, all levers are not straight. Look at figure 1-3 and you may wonder how to measure the length of the two arms about the fulcrum. This figure represents a curved pump handle. You do not measure around the curve—you use the straight-line distance. To determine the length of the effort arm, draw a straight line representing force F_1 through the point where the force, F_1 is applied and in the direction in which it is applied. Then, from the fulcrum, draw a line perpendicular to this force line. The length of this perpendicular, ℓ_1, is the length of the effort arm.

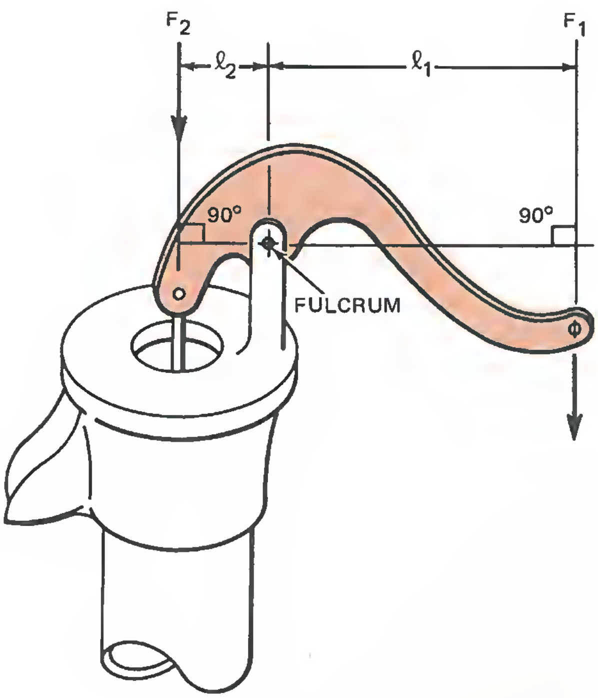


Fig. 1-3 Class-One Lever - Curved Lever Arm

To find the length of the resistance arm, the same method is used. Draw a line in the direction the resistance, F_2, is operating and from the fulcrum construct a perpendicular to this line. The perpendicular distance from the fulcrum to this line, ℓ_2, is the length of the resistance arm.

Regardless of how the curvature of a lever is formed, this method will find the lengths of the moment arms. Then, the same solutions described for straight-arm levers can be used.

In summary, levers are machines that help do work. They can change the size, direction, or speed of the force that you apply. The class-one lever has the effort and the resistance on opposite sides of the fulcrum. The effort (force applied) and the resistance or opposition (force output) move in *opposite* directions. The force is magnified, but with a corresponding decrease in speed or distance. It was seen that the mechanical advantage equaled the ratio of resistance to effort. This ratio is also equal to the moment arm (or lever arm) ratio. Further, when ignoring friction and the weight of the lever arm, the velocity or displacement ratio is equal to the mechanical advantage.

MATERIALS

- 1 Breadboard with legs
- 2 Shaft hangers, 1-1/2 in. with bearings
- 1 Shaft, 4" x 1/4"
- 2 Spring balances
- 2 Spring balance posts with clamps
- 1 Dial caliper (0 - 4 in.)
- 2 Collars
- 1 Lever arm, 2 in. long with 1/4 in. bore hub
- 1 Lever arm, 1 in. long with 1/4 in. bore hub

PROCEDURE

1. Inspect each of your components to be sure they are undamaged.
2. Mount the components on the breadboard as shown in figure 1-4.
3. Move both spring balance posts until the lower balance reads about 10 oz *and* the lever arm is *vertical.* Record the readings of both spring balances (F_1 and F_2).
4. Using the dial caliper, measure and record the distances from the *center* of the shaft to the point where each spring balance attaches to the lever arm (ℓ_1 and ℓ_2).
5. Manually twist the lever arm slightly away from the vertical, and observe the changes in the spring balance readings. Make notes as to the nature and size of the changes.
6. Move the upper spring balance closer to the fulcrum and repeat steps 3 and 4.
7. Again move the upper spring balance closer to the fulcrum and repeat steps 3 and 4.
8. Keep repeating the above process until the upper spring balance is quite close to the fulcrum.

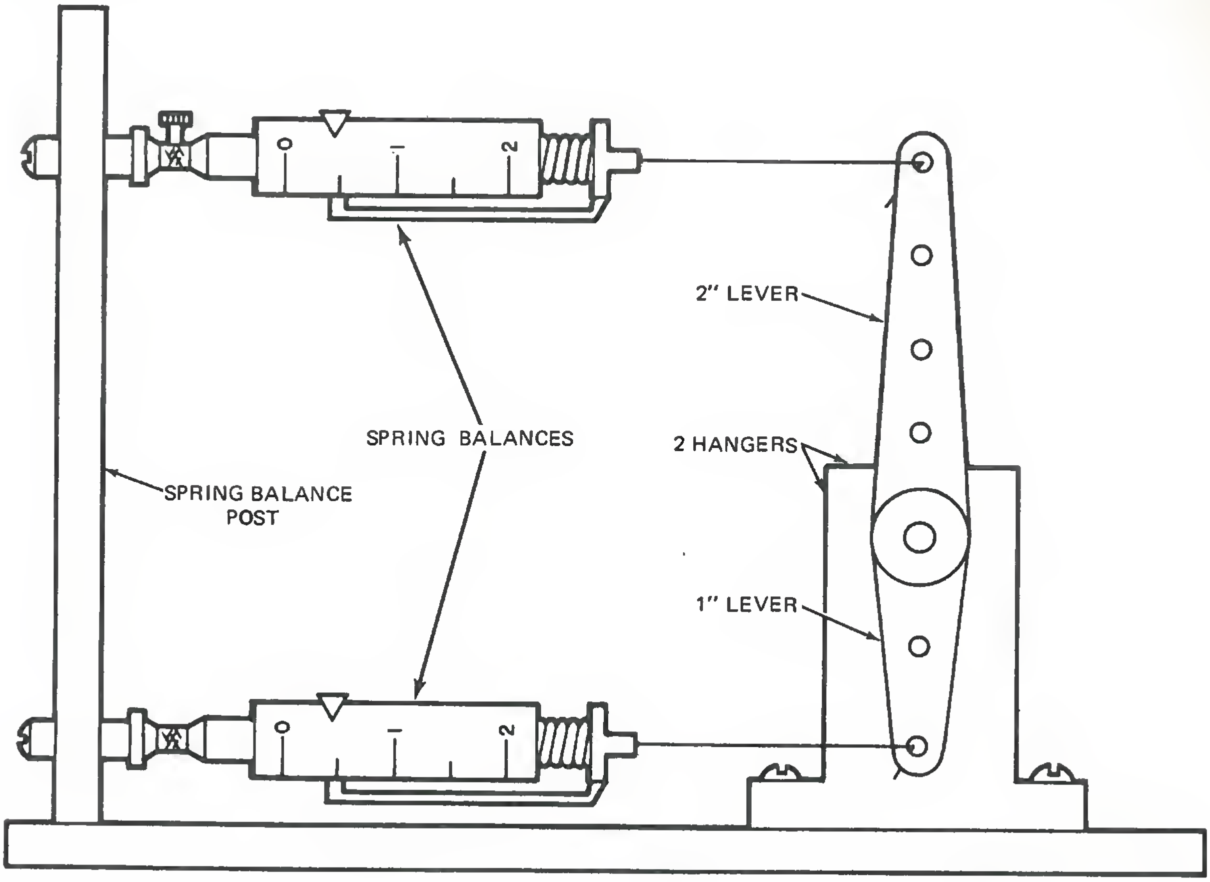


Fig. 1-4 The Initial Setup

9. Move the lower spring balance closer to the fulcrum and return the upper one to its original position. Then repeat steps 3 through 8.

10. Now move the longer lever arm to the other end of the shaft. Set up the lever so that you still have a class-one lever but the arms are at opposite ends of the shaft.

11. Repeat steps 3 through 9.

12. For each set of data, compute and record the moment of upper and lower lever arms (M_1 and M_2).

13. Compute the percent difference between M_1 and M_2 for each case.

ANALYSIS GUIDE. In your own words explain the action of the class-one lever. Compare the ratio of the lever-arm distances with the ratio of the forces for each case. Comment on the observed relative changes in force observed when the lever arm was moved away from the vertical.

Explain any large percent differences in the values of M_1 and M_2 for each case.

Were the results when the lever arms were separated the same as when they were together? Explain why you think this is reasonable.

F_1	S_1	M_1	F_2	S_2	M_2	% Diff. in M

Fig. 1-5 Data for Coplanar Arms

F_1	S_1	M_1	F_2	S_2	M_2	% Diff. in M

Fig. 1-6 Data for Noncoplanar Arms

PROBLEMS

1. A crowbar (figure 1-7) has a solid support at P; a load F_2 is to be lifted by a man's push at F_1. If force F_1 is 100 lbs, what load can be lifted for the dimensions shown?

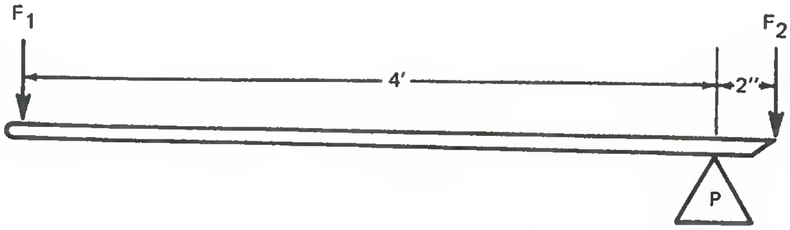


Fig. 1-7 Lever for Problem 1

2. A bar is placed under a two-ton stone with a fulcrum 16 inches from the point of application. How long will the rest of the lever be in order to raise the stone with a 150-lb pull?

3. It is desired to transmit motion by means of a class-one lever. The driver is attached 75 mm from the fulcrum and moves 4 cm. If the output motion desired is 1.5 cm, where would the output be attached?

4. A typewriter type bar is 8 in. long on one side of the fulcrum and has 1/2 in. on the other side. The typewriter linkage causes the 1/2-in. arm to move 90 degrees in 0.1 second. Compute the linear velocity of the type on the end of the 8-in. lever arm.

experiment 2 COMPOUND LEVERS

INTRODUCTION. In many practical cases it is desirable to connect two or more simple levers with a rigid linkage. In this experiment we shall examine a simple example of compound class-one levers.

DISCUSSION. One of the characteristics of a class-one lever is that it reverses the direction of the action. Figure 2-1 shows a simple class-one lever with the load and effort. Any downward motion of the effort will result in an upward motion of the load.

With a lever of this type, the moments-of-force acting on the effort and load side of the lever are

$$M_1 = F_1\ell_1 \quad \text{and} \quad M_2 = F_2\ell_2$$

respectively. When the lever is in equilibrium, the two moments are equal. That is,

$$M_1 = M_2 \quad \text{or} \quad F_1\ell_1 = F_2\ell_2$$

If we solve this relation for the ratio of F_2 to F_1, we have

$$\frac{F_2}{F_1} = \frac{\ell_1}{\ell_2}$$

This ratio is often called the *mechanical advantage* (MA) of the lever. In other words,

$$\boxed{MA = \frac{F_2}{F_1} = \frac{\ell_1}{\ell_2}} \qquad (2.1)$$

In some cases we want the load motion to be in the *same* direction as the effort motion. We can produce this result by using two class-one levers as shown in figure 2-2. Notice that the two levers are connected by a rigid link. If the effort force causes the left end of the lever system to move downward, then the load end is also moved downward.

The force (f) acting on the link can be determined using equation 2.1 and the left-hand lever when the connecting link is perpendicular to ℓ_2:

$$f = F_1\frac{\ell_1}{\ell_2}$$

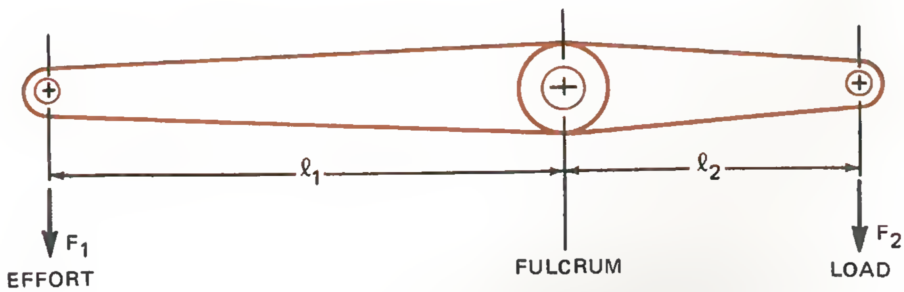


Fig. 2-1 A Class-One Lever

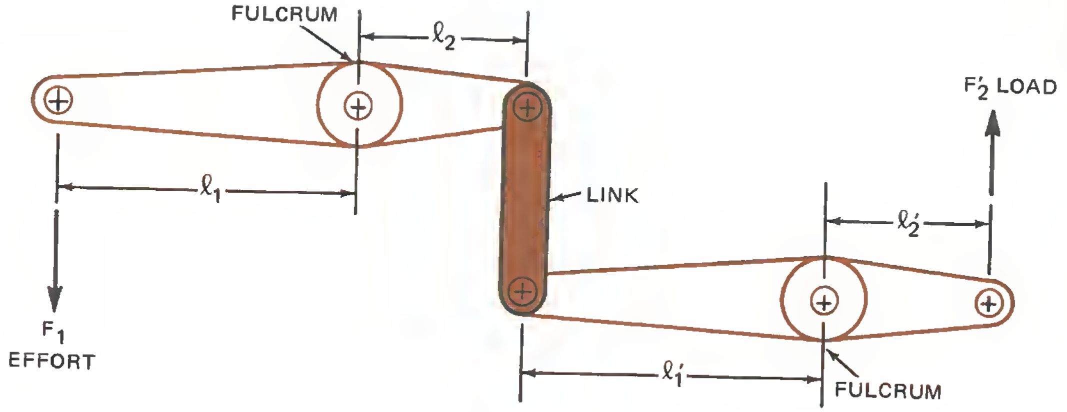


Fig. 2-2 A Compound Class-One Lever

Or, we could use the righthand lever and F'_2, in which case the link force is

$$f = F'_2 \frac{\ell'_2}{\ell'_1}$$

Equating these two equations for the link force gives us

$$F_1 \frac{\ell_1}{\ell_2} = F'_2 \frac{\ell'_2}{\ell'_1}$$

Then, solving for the ratio F'_2/F_1, we have

$$\frac{F'_2}{F_1} = \left(\frac{\ell_1}{\ell_2}\right)\left(\frac{\ell'_1}{\ell'_2}\right) \qquad (2.2)$$

Notice that ℓ_1/ℓ_2 is the mechanical advantage (MA_1) of the first lever, ℓ'_1/ℓ'_2 is the mechanical advantage (MA_2) of the second lever, and F'_2/F_1 is the mechanical advantage (MA_T) of the whole compound system. Consequently, we see that the mechanical advantage of a compound lever system is the product of the individual mechanical advantages:

$$\boxed{MA_T = (MA_1)(MA_2)} \qquad (2.3)$$

As you will recall, the velocity ratio (VR) of a class-one lever is given by

$$VR = \frac{S_1}{S_2} = \frac{\ell_1}{\ell_2} = \frac{F_1}{F_2} \qquad (2.4)$$

when friction and the weight of the arms are neglected. Comparing this relationship to equation 2.3, we see that

$$\boxed{VR_T = (VR_1)(VR_2)} \qquad (2.5)$$

will give us the velocity ratio of a compound lever system.

It is worth mentioning at this point that the work done at the load end of a lever is

$$W_2 = F_2S_2$$

where S_2 is measured in the direction of the force F_2. At the input the work done is

$$W_1 = F_1S_1$$

If we define lever efficiency as

$$eff = \frac{W_2}{W_1}$$

then we have

$$eff = \frac{F_2S_2}{F_1S_1}$$

Comparing this equation to 2.2 and 2.5, we see that we can express efficiency as

$$\boxed{\text{eff} = \frac{MA}{VR}} \qquad (2.6)$$

In most practical cases the efficiency of a lever system is so near unity that measuring it is quite difficult.

MATERIALS

1 Breadboard with legs and clamps

2 Bearing plates with spacers

2 Bearing holders with bearings

2 Spring balance posts and clamps

2 Shaft hangers, 1-1/2 in. with bearings

2 Shafts, 4" x 1/4"

1 Collar

2 Lever arms, 2 in. long with 1/4 in. bore hub

2 Lever arms, 1 in. long with 1/4 in. bore hub

*1 Straight link, 6 in. long

2 Spring balances

1 Dial caliper (0 - 4 in.)

*If the straight link is not available it can be fabricated as shown in appendix A.

PROCEDURE

1. Inspect each of your components to be sure they are undamaged.
2. Assemble the mechanism shown in figure 2-3. The link should be at the last hole in each lever arm and it should be parallel to the breadboard.
3. Adjust the spring balances so that they are *parallel* to the breadboard and the input force is about 4 oz. All of the lever arms *should be perpendicular* to the breadboard.
4. Record the input force F_1 and the output force F'_2.
5. Measure and record the effective length of each lever arm (ℓ_1, ℓ_2, ℓ'_1, and ℓ'_2).
6. Compute the force (f) acting in the link.
7. Compute the moment of force acting on each lever arm (M_1, M_2, M'_1, and M'_2).
8. Compute the mechanical advantage of each class-one lever (MA_1 and MA_2).
9. Record the product $(MA_1)(MA_2)$ of the two mechanical advantages.
10. Compute the total mechanical advantage using $MA_T = F'_2/F_1$.
11. Compute the percent difference between the results of steps 9 and 10.
12. Move the input spring balance down to the hole nearest the fulcrum and repeat steps through 11.

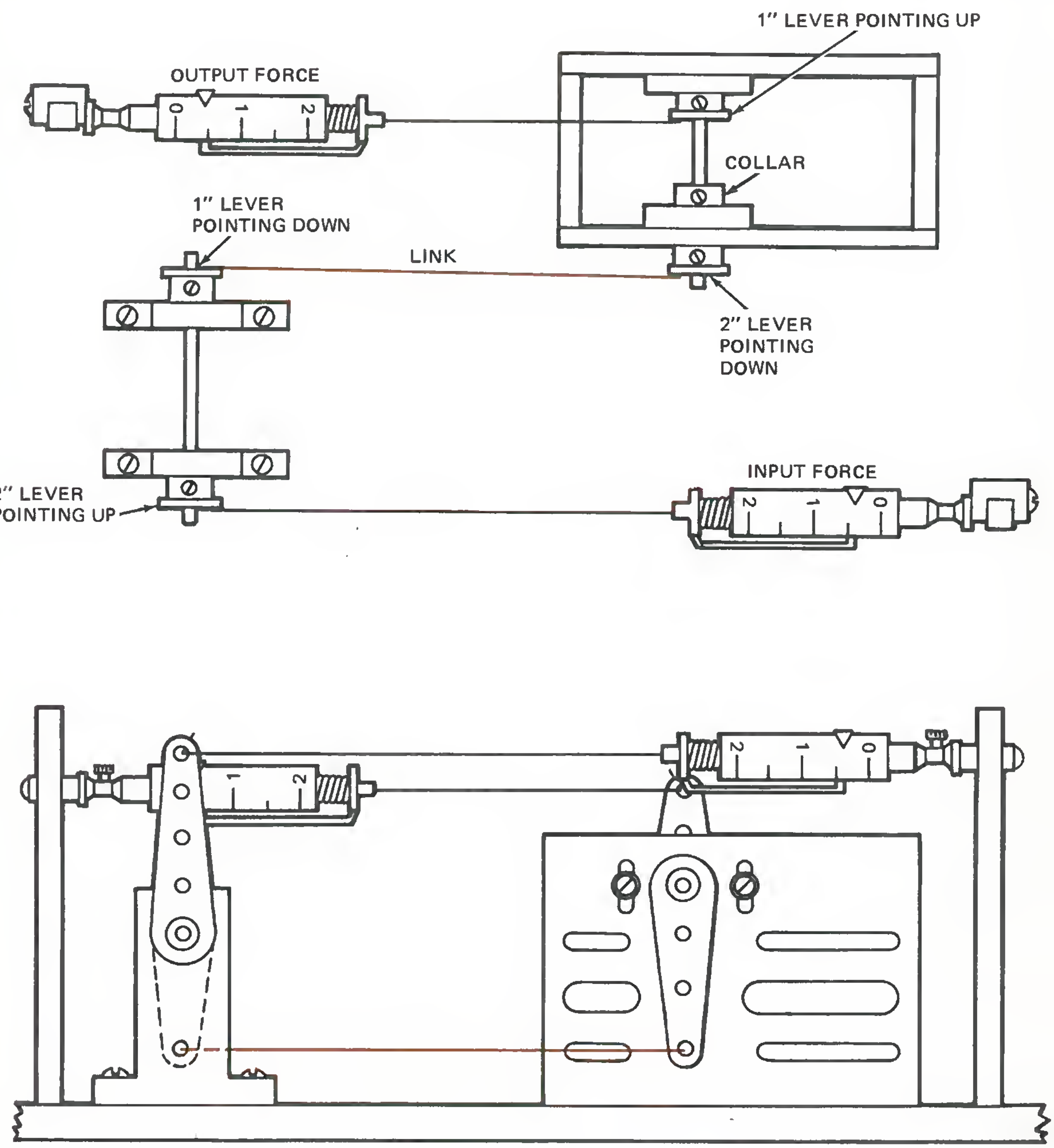


Fig. 2-3 The Experimental Mechanism

ANALYSIS GUIDE. In analyzing your results you should consider the following point

1. Did all of the values for moment-of-force agree in each trial?
2. Did your values of total mechanical advantage agree?
3. When would a compound lever be more suitable than a single class-one lever?
4. What do you think caused the errors in this experiment?

Qty.	First Trial	Second Trial
F_1		
F'_2		
ℓ_1		
ℓ_2		
ℓ'_1		
ℓ'_2		
f		
M_1		
M_2		
M'_1		
M'_2		
MA_1		
MA_2		
$(MA_1)(MA_2)$		
F'_2/F_1		
% Diff.		

Fig. 2-4 The Data Table

PROBLEMS

1. A compound lever of the type shown in figure 2-2,has an input force of 125 oz. The lever arm lengths are: $\ell_1 = 1.2$ ft, $\ell_2 = 4$ in., $\ell'_1 = 11$ in., and $\ell'_2 = 3.5$ in. What is the output force?
2. What is the link force in problem 1?
3. If the input in problem 1 moves a short distance with a velocity of 1.8 ft/sec, how fast does the link move?
4. How fast does the output move in problem 3?
5. What is the total mechanical advantage in problem 1?

experiment 3 CLASS–TWO LEVERS

INTRODUCTION. A second-class lever has the pivot point or fulcrum at one end—the force is applied at the other end. The resistance is somewhere between these two points. In this experiment we shall investigate the characteristics of this type of lever.

DISCUSSION. If the weight or resistance is placed between the fulcrum and the force as shown in figure 3-1, the result is known as a class-two lever.

A good example of a class-two lever is the wheelbarrow. The wheel will be the fulcrum, the load is represented by F_2, and the lifting force by F_1 in figure 3-1. If distance ℓ_1 equals 4 feet and distance ℓ_2 equals 1 foot, applying 50 lbs on the handles (force F_1) would give a lifting power of 200 lbs at $F_2 \cdot$ [50 lb x 4/1]. If the weight were placed farther back from the wheel, would it be easier or harder to lift?

Again referring to figure 3-1 and applying the principle of moments, we see that if the fulcrum is the center of moment, then

$$F_2 \times \ell_2 = F_1 \times \ell_1 \qquad (3.1)$$

The mechanical advantage, F_2/F_1, equals the ratio of the moment arms, ℓ_1/ℓ_2. It should be noted that the length, ℓ_1, is the entire length of the lever since the fulcrum is at the other end of the lever.

For a class-one lever, the direction of motion of the output force was opposite to the direction of the input force—when F_1 moved down, F_2 moved up. A look at figure 3-1 shows that for the class-two lever, the motions are in the same direction; when F_1 moves up (counterclockwise), F_2 moves up.

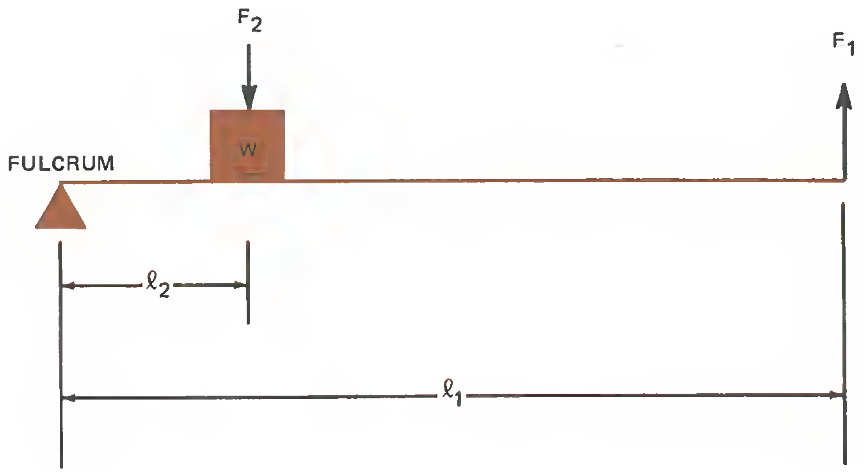


Fig. 3-1 A Class-Two Lever

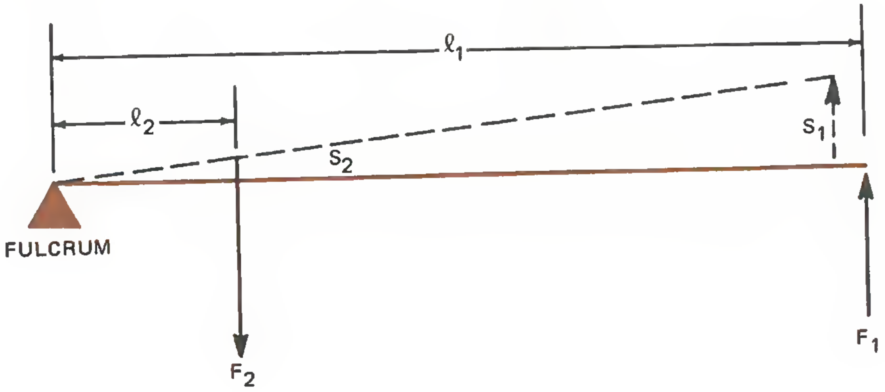


Fig. 3-2 Class-Two Lever – Relative Motions

What happens to the relative distances moved in the second class lever? Referring to figure 3-2, it can be seen that the force of the output, F_2, is greater than the input force, F_1, by a factor equal to the lever-arm ratio, ℓ_1/ℓ_2; in this case, 4/1 or 4. Graphically, it can be seen that force F_1 will move more than force F_2 when the lever moves from its initial position to the dashed-line position in figure 3-2. Ignoring friction, the work (force x distance) input will equal the work output. That is,

$$F_2 \times S_2 = F_1 \times S_1$$

Since F_1 is 1/4 F_2, then S_1 must be equal to 4 times S_2 if this equality is to be maintained. Again note that the *direction* of the motion in the class-two lever is the same for F_2 and for F_1.

A basic law of mechanics states that the sum of forces acting on a body in any plane or direction equals zero if the body is in equilibrium. Another way of saying this is that the forces acting downward must equal the forces acting upward, or, the forces acting to the right must equal the forces acting to the left. Referring to figure 3-2, two forces are shown (F_2 and F_1). F_2 is four times as large as F_1 and is acting downward. For equilibrium, the forces downward must be balanced by upward forces. Assume that F_2 is 200 lbs and that F_1 is 50 lbs. Where is the other 150 lbs that must be acting upward along with F_1? If you answered that the fulcrum point must be supporting 150 lbs, you are correct. This force—not one of the applied forces—is called a *reaction* and would have to be mechanically capable of holding that much weight.

What if F_1 were to be applied at an angle of, say, 45 degrees to the horizontal lever? Would the full 50 lbs of F_1 be felt in the vertical direction? Would the full 50 lbs of F_1 be felt in the horizontal direction? With the application of a force at an angle, the amount of force felt in the vertical direction equals the applied force times the sine of the angle of application. Figure 3-3 shows this relationship. In other words, applying 50 lbs at 45°, as shown, is equivalent to applying simultaneously 35.35 lbs horizontally to the right and 35.35 lbs vertically because sin 45° = cos 45° = .707.

When applied to the second-class lever, it can be seen in figure 3-3, that applying a force at an angle is the same as applying a lesser force ($F_1 \times \sin \Theta$) at the end of the lever arm.

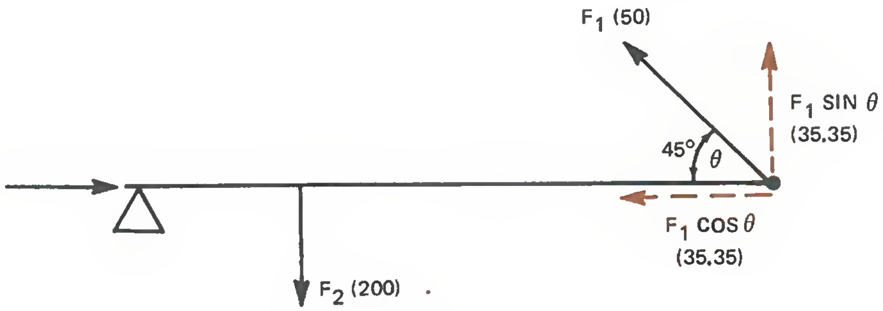


Fig. 3-3 Force Applied at an Angle

What happens to the reactive force at the fulcrum when F_1 is applied at an angle? Remember that when force F_1 was applied at right angles, there was a reactive force that acted upward. There will still be an upward reactive force, but now there must be a force that will balance the *horizontal component* ($F_1 \times \cos\ \theta$) caused by F_1 operating at an angle (if there were not, the lever would be pushed to the left in figure 3-3).

In figure 3-3, what weight, F_2, can be lifted (balanced) by a pull, F_1, of 50 lbs at an angle of 45°, if the lever-arm ratio is 4:1? F_1 at 45° is equivalent to a vertical pull of 35.35 lbs. Multiplying this 35.35 lbs by the lever-arm ratio gives a weight of 141.4 lbs. Another technique is to use the "effective lever-arm distance." Be sure that you can check the accuracy of the above using this technique. *[Note: The effective lever-arm or moment-arm of the 50-lb pull is the perpendicular distance of the 50 lbs line of action to the fulcrum point.]*

Scales often use combinations of lever arrangements similar to the compound lever system shown in figure 3-4.

With the dimensions shown in the figure 3-4, what must the value of F_1 be to balance the system? First, what is the value of f for the lower lever? The lever-arm ratio is 1/6 (*remember that the fulcrum is 6 ft from f, not 5 ft*). So, f will equal 1/6 $\times$ 300 lbs or 50 lbs. This will be the load on the upper lever. The lever-arm ratio of the upper lever is 1/7. Therefore, F_1 will be 1/7 $\times$ 50 lbs or 7.14 lbs. Another solution is to first compute

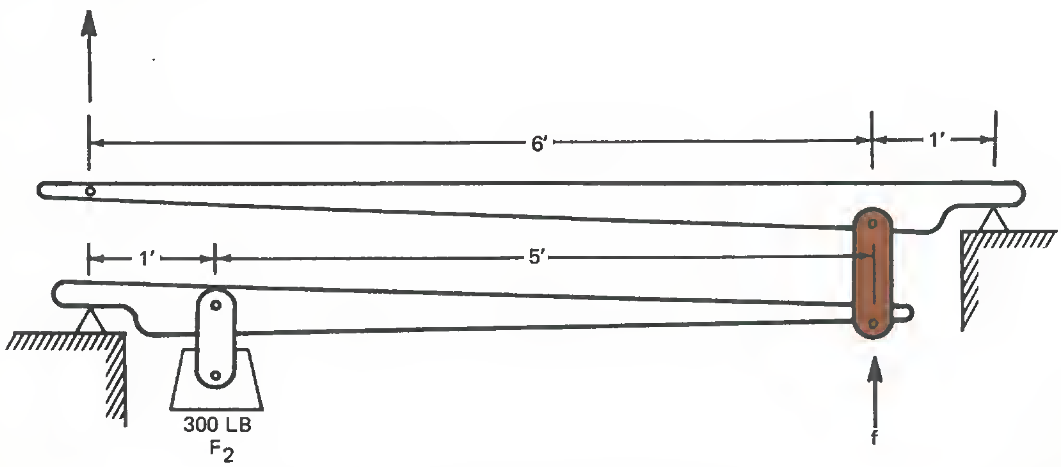


Fig. 3-4 Class-Two Compound Levers

the overall lever-arm ratio. The lower lever has a 1/6 ratio; then the upper lever ratio is 1/7. This gives an overall lever ratio of 1/42 (1/6 X 1/7). Multiply the 1/42 by 300 lbs and, again, F_1 = 7.14 lbs.

Now, let's look at the relative motions in this compound lever system. Assume that you wish the 300-lb weight to move 0.1 in. How far must F_1 move to accomplish this motion? Moving 300 lbs 0.1 in. upward accomplishes 30 in.-lbs of work; therefore, f, which is 50 lbs, must accomplish at least this much work [50 X S = 30 in.-lb; S = 0.6 in.]. You may note that this is the same as the lever arm ratio, but inverse to the force ratio; that is, 6 to 1 rather than 1 to 6. F_1 must, then, move 7 times S, or 4.2 in. Since the overall system ratio was 1/42 for force, then the motion ratio is the inverse of 1/42, or 42 to 1.

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 2 Bearing holders with bearings
- 2 Shaft hangers, 1-1/2 in. with bearings
- 2 Spring balance posts with clamps
- 2 Spring balances
- 2 Shafts, 4" x 1/4"
- 2 Lever arms, 2 in. long with 1/4 in. bore hub
- 2 Lever arms, 1 in. long with hubs
- 2 Collars
- 1 Dial caliper (0 - 4 in.)
- *1 Straight link, 6 in. long

*For link construction details see appendix A.

PROCEDURE

1. Inspect each of your components to be sure they are undamaged.
2. Mount the bearing plates and shaft hangers as shown in figure 3-5.
3. Mount a 1-in. lever and a 2-in. lever, both pointing downward from a shaft through the bearing plates.
4. Similarly mount two levers on a shaft through the hangers.
5. Install the 6-in. link between the small lever on the hanger shaft and the long lever on the bearing plate shaft. The bearing plate levers should both point downward and both hanger levers should point upward.
6. Adjust the bearing plate location and shaft height *so that all levers are vertical and the link is horizontal.*
7. Install the spring balances so that one is between the small bearing plate lever and a post. The other is between the long hanger lever and a post.
8. Adjust the spring balances so that they are in the end holes of the levers.
9. Set the input force for about 4 oz. *being sure that the spring balances are horizontal.*
10. Record both forces, F_1 and F_2.

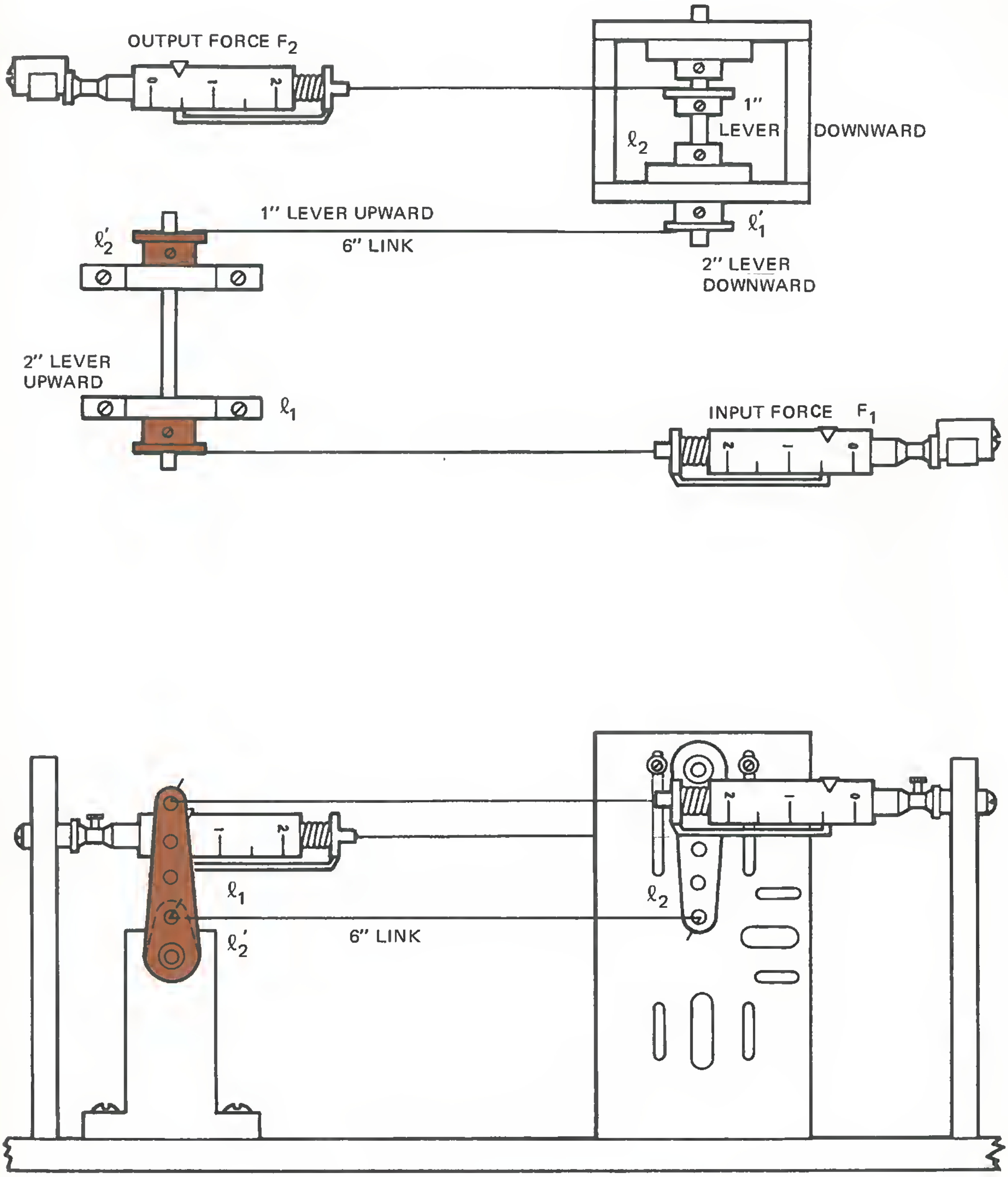


Fig. 3-5 The Experimental Setup

11. Measure and record the length of each lever, ℓ_1, ℓ'_2, ℓ'_1 and ℓ_2. (*The identity of each length is shown in figure 3-5.*)

12. Compute the force (f) acting in the link using F_1, ℓ_1, and ℓ'_2.

13. Compute the force (f′) in the link using F_2, ℓ_2 and ℓ'_1.

14. Compute the percent difference between f and f′.

15. Move the input spring balance down to the next hole in the input lever and repeat steps 9 through 14. Record data.

16. Again move the input spring balance down to the next hole in the lever and repeat steps 9 through 14. Record data.

Qty.	F_1	F_2	ℓ_1	ℓ'_2	ℓ'_1	ℓ_2	f	f′	% Diff.
First Trial									
Second Trial									
Third Trial									

Fig. 3-6 The Data Table

ANALYSIS GUIDE. Draw a simplified sketch of each of the class-two levers illustrated during this experiment. Compute the lever-arm ratio from the measured distances and compare this ratio with the force ratio. Explain the difference in the force readings obtained from the three different trials. In your own words discuss other aspects of class-two levers.

PROBLEMS

1. Assume that a sign hinged to a wall has a length of 12 feet and an effective weight of 100 pounds when 3 feet from the wall. What upward force, F_1, is necessary to support the sign? (See figure 3-7.)

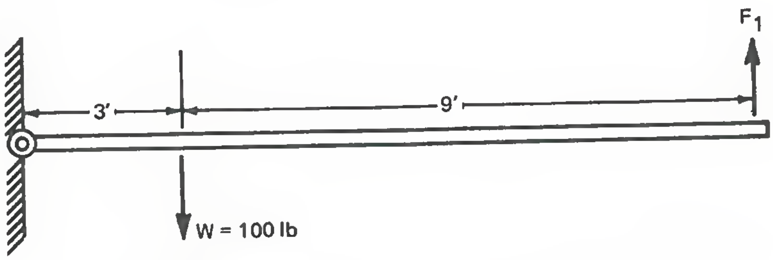


Fig. 3-7 Diagram of Problem 1

2. In the same problem, assume that there is no place to support F_1 in a vertical position, and that a cable is attached to that end of the sign and brought over to the building making an angle of 30° with the horizontal. How much force will the cable be required to support? [*Note: You already know the vertical component from problem one.*]

3. A safety valve on a boiler (figure 3-8) has a 2-inch diameter and a steam pressure of 200 lb/in^2. If the lever arm is 15 inches long and the valve is 3 inches from the pivot point, what is the value of W that is required?

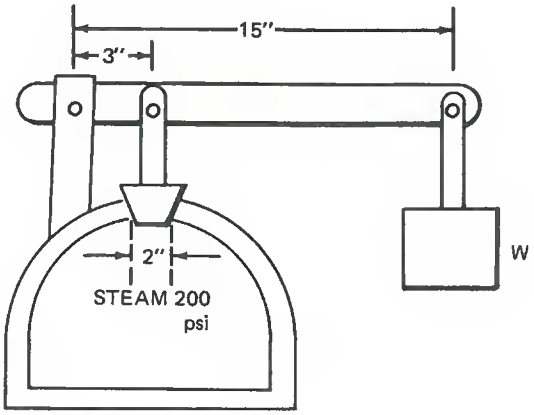


Fig. 3-8 Valve in Problem 3

4. In the discussion, a problem was solved which used the "equivalent vertical pull" of a 50-lb force. Draw a sketch showing that 50 lb times the "effective lever arm distance" gives identical results.

5. In problem 3, if the safety valve moves 1/32 in., how far does the weight move?

6. Using a class-two lever, where would you place the output on a 6-inch lever to achieve a motion of 0.1 in. if the input motion is 0.4 in? This may be necessary, for example, if the error in the input motion is 0.4 in. and your allowable error in a work device is 0.1 in. Draw a sketch of this lever arrangement.

7. Discuss the similarities and the differences between class-one and class-two levers. Give three practical examples of each type.

experiment 4 CLASS-THREE LEVERS

INTRODUCTION. A class-three lever is very similar to a class-two lever in that both the resistance and the effort are on the same side of the fulcrum. However, the effort or input force of the class-three lever is closer to the fulcrum than is the load or output force. In this experiment we will investigate the characteristics of this type lever. Also, combinations of load positions will be examined.

DISCUSSION. There are times when you will want to speed up the motion of the output force even though you will have to use a large amount of input force to accomplish this. Levers that help do this are called class-three levers. As shown in figure 4-1, the fulcrum of a class-three lever is at one end, and the weight or output force to be overcome is at the other end, with the effort or input force applied at some point between.

It is easy to see that, while the input force, F_1, moves the short distance, S_1, the output load, F_2, moves the greater distance, S_2. Since the whole lever moves during the same time interval, then the speed of F_2 must be greater than F_1 because F_2 covers a greater distance in the same period of time.

Your arm, as illustrated in figure 4-2, is a class-three lever. It is this lever action that makes it possible for you to flex your arm. Your elbow is the fulcrum. Your biceps muscle applies the input force about one inch from your elbow. The output force to be overcome is in your hand located some 13 inches from your elbow.

If you contract your biceps muscle one inch, your hand swings through a thirteen-inch arc. This illustrates the major use of the class-three lever—*to gain speed or displacement.*

Referring back to figure 4-1, assume that F_2 is 100 pounds. How much force F_1 will be required to lift this weight? The distance from the fulcrum to F_2 is 1 ft + 3 ft or 4 ft. So, a moment of 100 lb X 4 ft or 400 lb-ft is created by this weight. For equilibrium, F_1 must overcome this clockwise moment. F_1 operates a distance of 1 ft from the fulcrum,

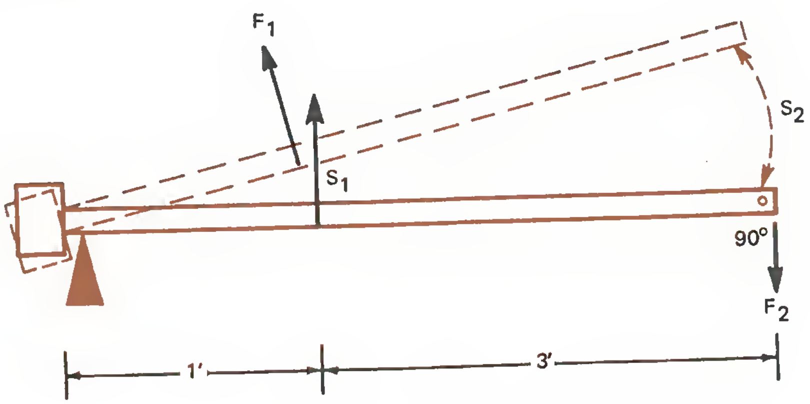


Fig. 4-1 Class-Three Lever

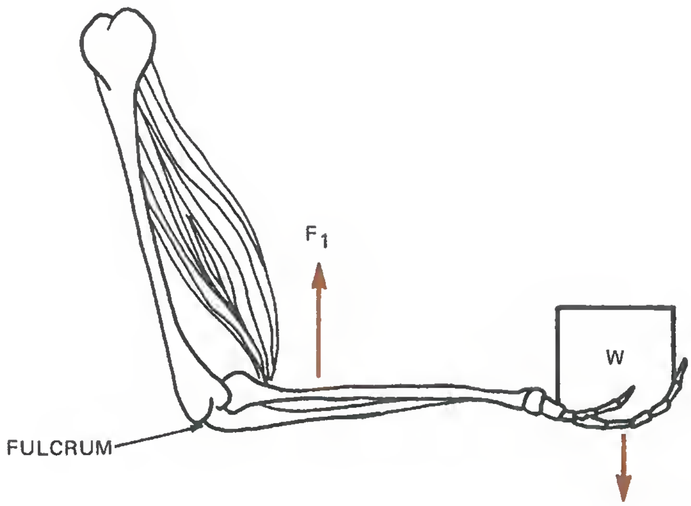


Fig. 4-2 The Forearm – A Class-Three Lever

so

$$F_1 \times 1 \text{ ft} = 100 \text{ lb} \times 4 \text{ ft}$$

and

$$F_1 = \frac{400 \text{ lb-ft}}{1 \text{ ft}} = 400 \text{ lb}$$

It can be seen that MORE force input is required than will be lifted. However, if F_1 is moved one inch, then F_2 will move 4 inches.

What is the mechanical advantage of a class-three lever? Remember that *mechanical advantage* is the ratio of the output force to the input force. In the illustration just given, the output force was 100 lb and the input force was 400 lb, giving a mechanical advantage of 100/400 or 1/4. Class-three levers will have a fractional mechanical advantage which means that more force must be applied than is to be moved or lifted.

Again, referring to figure 4-1, if F_2 is 100 pounds acting in a downward direction and F_1 is 400 pounds acting upward, what force must be present at the fulcrum? Since the upward forces must equal the downward forces, then the fulcrum must have a reaction force of 300 lbs acting downward (*the clamp arrangement shown in the figure is necessary to keep the lever on the fulcrum!*).

It is important to notice that the weight of the lever has not been considered in any of our computations. In some applications, the weight of the lever is so small that it can be ignored. However, in other applications, the weight of the lever may be large enough to be an important consideration. When the weight must be considered in the computation, an additional moment is determined by the product of the weight of the lever arm, and the lever arm distance to its center of gravity from the fulcrum. You will remember that an object's *center of gravity* is the point where the weight may be considered to be concentrated.

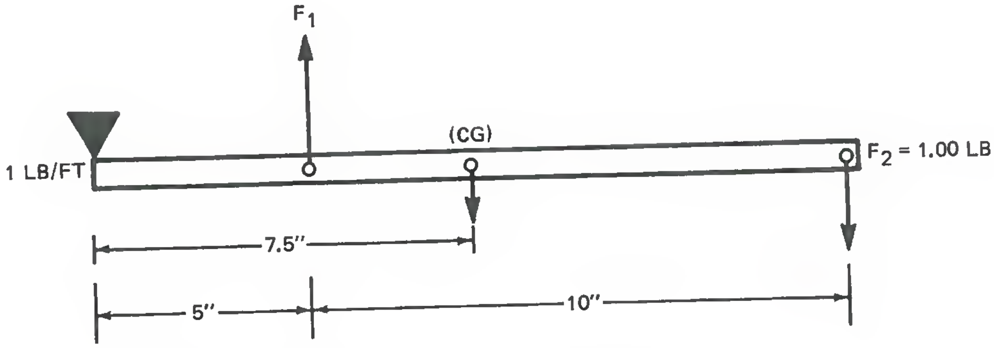


Fig. 4-3 Class-Three Lever - Lever Weight Consideration

For example, the 15-in. lever shown in figure 4-3 weighs 1 lb per foot uniformly. With the dimensions as shown, what force F_1 is necessary to place the lever and the one-pound weight in equilibrium?

The weight of the bar can be considered to be at the center of gravity (CG) which will be located 7.5 in. from the fulcrum. Since the bar weight is 1 lb/foot or 1 lb/12 inches, then 15 inches will equal

$$\frac{1\text{ lb}}{12\text{ in.}} \times 15\text{ in.} = 1.25\text{ lb}$$

It can be seen that there are two forces tending to rotate the lever clockwise: the weight of the lever at the CG point and the 1-pound weight. The pull, F_1, must equal these two moments:

$$F_1 \times 5\text{ in.} = (1.25\text{ lb} \times 7.5\text{ in.}) + (1.00\text{ lb} \times 15\text{ in.})$$

$$F_1 = \frac{9.38\text{ lb-in.} + 15\text{ lb-in.}}{5\text{ in.}}$$

$$= \frac{24.38\text{ lb-in.}}{5\text{ in.}} = 4.88\text{ lb}$$

In this case, the weight of the lever is significant and must be included in the computations. The lever arm ratio is 1:3, which would indicate a pull, F_1. pf 3 lb if the lever weight is neglected. Be sure to notice that the force, F_1, to balance the system is not based just on the opposing weights, but on each weight times its respective lever arm.

As an exercise for you, compute the reactive force that must be present at the fulcrum in the example given in figure 4-3. See if your answer is 2.63 lb downward.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Bearing holders with bearings
2 Shaft hangers, 1-1/2 in. with bearings
2 Spring balance posts with clamps
2 Spring balances
2 Shafts, 4" x 1/4"
2 Lever arms, 2-in. long with 1/4-in. bore hubs
2 Lever arms, 1-in. long with 1/4-in. bore hubs
2 Collars
1 Dial caliper (0 - 4 in.)
*1 Straight link, 6-in. long

*For link construction details see appendix A.

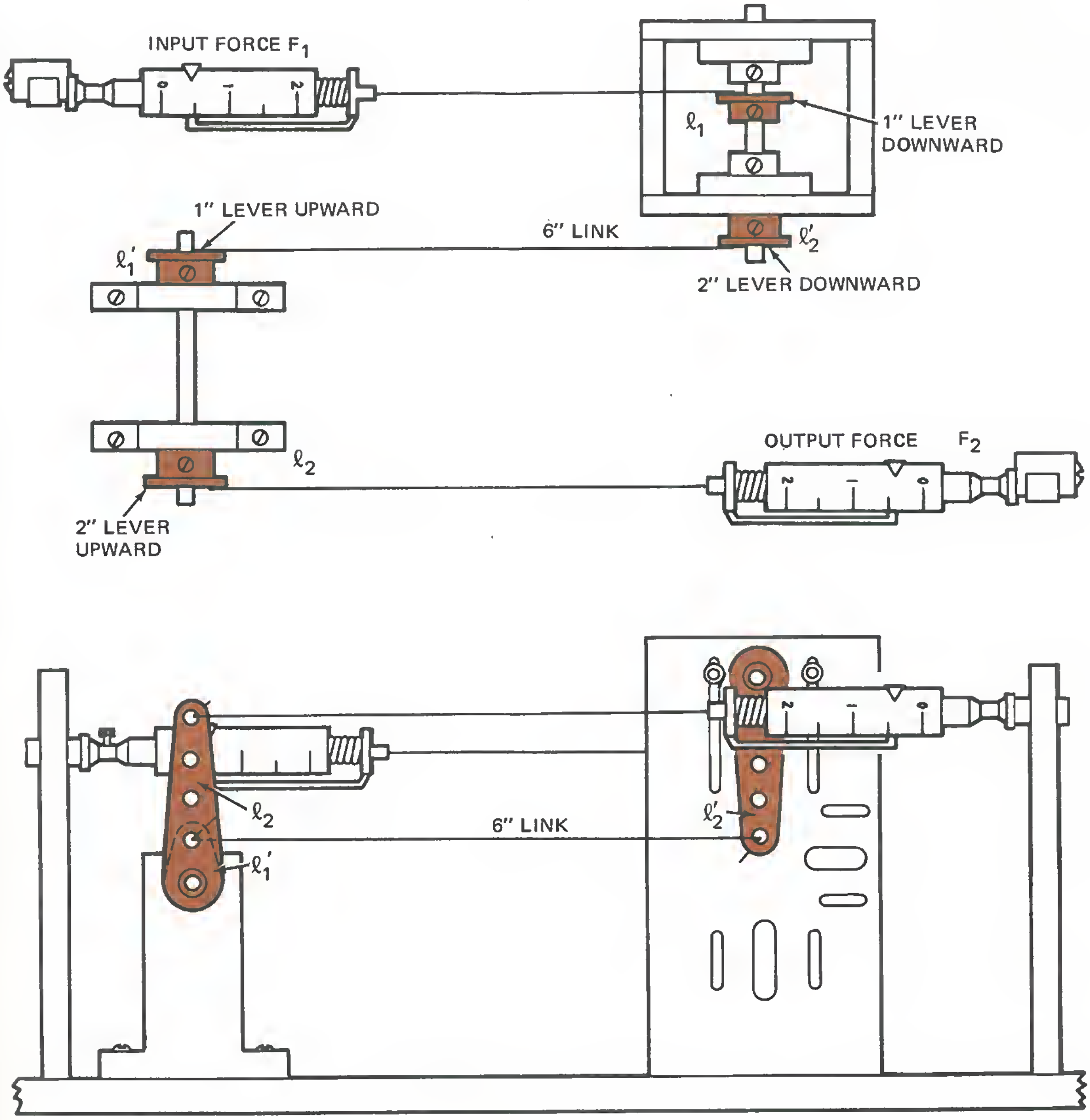


Fig. 4-4 The Experimental Setup

PROCEDURE

1. Inspect each of your components to be sure they are undamaged.
2. Mount the bearing plates and shaft hangers as shown in figure 4-4.
3. Mount a 1-in. lever and a 2-in. lever both pointing downward from a shaft through the bearing plates.
4. Similarly mount two levers on a shaft through the hangers.

5. Install the 6-inch link between the small lever on the hanger shaft and the long lever on the bearing plate shaft. The bearing plate levers should both point downward and both hanger levers should point upward.
6. Adjust the bearing plate location and shaft height so that *all levers are vertical and the link is horizontal.*
7. Install the spring balances so that one is between the small bearing plate lever and a post; the other is between the long hanger lever and a post.
8. Adjust the spring balances so that they are in the end holes of the levers.
9. Set the input force for about 20 oz being sure that the *spring balances are horizontal.*
10. Record both forces F_1 and F_2.
11. Measure and record the length of each lever, ℓ_1, ℓ'_2, ℓ'_1 and ℓ_2. (*The identity of each length is shown in figure 4-4.*)
12. Compute the force (f) acting in the link.
13. Compute the moment of force acting on each lever, M_1, M'_2, M'_1, and M_2.
14. Compute the percent difference between M_1 and M'_2 and between M'_1 and M_2.
15. Compute the mechanical advantage of each class-three lever, MA_1 and MA_2.
16. Compute the total mechanical advantage (MA_T) using the results of step 15.
17. Compute the total mechanical advantage (MA'_T) using only F_1 and F_2.
18. Compute the percent difference between your two values for total mechanical advantage.

F_1	F_2	ℓ_1	ℓ'_2	ℓ'_1	ℓ_2

f	M_1	M'_2	% Diff.	M'_1	M_2	% Diff.

MA_1	MA_2	MA_T	MA'_T	% Diff.

Fig. 4-5 The Data Tables

ANALYSIS GUIDE. Using the measured moment-arm lengths, compute the moment-arm ratio for each set of data. Compare this ratio with the ratio of the forces. In your own words, summarize the characteristics of class-three levers. Give five examples of how lever class may be applied in a practical situation. Distinguish between class-two and class-three levers. Add any comments you believe appropriate.

PROBLEMS

1. Is the lever shown in figure 4-6 a class-three lever, or is it a class-two lever? Explain in detail your answer.

2. In figure 4-6 how much pull must be exerted to overcome the load of 6 oz?

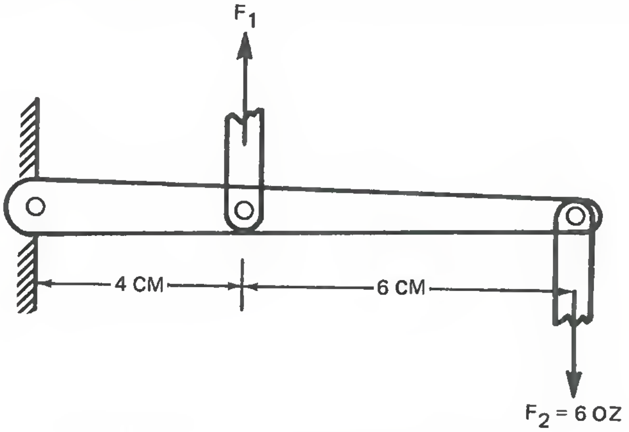


Fig. 4-6 Lever for Problems 1, 2 and 3

3. Assume that the lever in Problem 2 moves upward (counterclockwise two degrees). Compute the true distances (arc length) that the attaching points of F_1 and of F2 travel. Compare the ratio of these distances with the lever-arm ratio.

4. A crimping tool is pinched together with a pair of forces of 10 pounds each as shown in figure 4-7. What is the force exerted on the connector? What is the force in the link X between the two levers of the crimping tool? *Hint: If one side of the tool were placed on the bench, the bench would push with 10 lbs.*

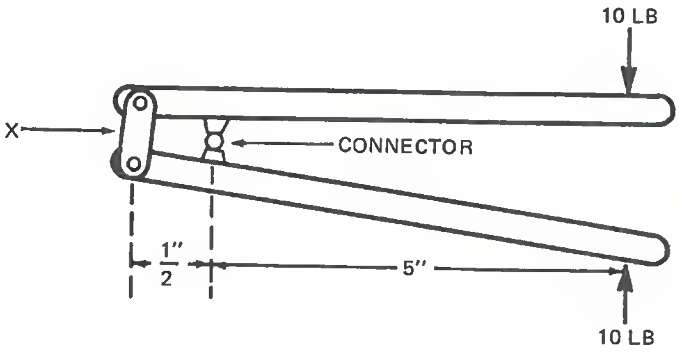


Fig. 4-7 A Crimping Tool

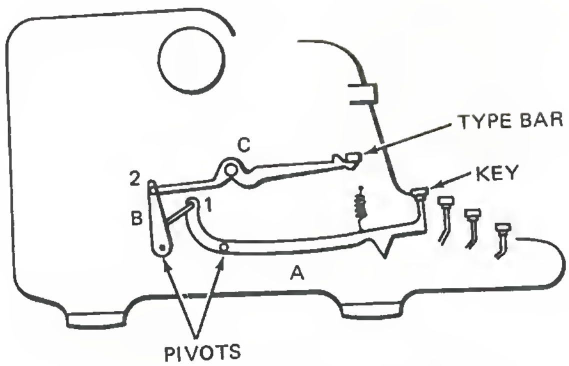


Fig. 4-8 Typewriter Mechanism

5. Figure 4-8 is a sketch of a typewriter mechanism. Identify the class of the levers that are marked A, B, and C. Discuss the relative motion (speed) of point 1 and point 2.

6. Discuss the relative forces between that applied to the key and that delivered by the type bar, based on the typewriter sketch. Which of the three levers (A, B, or C) is used primarily to increase speed?

7. An eight-inch long lever is pivoted at one end and has its input force one inch from the pivot. Three loads of 2, 3, and 4 ounces are 6, 7, and 8 inches, respectively. from the pivot. What input force is necessary to establish equilibrium?

8. In figure 4-1, let the 1-foot distance be X and the 3-foot distance be Y. Assume that the lever rotates through Θ degrees. In terms of X, Y, and Θ, express the vertical distance and the arc distance that F_1 and F_2 travel.

experiment 5 ROCKER ARMS AND BELL CRANKS

INTRODUCTION. This experiment summarizes the characteristics of the three basic types of levers and investigates the characteristics of two common ways of linking basic mechanical parts: the rocker arm and the bell crank.

DISCUSSION. Levers can be used: to change the direction of the force being applied, or to change the speed of a force applied.

Class-one levers have the applied or input force and the output force on opposite sides of the pivot point or fulcrum. These two forces move in opposite directions. The relative speed and relative magnitudes of the two forces depend upon the moment-arm lengths.

Class-two levers have the input force and the output force on the same side of the fulcrum, but the input force is farther from the fulcrum than is the output. Both forces move in the same direction. The output force is greater than the input, and the linear speed of the output is less than the input.

Class-three levers have the two forces on the same side of the fulcrum but the input force is applied between the output and the fulcrum. Both forces move in the same direction. The linear speed of the output is greater than the input, but the magnitude of the output force is less than the applied force.

The method used to analyze all levers is the relationship that clockwise moments must equal counterclockwise moments for equilibrium to exist.

This relationship is expressed mathematically as:

$$\boxed{F_1 \times \ell_1 = F_2 \times \ell_2} \qquad (5.1)$$

where F_1 and F_2 are the input and output forces, ℓ_1 is the moment arm of the input force, and ℓ_2 is the moment arm of the output force. This relationship is frequently used in the form

$$\boxed{F_2/F_1 = \ell_1/\ell_2} \qquad (5.2)$$

which shows that the ratio of force out to force in is equal to the moment-arm (or lever-arm) ratio. This ratio is known as the lever's *mechanical advantage.*

For equilibrium to exist, forces in any plane or direction (e.g. – horizontal, vertical, etc.) must be equal. This is, forces pushing downward must be counterbalanced by forces pushing upward. Forces pushing to the right must be balanced by forces pushing to the left.

In machines it is often necessary to transmit *limited* rotary or linear motion from one place to another. This is accomplished by one or more of the following basic machine parts:

1. Rocker Arms
2. Bell Cranks
3. Levers
4. Rods or Shafts

Typical of the use of the first of these is in an automobile engine where a rocker arm moves a valve assembly, thus, opening it. This basic mechanism is shown in figure 5-1. When the push rod moves up, the other end of the rocker arm must move down, causing the valve to move down. This action is diagramed

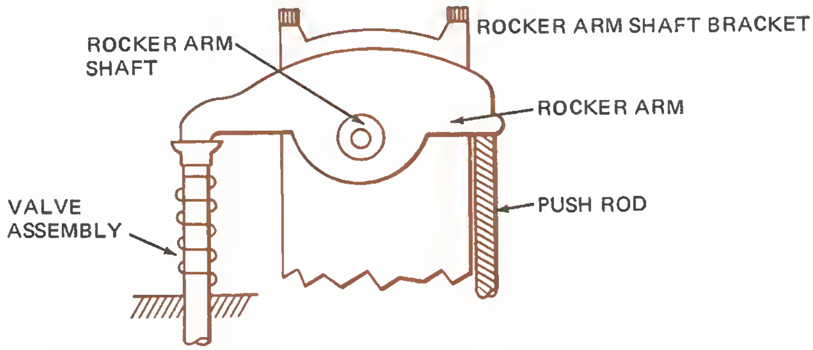


Fig. 5-1 Automobile Rocker Arm Assembly

in figure 5-2 showing that a rocker arm is a class-one lever.

Referring to figure 5-2, what is the force applied to the valve assembly if the push rod force is 30 lb? If the push rod moves upward 3/8 in., how far downward does the valve assembly move? What do you need to know to answer these two questions? If you stated lever-arms or moment-arms, you are correct. If this point did not occur to you, refer back to equations 5.1 and 5.2.

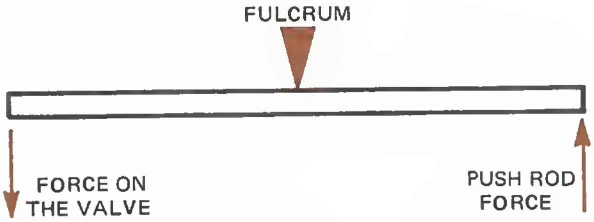


Fig. 5-2 Rocker Arm Force Diagram

Assume that the push rod force acts 1/2 in. from the pivot and the valve assembly is 5/8 in. from the pivot. *Note: These distances are the perpendicular distances from the pivot to the line of action of the two forces.* Now, we can compute the output force by using equation 5.1:

$$F_2 = F_1 \times \ell_1/\ell_2 = 30 \text{ lb} \times 4/5 = 24 \text{ lbs}$$

which states that when a 30-lb force is applied 1/2 in. from the pivot in an upward direction, a 24-lb force is felt 5/8 in. from the pivot in a downward direction.

Remember that the distance ratio varies inversely as the moment arm ratio: so, to solve for the relative motions of the two forces,

$$S_2 = S_1 \times \ell_2/\ell_1 = 3/8 \times 5/4 = 15/32 \text{ in.}$$

The push rod moves upward 3/8 in. and the valve moves downward 1.25 times that distance, or 15/32 in.

An easy check of motions and forces of a class-one lever is to remember the action of a see-saw (teeter-totter) when a very heavy person is on one end and a light person is on the other. The heavy person must sit quite close to the fulcrum and will not move up or down much — the light person will be much farther away from the pivot point and will move a large distance up and down.

What is the result of applying the forces at an angle other than 90° to the moment rocker arm? This application is shown in figure 5-3. There are two ways of analyzing the force relationships. The first is illustrated in the righthand sketch in figure 5-3. Here, the lines of action of the input and output forces

Fig. 5-3 Rocker Arm, Non-Perpendicular Forces

are shown by dashed lines along the force vectors. The moment or *torque* produced by F_2 is equal to that force multiplied by the *perpendicular* distance from the pivot, O, which is shown as ℓ_2'. You can see that this distance is less than the rocker arm length ℓ_2. This torque must be equal to that produced by F_1 so: $F_2 \times \ell_2' = F_1 \times \ell_1'$. Practically, however, it is often difficult to measure the true perpendicular distance from the pivot point to the line the force is acting upon. But, we usually can measure the angle with the lever. This is shown in figure 5-4.

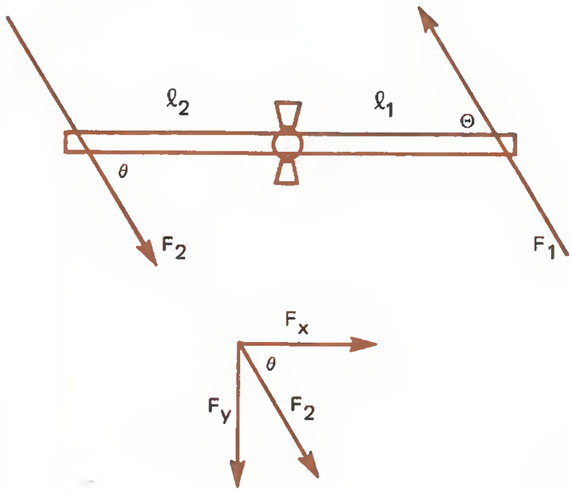


Fig. 5-4 Angular Force Analysis

We can also measure the distance ℓ_1 and ℓ_2 quite easily in most practical situations. If distance ℓ_2 is used, what moment or torque is being produced? Only that force which is perpendicular to that moment arm is considered. To say it another way, force F_2 can be broken into two components: one acting horizontally, F_x, and one acting vertically, F_y. If two true forces, F_x and F_y, were applied they would equal F_2. F_x generates NO torque because it is applied in line with the fulcrum which makes the moment arm zero. F_y acts perpendicularly and generates a torque equal to $F_y \times \ell_2$.

How do you compute these components? If the angle with the horizontal of F_2 is θ, then $F_y/F_2 = \sin\theta$. Thus, $F_y = F_2 \sin\theta$. The moment generated by F_y must be counterbalanced by the moment generated by the vertical component of F_1; let's call it, $F_y' \times \ell$. From this, our basic equilibrium equation becomes:

$$\boxed{\begin{array}{c} F_y \times \ell_2 = F_y' \times \ell_1 \\ \text{OR} \quad \ell_2 \times F_2 \sin\theta = \ell_1 \times F_1 \sin\Theta \end{array}} \quad (5.3)$$

It should be noted that the parameters of equation 5.3 are usually quite easily measured: the force, the angles with the lever, and (*from trigonometry tables*) the cosines of the angles.

Another device used primarily to transmit motion from a link traveling in one direction to another link which is to be moved in a different direction is the *bell crank.* The name of this device came from the linkage used to operate our grandparent's doorbells. The bell crank is mounted on a fixed pivot and the two links are connected at two points in different directions from the pivot. By properly locating the connecting points, the output links can be made to move in any desired direction. One type of bell crank is shown in figure 5-5.

In this figure the two arms are perpendicular to each other and the connecting links are perpendicular to the arms. Since the forces are applied on a line perpendicular to the pivot point, then the clockwise moment, $F_1 \times \ell_1$, must be equalled by the counterclockwise moment, $F_2 \times \ell_2$. Thus,

$$\ell_1 \times F_1 = \ell_2 \times F_2$$

or

$$F_1/F_2 = \ell_2/\ell_1$$

Does this look familiar?

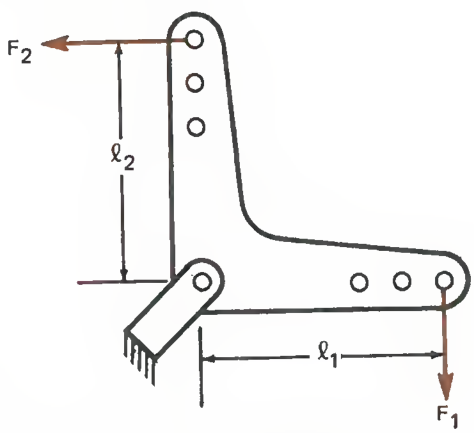


Fig. 5-5 Simple Bell Crank

Now, let's analyze the movements of these two forces – sometimes these forces are actually tensions in taut cables. In figure 5-6, the solid lines indicate the initial position of the bell crank and the dotted lines indicate the bell crank position after F_1 has been applied to rotate the crank through Θ degrees. The distances S_1 and S_2 are the linear distances rather than the arc distances.

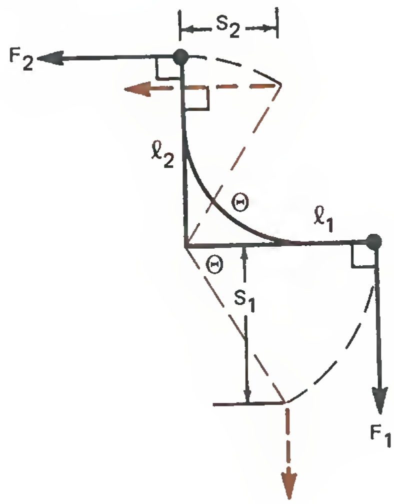


Fig. 5-6 Bell Crank Movements

It can be seen in the lower triangle that $\sin \Theta = S_1/\ell_1$ (*ℓ_1 is the length of the arm rotated to the dotted position and is the hypotenuse of the small right triangle*). Also, in the upper triangle, $\sin \Theta = S_2/\ell_2$. Equating these two expressions gives:

$$S_1/\ell_1 = S_2/\ell_2$$

or

$$S_1/S_2 = \ell_1/\ell_2$$

The linear distance moved by one of the forces is equal to the distance moved by the other force multiplied by the moment-arm ratio. It should be noted that there is also movement toward the pivot which was not considered in the above.

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 2 Bearing holders with bearings
- 1 Shaft, 4″ x 1/4″
- 1 Lever arm, 2-in. long with 1/4-in. bore hub
- 1 Lever arm, 1-in. long with 1/4-in. bore hub
- 2 Spring balances
- 2 Spring balance posts with clamps
- 1 Dial caliper (0 - 4 in.)
- 1 Protractor
- 2 Collars

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 5-7.

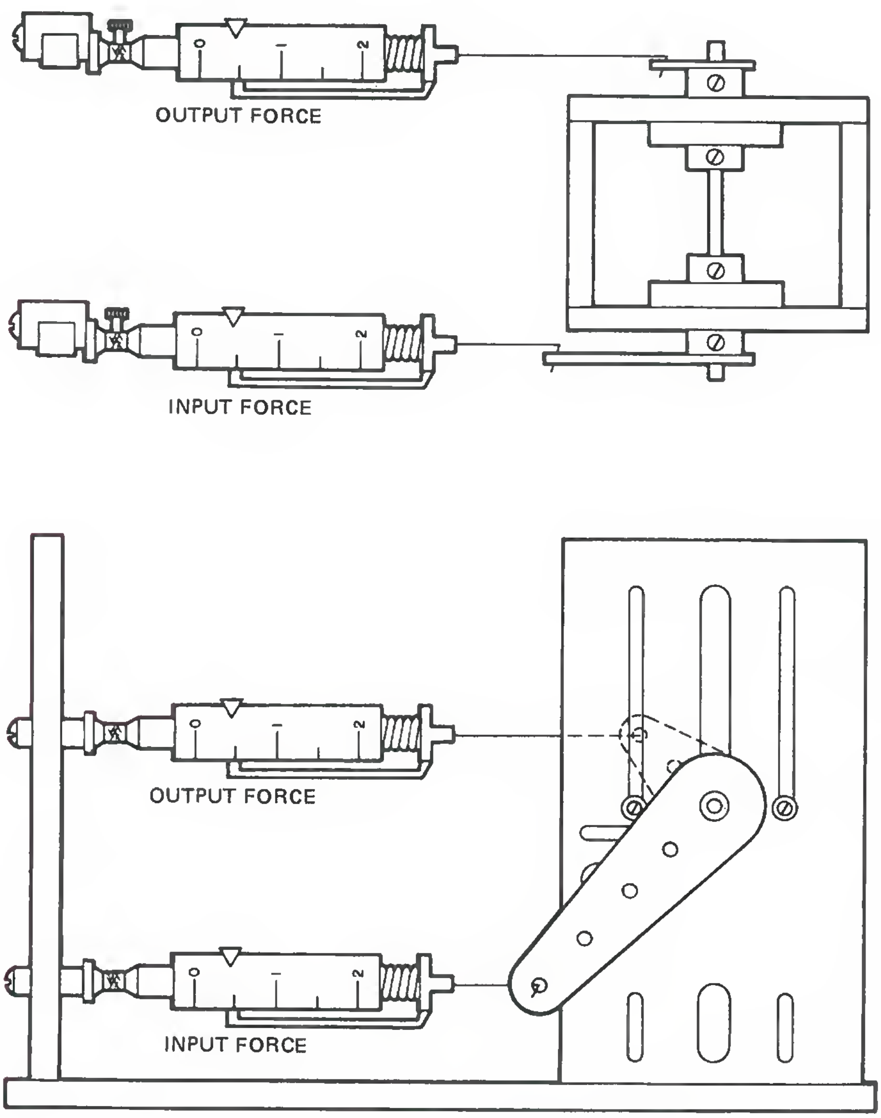


Fig. 5-7 The Experimental Setup

3. Adjust the two lever arms so that they are approximately 90 degrees apart.
4. Adjust the two spring balances *so that they are horizontal* and the input force (F_1) is about 6 oz.
5. Record the values of both forces (F_1 and F_2).
6. Measure and record the lengths of the lever arms (ℓ_1 and ℓ_2).
7. Measure and record the angle between each lever arm and its spring balance (Θ_1 and Θ_2).
8. Compute the component of force acting at right angles to each lever arm (f_1 and f_2).
9. Compute the moment of force acting on each lever (M_1 and M_2).
10. Compute the percent difference between the two moments.
11. Repeat steps 4 through 10 using input forces of 8, 10, and 12 oz.

F_1	F_2	ℓ_1	ℓ_2	Θ_1	Θ_2	f_1	f_2	M_1	M_2	% Diff.

F_1/F_2	f_1/f_2	ℓ_2/ℓ_1

Fig. 5-8 The Data Tables

ANALYSIS GUIDE. Compute the force ratios as indicated in the data table. Compare these force ratios with the lever-arm ratio, and note and explain any deviations. Summarize your understanding of rocker arms and bell cranks.

PROBLEMS

1. Assume that a rocker arm is 3-in. long with the pivot point 1-1/4 in. from the load side. The input force occurs 2000 times per minute and travels 1/2 in. What is the average angular speed in radians-per-second of the rocker arm? What is the average linear speed of the input force and of the output force?

2. In the previous problem, the input force is increased linearly to 3000 per minute in a time of 30 seconds. What is the linear acceleration of the input and of the output? What is the new linear velocity (speed) of the output?

3. A bell crank has one arm 3-in. long and another 2-in. long which are separated by 75°. A force of 6 oz is applied at 60° (see figure 5-9)) to the 3-in. arm. What force is felt at 30° output at the end of the 2-in. arm?

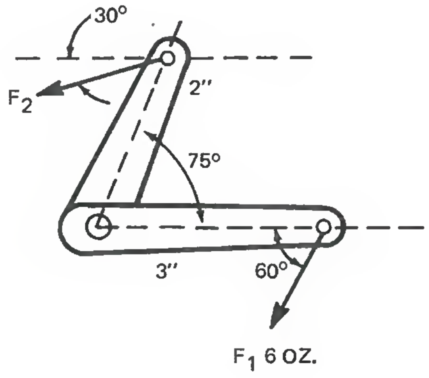


Fig. 5-9 Levers for Problem 3

experiment 6 *COMBINED MECHANISMS*

INTRODUCTION. As we have seen, levers may be compounded using rigid links. They may also be used in combination with a wide variety of other mechanisms. In this exercise we will examine one of these possibilities.

DISCUSSION. In many practical lever applications, the input and output lever arms are coupled through a gear train as shown in figure 6-1.

We can use any one of several different approaches in analyzing the operation of such a mechanism.

One way is to consider the forces required to produce equilibrium. Starting with F_1 acting on ℓ_1, we see that the force component acting perpendicular to the lever arm is

$$f_1 = F_1 \sin \Theta_1$$

where Θ_1 is the angle between the input force and the lever arm centerline. This perpendicular force produces a torque of

$$T_1 = f_1 \ell_1 = F_1 \ell_1 \sin \Theta_1$$

This amount of torque is transmitted through the gear mesh and transformed according to

$$\frac{T_1}{T_2} = \frac{n}{N}$$

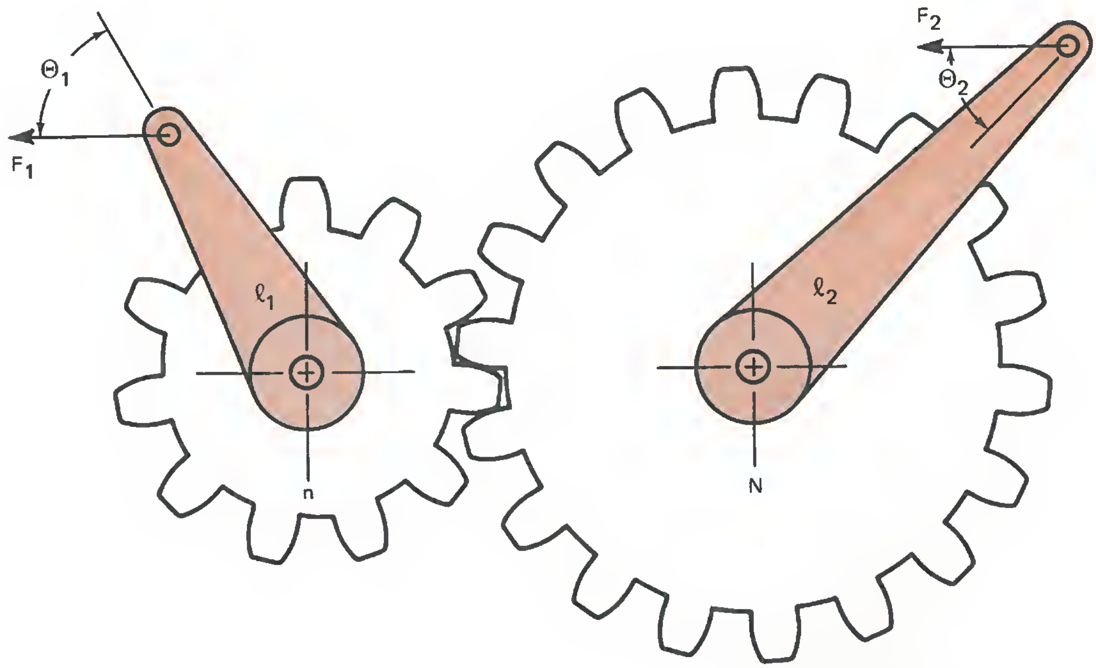


Fig. 6-1 Coupled Lever Arms

or

$$T_2 = T_1 \frac{N}{n}$$

which gives us

$$T_2 = F_1 \ell_1 \frac{N}{n} \sin \Theta_1$$

as the amount of torque acting on the second lever arm where n and N are the numbers of teeth on the gears. This second lever arm torque must also be equal to

$$T_2 = f_2 \ell_2$$

where f_2 is the force component perpendicular to the lever arm. Equating these last two equations for torque gives us

$$f_2 = F_2 \sin \Theta_2$$

Substituting this value of f_2 into the torque equation gives us

$$F_2 \ell_2 \sin \Theta_2 = F_1 \ell_1 \frac{N}{n} \sin \Theta_1$$

or

$$\boxed{F_2 = F_1 \frac{\ell_1}{\ell_2} \frac{N}{n} \frac{\sin \Theta_1}{\sin \Theta_2}} \qquad (6.1)$$

as the equation for the output force.

It is worth mentioning that both Θ_1 and Θ_2 are the angles between the applied force and the lever arm centerline.

This process seems somewhat lengthy and involved; however, in actual practice it is easier than you might expect. Let's work through an example to illustrate how it is done. Suppose that the mechanism in figure 6-1 has the following parameters:

$F_1 = 10$ oz $\qquad \ell_1 = 2.5$ in.

$n = 36$ teeth $\qquad N = 60$ teeth

$\Theta_1 = 45°$ $\qquad \Theta_2 = 30°$ $\qquad \ell_2 = 3.4$ in.

$F_2 = ??$

First we determine the force acting perpendicularly to the input lever arm:

$$f_1 = F_1 \sin \Theta_1 = 10 \times 0.707 = 7.07 \text{ oz}$$

Then the input torque is

$$T_1 = f_1 \ell_1 = 7.07 \times 2.5 = 17.65 \text{ in.-oz}$$

The output torque transformed by the gear mesh is

$$T_2 = T_1 \frac{N}{n} = 17.65 \frac{60}{36} = 29.45 \text{ in.-oz}$$

$$f_2 = \frac{T_2}{\ell_2} = \frac{29.45}{3.4} = 8.65 \text{ oz}$$

And finally the output force is

$$F_2 = \frac{f_2}{\sin \Theta_2} = \frac{8.65}{0.5} = 17.3 \text{ oz}$$

You may wish to compare this result to that produced by equation 6.1 to verify that they are the same.

We can analyze the lever displacements in much the same manner. That is, if the end of the input lever moves an arc distance S_1, then the input gear rotates through an angle Θ_1 of

$$\Theta_1 = \frac{S_1}{\ell_1} \text{ radians}$$

which results in a rotation θ_2 of the output gear equal to

$$\Theta_2 = \Theta_1 \frac{n}{N} = \left(\frac{S_1}{\ell_1}\right)\left(\frac{n}{N}\right) \text{ radians}$$

Then since the output lever notion is related to output gear rotation by

$$\Theta_2 = \frac{S_2}{\ell_2}$$

we have

$$\frac{S_2}{\ell_2} = \frac{S_1}{\ell_1}\frac{n}{N}$$

or

$$S_2 = S_1 \frac{\ell_2}{\ell_1}\frac{n}{N} \qquad (6.2)$$

As before, the practical application of this type of analysis is easier than it might seem.

For example, if the end of input lever in the previous example moves 1.0 in., then the input gear rotates

$$\Theta_1 = \frac{S_1}{\ell_1} = \frac{1}{2.5} = 0.4 \text{ radians}$$

while the output gear rotates

$$S_2 = \ell_2\Theta_2 = 3.4 \times 0.24 = 0.816 \text{ in.}$$

Since linear velocity at the end of the lever is equal to

$$V = S/t,$$

we can also use this approach to determine output velocity if the input velocity is known. The gears here, as in the rest of this text unless otherwise stated, are considered to have a constant speed.

Similarly, ratios such as the mechanical advantage can be found using the same type of analysis methods.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Shaft hangers, 1-1/2 in. with bearings
4 Bearing holders with bearings
2 Spring balance posts with clamps
3 Shafts, 4″ x 1/4″
2 Spring balances
1 Spur gear, approx. 1-1/2 in. OD with 1/4 in. bore hub
1 Spur gear, approx. 1 in. OD with 1/4 in. bore hub
4 Collars
2 Lever arms, 2 in. long with 1/4 in. bore hubs
2 Lever arms, 1 in. long with 1/4 in. bore hubs
1 Dial caliper (0 - 4 in.)
1 Protractor
*1 Straight link, 6 in. long

*See appendix A for link construction details

PROCEDURE

1. Inspect each of your components to be sure it is not damaged. Record the gear tooth counts, n and N.

2. Assemble the mechanism shown in figure 6-2.

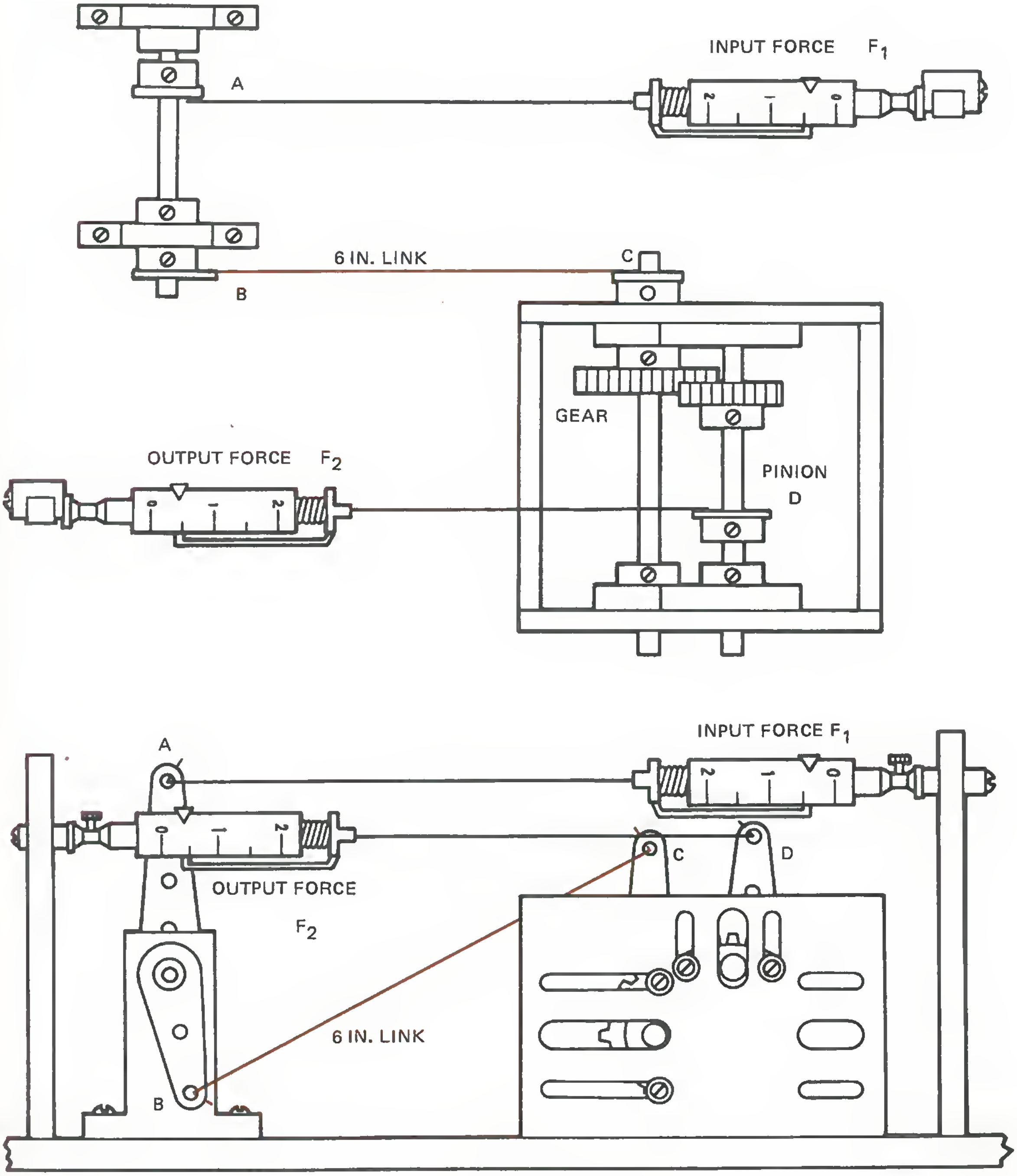


Fig. 6-2 The Experimental Setup

3. Adjust the mechanism so that levers A, C, and D are vertical. Lever B should make a right angle with the 6-in. link. Both spring balances should be horizontal and the input force should be about 6 oz.
4. Record the input and output forces, F_1 and F_2.
5. Measure and record the length of each lever arm, ℓ_A, ℓ_B, ℓ_C, and ℓ_D.
6. Measure and record the angle between each lever arm and its applied force, Θ_A, Θ_B, Θ_C, and Θ_D.
7. Using F_1, ℓ_A, ℓ_B, Θ_A, and Θ_B, compute and record the force acting on the 6-in. link. (f)
8. Using F_2, ℓ_C, ℓ_D, Θ_C, Θ_D, n, and N, compute and record the force acting on the 6-in. link. (f′)
9. Compute the percent difference between f and f′.
10. Hold the gear securely while you slip the pinion out of mesh. Then rotate the pinion about 30 degrees in the direction which increases F_2. Re-engage the gear teeth.
11. Repeat steps 4 through 9.

Qty Trial	n	N	F_1	F_2	ℓ_A	ℓ_B	ℓ_C	ℓ_D
1								
2								

Fig. 6-3 Data Table A

Qty Trial	Θ_A	Θ_B	Θ_C	Θ_D	f	f′	% Diff.
1							
2							

Fig. 6-3 Data Table B

ANALYSIS GUIDE. In analyzing your results you should focus primarily on the analysis methods used. Did they result in good agreement between f and f′? Why do you think this occurred? What do you think were the main causes of error in this experiment? How could the errors be reduced?

PROBLEMS

1. Figure 6-4 represents a lever with angular forces applied to each end. From the dimensions given, compute the pull, P, necessary to achieve equilibrium.

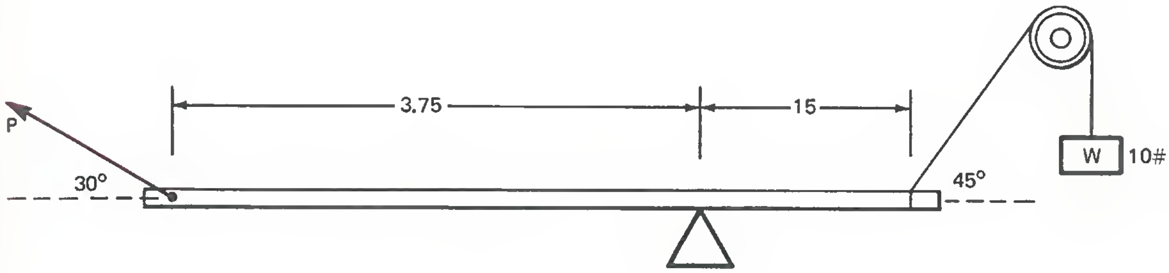


Fig. 6-4 Lever for Problem 1

2. Referring to figure 6-1, assume the lever parameters are:

$F_1 = 14$ oz	$\Theta_1 = 40°$	$\ell_1 = 2.7$ in.
$F_2 = 91$ oz	$\Theta_2 = 25°$	$\ell_2 = 1.9$ in.

   A. If one of the gears has 36 teeth, what are the two possible tooth counts that the other gear could have?

   B. If the righthand gear in figure 6-1 has 36 teeth, how many does the lefthand gear have? (*Use your results from 2A.*)

3. A certain lever system is composed of a class-two lever, a link, and a bell crank as diagramed in figure 6-5. What is the mechanical advantage of the system?

4. What is the value of F_2 in figure 6-5?

5. What is the angle between F_1 and F_2 in figure 6-5?

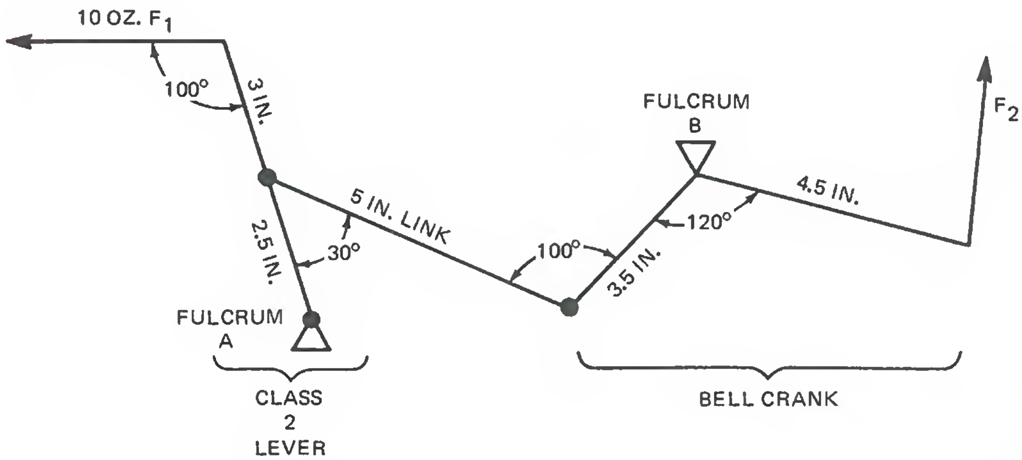


Fig. 6-5 Figure for Problems 3 to 5

experiment 7 FOUR-BAR INTRODUCTION

INTRODUCTION. The four-bar mechanism is one of the most basic practical mechanisms. In this experiment we shall examine the various classes of operation of this important mechanism.

DISCUSSION. A four-bar mechanism is a system in which four rigid links are interconnected in such a way as to allow predictable relative motion. You should notice at this point that it is possible to assemble four rigid links in such a way that relative motion cannot occur without deforming the links. Figure 7-1 shows one such arrangement. A construction of this type is NOT a mechanism, it is a structure. The members of a structure normally do not move relative to each other.

Strictly speaking a linkage will have a number of members and there may be relative motion between them. When one link of a linkage is fixed and the others have specified motions, it becomes a *mechanism.* Actually this text deals only with mechanisms but the words linkages and mechanisms are used interchangeably.

In a four-bar mechanism the links or members do move relative to each other. Figure 7-2 show one possible arrangement for a four-bar mechanism. In this case, when the input link (ℓ_1) rotates through a complete revolution, the output link swings through an arc and back to its starting point. The pins used to join the links must, of course, be free to allow link motion.

The type of mechanism shown in figure 7-2 can only be constructed if the longest link (ℓ_o in this case) is shorter than the sum of the other three links. This is usually the first test we apply to a proposed mechanism.

For example, suppose we wish to build a mechanism using a 4-in. fixed frame link, a 2-in. input link, a 3-in. output link and a 10-in. coupling link. We first test to see if the mechanism is possible by adding the three

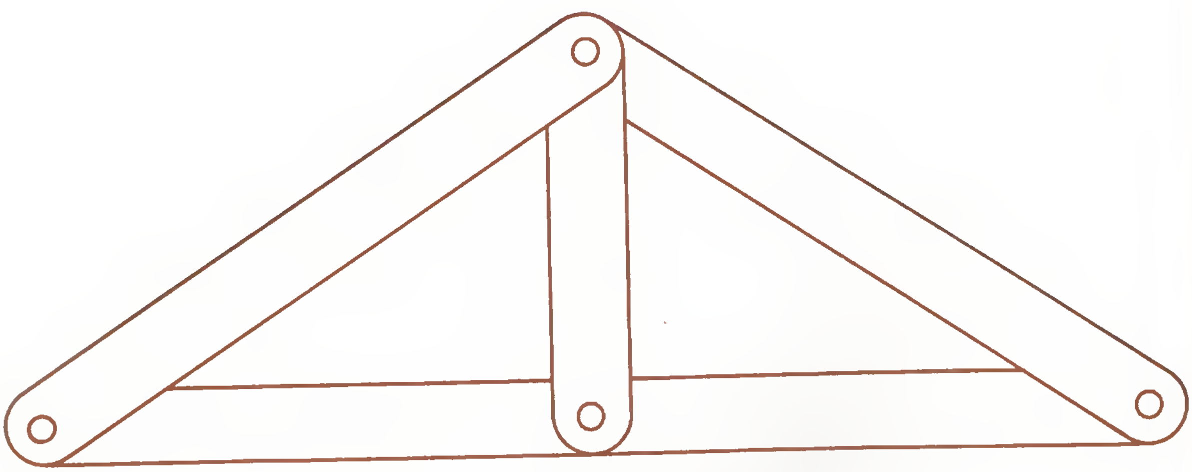


Fig. 7-1 A Four-Bar Structure

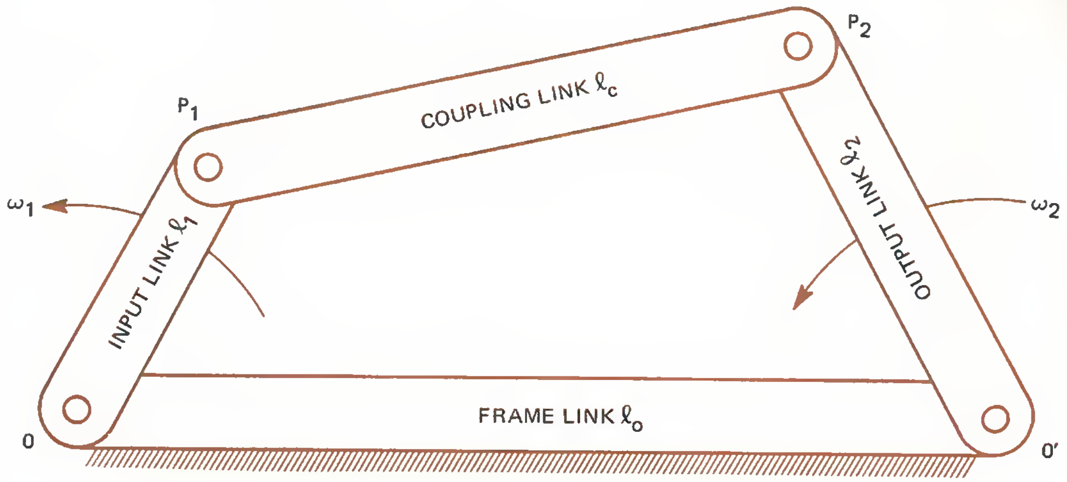


Fig. 7-2 A Four-Bar Mechanism

smallest link lengths and comparing the sum to the longest length:

$$4 + 2 + 3 = 9 < 10$$

In this example the total of the three short lengths is less than the longest length. So such a four-bar mechanism *can not be built.* If you think about this for a while you will realize that this mechanism is impossible because the three small links just aren't long enough to reach the ends of the long link.

On the other hand suppose that we wish to build a mechanism using a 2-in. input link, a 5-in. coupling link, a 4-in. output link and a 6-in. fixed frame link. Testing this mechanism as before we have

$$2 + 4 + 5 = 11 > 6$$

The sum of the shorter links is greater than the longest link so the mechanism is possible.

Figure 7-3 shows a mechanism of this type. In this case, as before, when the input link rotates through a complete circle the output link swings through an arc.

A four-bar mechanism which acts in this way is called a *type I four-bar mechanism* or a *crank-rocker mechanism.*

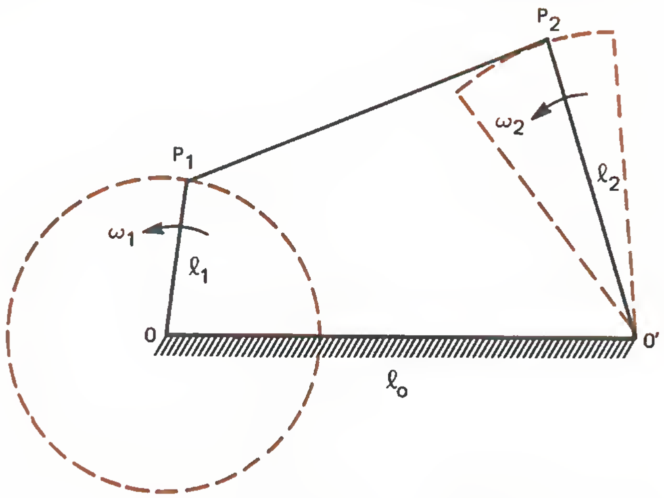


Fig. 7-3 Crank-Rocker Mechanism

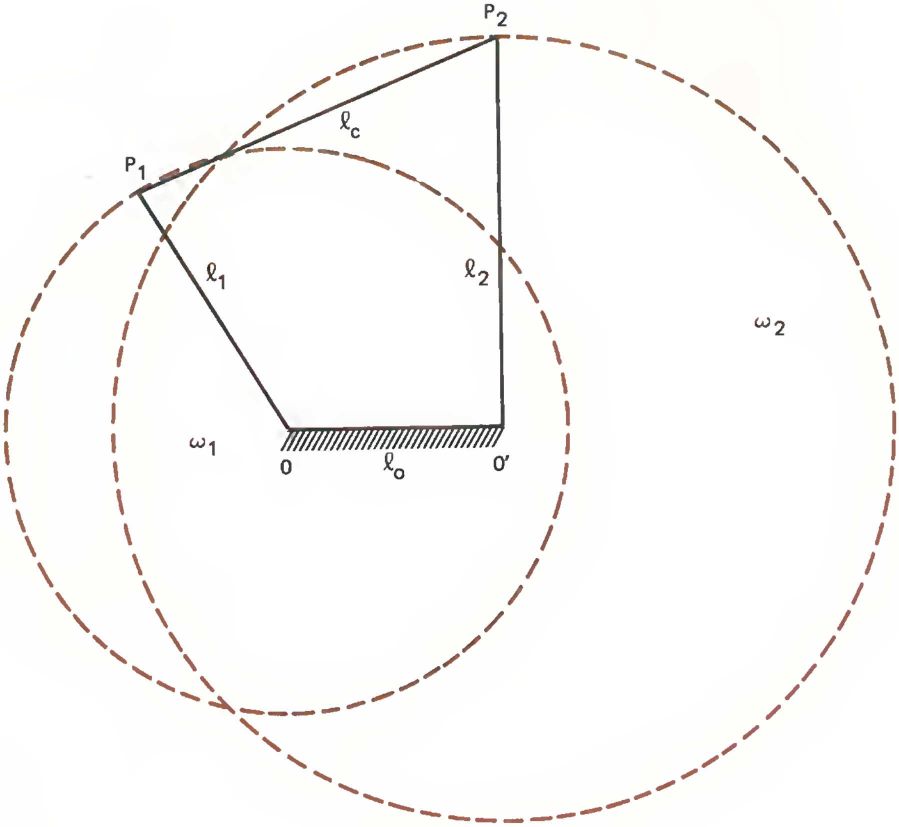


Fig. 7-4 Drag-Link Mechanism

By changing the dimensions of the links we can get a different type of relative motion. Suppose that we have an input link of 3 in., a coupling link of 4 in., an output link of 4 in., and a fixed frame link of 2 in.

You can check to insure that such a mechanism is possible

$$2 + 3 + 4 = 9 > 4$$

Figure 7-4 is a diagram of this type of mechanism.

This time when we rotate the input link a full revolution, the output link also rotates a full revolution. A mechanism which does this is called a *type II four-bar mechanism* or a *drag-link mechanism.*

Again, we can change the link dimensions to get another type of mechanism. Suppose we use an input link of 3 in., a coupling link of 2 in., an output link of 4 in., and a fixed frame link of 5 in. We still have a possible mechanism,

$$2 + 3 + 4 = 9 > 5$$

and it will look somewhat like figure 7-5. In this case the input cannot go through a complete revolution because the lengths of ℓ_c and ℓ_2 will not permit it. Similarly the output link can't go through a complete revolution because the lengths of ℓ_c and ℓ_1 won't permit it. However, when the input link does move, the output link must also move in the same manner. A mechanism of this type is called a *type III, four-bar mechanism* or a *double-rocker mechanism.*

The last type of four-bar mechanism that we will consider is somewhat more difficult to illustrate. Suppose that we have link lengths as follows: The input link is 3 in., the coupling link is 5 in., the output link is 4 in., and the fixed frame link is 6 in. Testing the possibility of building such a mechanism we see that since

$$3 + 4 + 5 = 12 > 6$$

it is possible.

Figure 7-6 shows a sketch of such a mechanism. Let's suppose that the input link is rotating counterclockwise at a constant rate. Now let's examine the mechanism at several different positions. First, in the position shown in figure 7-6, the output must follow the input in its counterclockwise motion.

However, as the links become colinear with the fixed frame as shown in figure 7-7, there is no vertical force coupled to the output link from the input link. Due to gravity or flywheel effect the output link may continue downward. Or, if it is spring-loaded upward, it may reverse its direction and swing upward as the input link continues its counterclockwise motion.

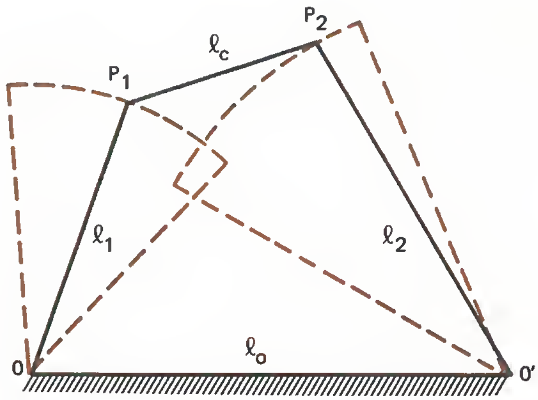


Fig. 7-5 Double-Rocker Mechanism

Such a condition produces a *type IV, four-bar mechanism.* Sometimes this type is called an *indefinite four-bar* operation since we can't tell from the linkage alone what it will do.

The condition shown in figure 7-7 is called a *critical position* in the cycle of the mechanism.

As a result of these link dimensions we can tell whether the mechanism will operate in type I or type II. External forces could cause it to do either.

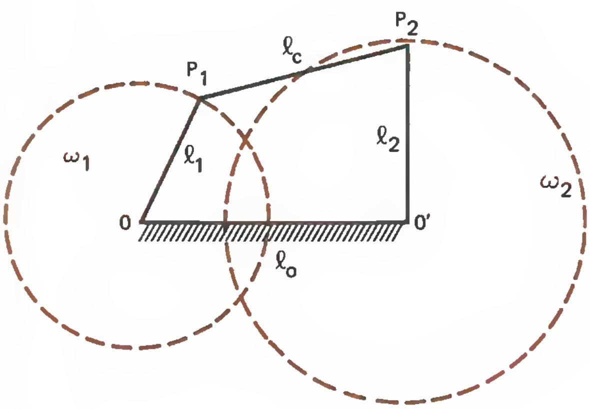


Fig. 7-6 A Class IV, Four-Bar Mechanism

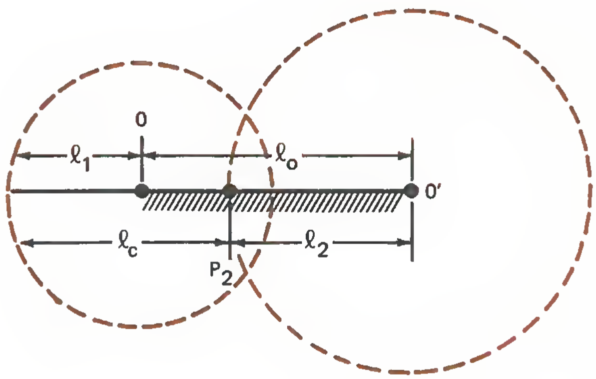


Fig. 7-7 A Critical Point in Mechanism Operation

In conclusion then we can summarize this discussion as follows:

1. Four-bar mechanisms are possible only if the sum of the three shorter links exceeds the length of the longest link.
2. If the input link rotates through a full circle while the output swings through an arc of less than a full circle, the mechanism is a crank-rocker or type I mechanism.
3. If the input and output links both rotate through full circles, the mechanism is a drag-link or type II mechanism.
4. If neither the input or output link can rotate through a full circle, then the mechanism is a double-rocker or type III mechanism.
5. If link dimensions alone make it impossible to classify a four-bar mechanism, it is considered an indefinite or type IV mechanism. External conditions may cause such a mechanism to operate as any of the other three classes.

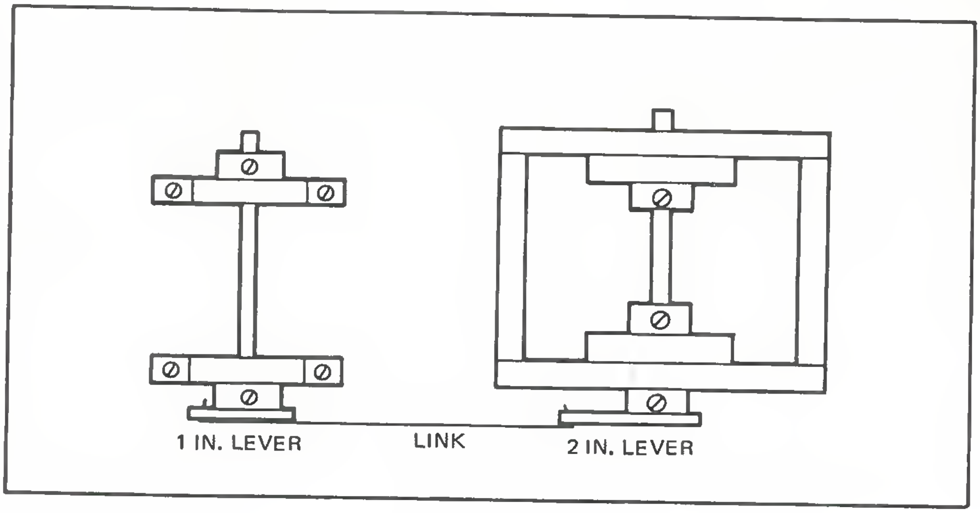
MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Bearing holders with bearings
2 Shaft hangers 1-1/2 in. with bearings
2 Shafts 4" x 1/4"
3 Collars
1 Lever arm 2 in. long with 1/4 in. bore hub
1 Lever arm 1 in. long with 1/4 in. bore hub
*1 Straight link 6 in. long
*1 Reverse link 2 in. long
1 Steel rule 6 in. long

*For link construction details see appendix A.

PROCEDURE

1. Inspect each of your components to be sure they are not damaged.
2. Assemble the mechanism shown in figure 7-8. The shaft through the bearing plates should be 2-3/4 in. above the breadboard.
3. Adjust the spacing between the shaft hangers and bearing plates so that when the shorter lever arm is straight down, the longer arm points to the right and is horizontal.
4. Make a simple diagram of the mechanism as it now appears.
5. Measure and record the length of each link ℓ_o, ℓ_1, ℓ_c, and ℓ_2. (*Note: Use the shortest lever arm for* ℓ_1 *and the distance between shaft centers as* ℓ_o.)
6. Using the measured link lengths verify that this mechanism satisfies the possibility test used in the discussion.
7. Carefully rotate the input link and observe the motion of ℓ_1, ℓ_c, and ℓ_2. Describe each of these motions in your own words.



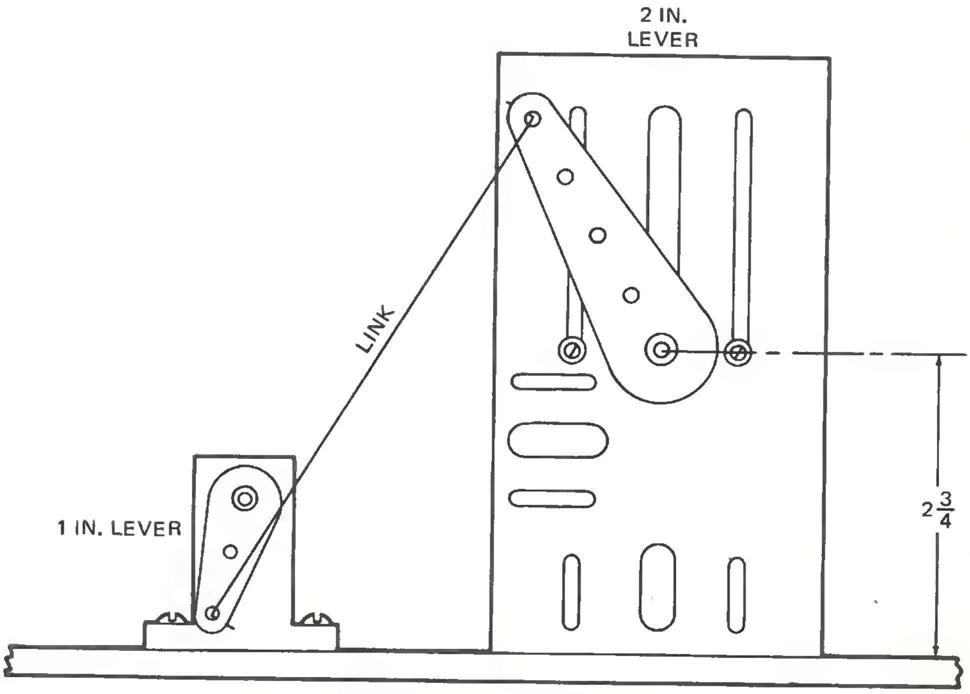


Fig. 7-8 The First Experimental Mechanism

8. Based on your observations identify the type and name of this mechanism.

9. Loosen the bearing plate clamps and readjust the spacing between the shaft hangers and bearing plates so that both lever arms point straight up.

10. Now repeat steps 4 through 8.

11. Readjust the spacing between the shaft hangers and the bearing plates so that both lever arms point to the right and are horizontal. The coupling should not bind on the lever arm holes.

12. Repeat steps 4 through 8.
13. Now loosen the shaft hangers and bearing plates and turn them around on the breadboard as shown in figure 7-9.
14. Repeat steps 4 through 8.

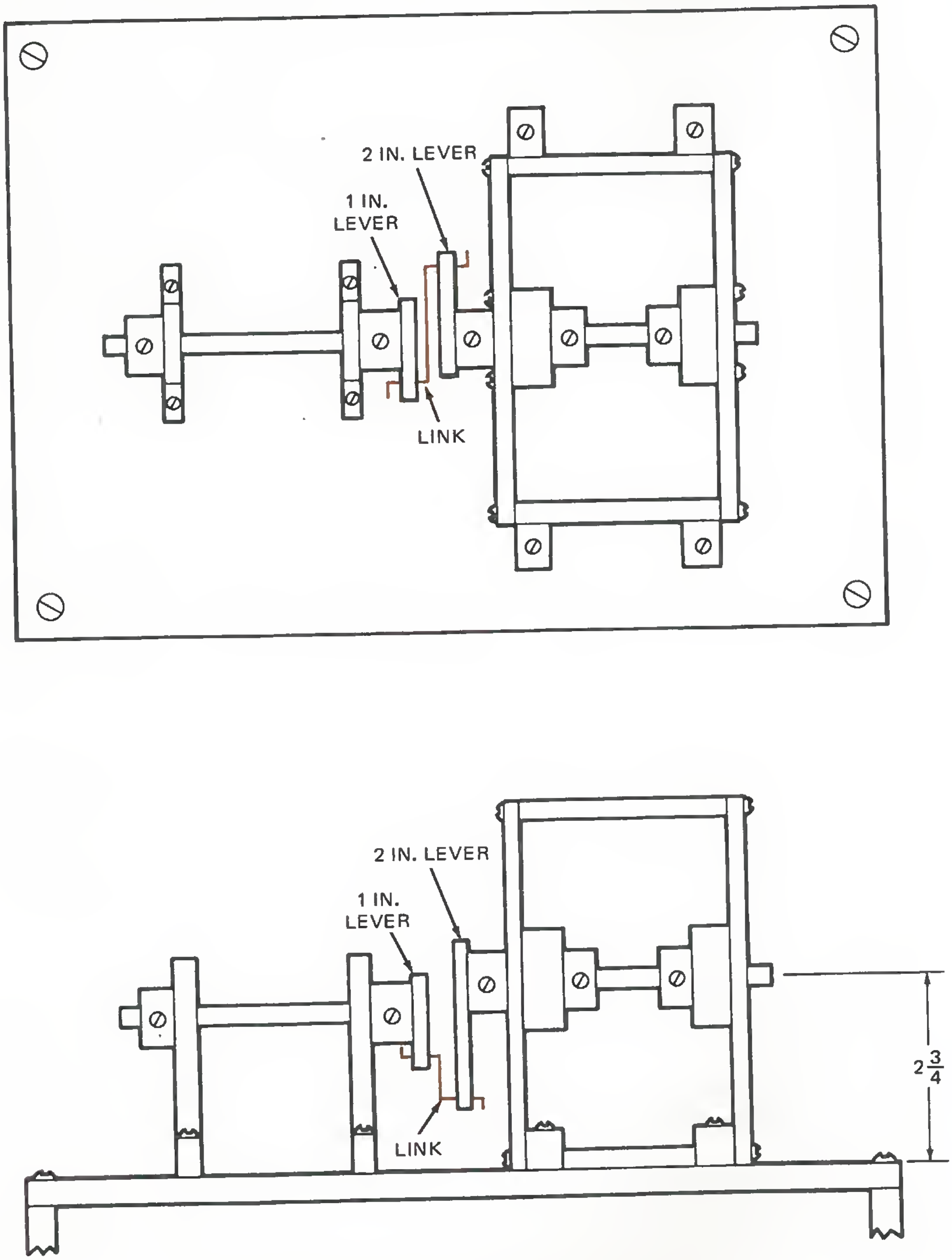


Fig. 7-9 The Second Experimental Setup

Type of Mechanism	Name of Mechanism			
Dimensions of Mechanism	$\ell_o =$	$\ell_1 =$	$\ell_c =$	$\ell_2 =$
Sketch of Mechanism		Description of Motion		
Test of Mechanism's Possibility				

Fig. 7-10 Data for the First Mechanism

Type of Mechanism	Name of Mechanism			
Dimensions of Mechanism	$\ell_o =$	$\ell_1 =$	$\ell_c =$	$\ell_2 =$
Sketch of Mechanism		Description of Motion		
Test of Mechanism's Possibility				

Fig. 7-11 Data for the Second Mechanism

<table>
<tr><td>Type of Mechanism</td><td colspan="4">Name of Mechanism</td></tr>
<tr><td>Dimensions of Mechanism</td><td>ℓ_o =</td><td>ℓ_1 =</td><td>ℓ_c =</td><td>ℓ_2 =</td></tr>
<tr><td colspan="2">Sketch of Mechanism</td><td colspan="3" rowspan="2">Description of Motion</td></tr>
<tr><td colspan="2">Test of Mechanism's Possibility</td></tr>
</table>

Fig. 7-12 Data for the Third Mechanism

<table>
<tr><td>Type of Mechanism</td><td colspan="4">Name of Mechanism</td></tr>
<tr><td>Dimensions of Mechanism</td><td>ℓ_o =</td><td>ℓ_1 =</td><td>ℓ_c =</td><td>ℓ_2 =</td></tr>
<tr><td colspan="2">Sketch of Mechanism</td><td colspan="3" rowspan="2">Description of Motion</td></tr>
<tr><td colspan="2">Test of Mechanism's Possibility</td></tr>
</table>

Fig. 7-13 Data for the Fourth Mechanism

ANALYSIS GUIDE. The main objective of this experiment has been to introduce the four types of four-bar mechanism. In analyzing your results you should discuss each four-bar type and tell how you can identify each one. Also discuss any difficulty you encountered in getting the mechanisms to operate properly.

PROBLEMS

1. What is a four-bar mechanism?
2. What are the characteristics of a mechanism as opposed to a structure?
3. How can you test to be sure that four given link dimensions can be connected into a four-bar mechanism?
4. Test each of the following link dimension sets for possibility in a four-bar mechanism.

   A) $\ell_o = 2$ in., $\ell_1 = 2$ in., $\ell_c = 6$ in., $\ell_2 = 1$ in.
   B) $\ell_o = 6$ in., $\ell_1 = 2$ in., $\ell_c = 4$ in., $\ell_2 = 2$ in.
   C) $\ell_o = 6$ in., $\ell_1 = 1$ in., $\ell_c = 3$ in., $\ell_2 = 2$ in.
   D) $\ell_o = 5$ in., $\ell_1 = \frac{3}{4}$ in., $\ell_c = 2$ in., $\ell_2 = 1$ in.
   E) $\ell_o = 4$ in., $\ell_1 = 2$ in., $\ell_c = 1$ in., $\ell_2 = 1\frac{1}{2}$ in.
5. Make a sketch of the mechanisms in problem 4 which are possible.
6. Tell how each mechanism in problem 5 would act if ℓ_1 rotates at a constant rate clockwise.

experiment 8 CRANK–ROCKER MECHANISMS

INTRODUCTION. Mechanisms having elements pinned or pivoted to each other are known as *linkages.* One of the most elementary forms of linkages is the crank and rocker mechanism. In this experiment you will investigate the features and characteristics of this form of linkage.

DISCUSSION. Figure 8-1 illustrates the simplest possible plane linkage. This is called a four-bar linkage, although in the past it has been called a three-bar linkage because there are three movable connectors. Since the forces and strains felt by the frame are quite important, it is most appropriate and correct to call this a four-bar mechanism. The links can be of any form and shape so long as they do not interfere with the desired motion.

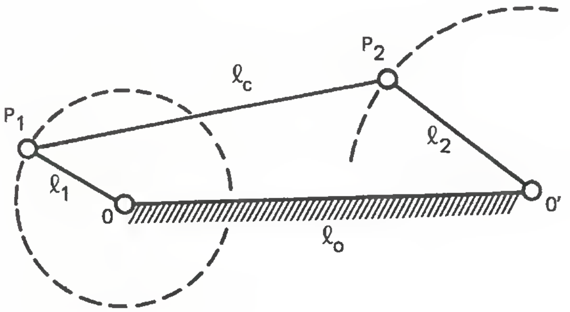


Fig. 8-1 Crank Rocker Mechanism

In figure 8-1 it is assumed that link ℓ_o is fixed and does not move. This means that with the link dimensions shown, link ℓ_1 will rotate through 2π radians. Link ℓ_1 is known as a *crank.* The word crank indicates a link that can rotate continuously. Link ℓ_2 is called the *rocker* (sometimes a beam or a lever) because it can only oscillate through a limited arc path. Link ℓ_c is the connecting link and is called a *coupler* or a *connecting rod.* This link connects the crank to the rocker. Link ℓ_o is fixed and frequently is the frame or foundation. You can see that if link ℓ_c were fixed instead of link ℓ_o, the same crank and rocker motion would result. When the fixed link is changed, this is known as an *inversion* of the mechanism.

The most general four-bar linkage has links of different lengths. This general linkage is shown in figure 8-1 and the links are lettered such that

$$\ell_1 < \ell_2 < \ell_c < \ell_o$$

Figure 8-2 shows one of the limiting positions of the linkage. This occurs when the crank and the connecting rod are colinear; that is, lie on the same line.

In the limiting position shown in figure 8-2, a triangle is formed having sides: link ℓ_o, link ℓ_2 and $(\ell_c - \ell_1)$. If a triangle is to be formed, as it must if we are to have a crank

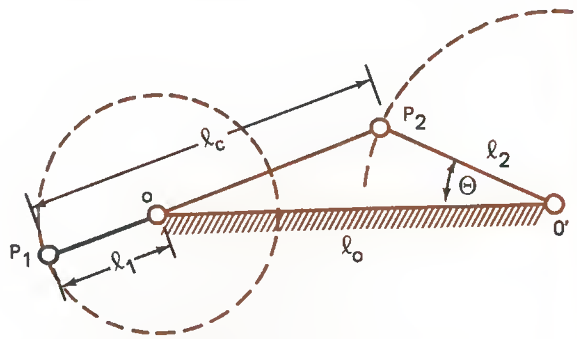


Fig. 8-2 Contracted Limiting Position

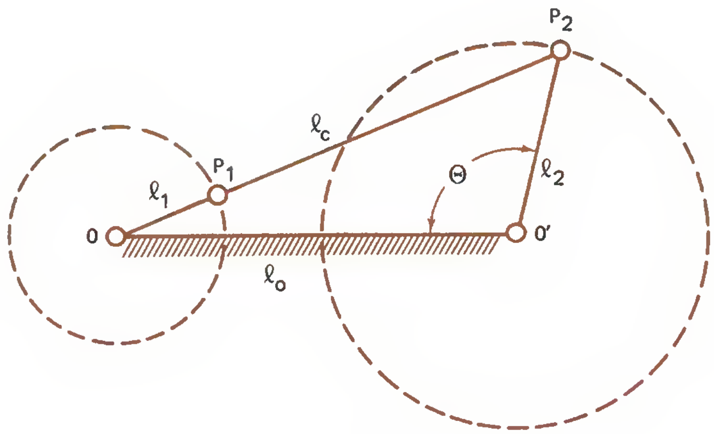


Fig. 8-3 Extended Limiting Position

and rocker, then you know that any one side of a triangle must be less than the sum of the other two sides. This leads to one significant criteria for this type mechanism.

$$\ell_o < ([\ell_c - \ell_1] + \ell_2)$$

or rewritten:

$$\boxed{\ell_o < (\ell_c + \ell_2 - \ell_1)} \qquad (8.1)$$

Equation 8.1 tells us that the distance on the frame between the pivot points of the crank and the rocker must be less than the lengths of the connecting rod (ℓ_c) plus the rocker (ℓ_2) minus the crank (ℓ_1).

Figure 8-3 illustrates the extended limiting position of the crank and rocker mechanism.

Using the same basic principle regarding the lengths of the sides of triangles, you can see from figure 8-3 that

$$\boxed{\ell_1 + \ell_c < \ell_2 + \ell_o} \qquad (8.2)$$

This equation tells us that the crank length added to the connecting rod length must be less than the length of the frame and rocker.

The limiting positions shown in figures 8-2 and 8-3 are quite useful in analyzing the motions and times of motions of the various lengths. If a scale drawing is made of the actual mechanism, the angle through which the rocker oscillates can be determined quite accurately. For many purposes a scale drawing will provide sufficient accuracy. For other purposes an analytical approach will be required. If the lengths of the links are known, you can use the cosine law to solve the angular positions shown in the limiting positions (figures 8-2 and 8-3). For example, in figure 8-3 the angle of the rocker (angle between links ℓ_o and ℓ_2) can be solved by substituting into the following relationship:

$$(\ell_1 + \ell_c)^2 = \ell_2^2 + \ell_o^2 - 2\ell_2\ell_o \cos\Theta$$

where Θ is the angle between ℓ_o and ℓ_2. Once this angle is known, the sine law can be used to find the angle at the crank. This process is repeated for the other extreme position which will give both limiting angles of the rocker and the angular position of the crank at these limits.

From the above analytical approach you can find out how many degrees of crank rotation are required for motion to the right and

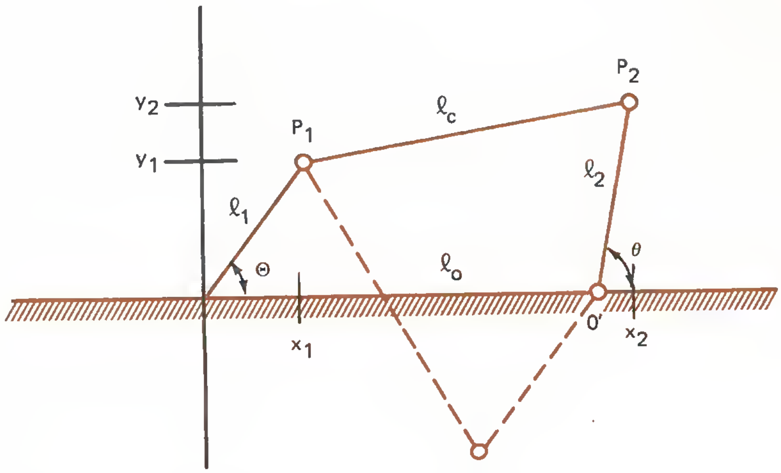


Fig. 8-4 Analytical Design of Crank-Rocker Mechanism

for motion to the left. Knowing these facts permits you to use the usual angular relationships to determine velocity and acceleration.

Now, let's look at the problem of determining where the follower will be for a given crank angular position. This general problem is outlined in figure 8-4.

The rectangular coordinates of point P_1 are

$$x_1 = \ell_1 \cos \Theta$$

$$y_1 = \ell_1 \sin \Theta$$

and of point P_2,

$$x_2 = \ell_o + \ell_2 \cos \theta$$

$$y_2 = \ell_2 \sin \theta$$

We know by the Pythagorean theorem that

$$\boxed{\ell_c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (8.3)$$

By substituting the f(Θ) values for the unknowns in equation 8.3 and expanding, you finally arrive at an equation for θ in terms of Θ.

$$\boxed{\theta = 2 \text{ arc tan } K \pm \left(\frac{\sqrt{K^2 + L^2 - M^2}}{L + M} \right)} \quad (8.4)$$

where

$$K = \sin \Theta$$

$$L = \frac{\ell_o}{\ell_2} + \cos \Theta$$

$$M = \frac{\ell_o}{\ell_1} \cos \Theta + \frac{\ell_1^2 + \ell_c^2 + \ell_2^2 + \ell_o^2}{2\ell_1\ell_2}$$

The plus sign in equation 8.4 will give the figure drawn with solid lines in figure 8-4. The minus sign will give the figure shown with dashed lines which is known as the cross condition.

We will now look at a linkage having dimensions as follows: ℓ_1 = 2.5 inches, ℓ_c = 10 inches, ℓ_2 = 4.5 inches, and ℓ_o = 9 inches. A scale drawing of this linkage is shown in figure 8-5 with the two extreme positions shown.

The problem is to determine the number of degrees that the rocker oscillates through as the crank revolves. Referring to figure 8-5, you can see that this equals angle $P_2'\ 0'\ P_2''$

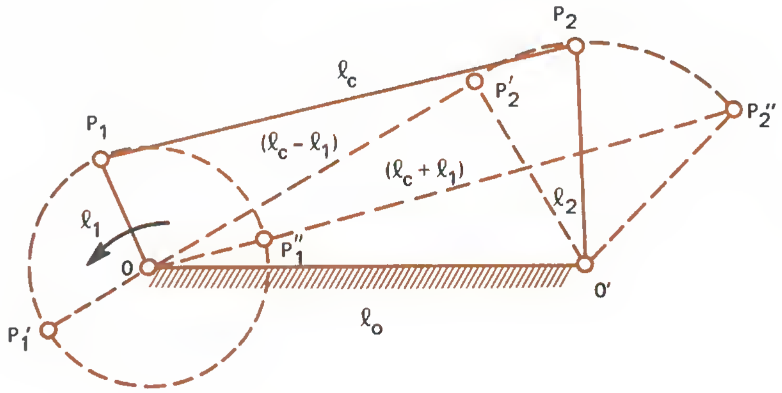


Fig. 8-5 Crank-Rocker Problem

and that this angle is equal to angle $00'P''_2$ - angle $00'P'_2$. You can see that we know the lengths of the sides of triangles $00'P''_2$ and triangles $00'P'_2$. Thus, we can use the law of cosines and determine the two angles desired. For the left position of link ℓ_2 which makes up triangle $00'P'_2$:

$$(\ell_c - \ell_1)^2 = \ell_2^{\,2} + \ell_o^{\,2} - 2\ell_2\ell_o \cos (00'P'_2)$$

Substituting, we have

$$7.5^2 = 9^2 + 4.5^2 - (2)(9)(4.5) \cos (00'P'_2)$$

$$00'P'_2 = \text{arc} \cos 0.5555$$

Angle $00'P'_2 = 56°15'$

In an identical fashion we can solve for angle $00'P''_2$ by using triangle $00'P''_2$ and the cosine law. From this we find that

$$\text{Angle } 00'P''_2 = 132°46'$$

And, angle $P''_2\,0'P'_2$ = angle $00'P''_2 - 00'P'_2 = (132°46') - (56°15') = 76°31'$ which is the total swing of the rocker.

You can see from figure 8-5 that the swing of link ℓ_2 from position P''_2 begins when crank ℓ_1 is in position P''_1 and ends at position P'_2 when the crank reaches P'_1. This takes more than 180 degrees of crank rotation. Also, the return swing of link ℓ_2 from P'_2 to P''_2 takes a crank rotation from P'_1 to P''_1 which is less than 180 degrees. If the crank is rotating with a constant angular velocity, this time is different. With rotation as shown in the figure, link ℓ_2 takes longer to swing left than to return to the right. Let's assume the crank is rotating counterclockwise at 120 RPM. What is the time for the left stroke and the time for the right stroke? This involves first determining the value of angle $P'_2\,0P''_2$ which is the difference in angular position. You can see that this angle is equal to

$$P'_2\,0P''_2 = 0'0P'_2 - 0'0P''_2$$

Again, using the cosine law, we determine that

$$P'_2\,0P''_2 = 29°56' - 15°20' = 14°36'$$

The left stroke (from P''_2 to P'_2) will take a movement of the crank of 180 degrees plus angle P'_2 to P''_2 of the 360 degrees of rotation. The right movement of the rocker will be 180 - angle $P''_2 0P'_2$ of the 360 degrees. The time involved will be this same fraction

of the time to revolve 360 degrees. 120 RPM means one revolution each half second. From this,

Time for left motion of

$$\ell_2 = \frac{180 + 14.6}{360} \times .5 = 0.27 \text{ seconds}$$

and

Time for right motion of

$$\ell_2 = \frac{180 - 14.6}{360} \times .5 = 0.23 \text{ seconds}$$

It is often important to know the velocities of the various motions of a linkage. Since we do have circular motion, the velocity is angular as is the acceleration. You may remember that in physics we define an angle as

$$\Theta = \frac{\text{arc length}}{\text{radius}}$$

which gives the angle, Θ, in radians. For the entire circle this expression becomes

$$\Theta = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

Angular velocity, ω, equals the angle covered per unit of time. All points on a given rotating body have the same angular velocity. Also,

$$\omega = \frac{\Theta}{t} = \frac{2\pi}{T} = (2\pi)\ \text{RPM}$$

where RPM is the revolutions per minute and time is in minutes. T is the time for one revolution. The relationship between angular velocity and linear velocity of a point at a given distance from the center of rotation is

$$\text{Velocity} = \frac{\text{distance}}{\text{time}} = \frac{\text{arc length}}{\text{time}} = r\frac{\Theta}{t} = r\omega$$

The relationship between the velocity and the acceleration for a crank rotating at a constant angular velocity is as shown in figure 8-6. You will note that the velocity is perpendicular to the radius from the center of rotation and that the acceleration is toward the center.

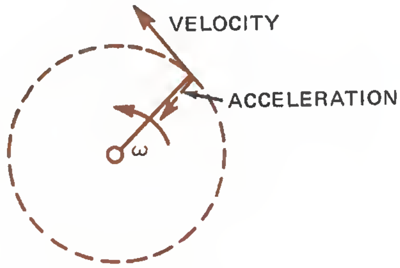


Fig. 8-6 Rotating Body Velocity and Acceleration

The angular acceleration, α, equals $d\omega/dt$ and is measured in units of radians per second, per second. In summary, all relationships found in linear displacement, velocity, and acceleration will remain true by substituting Θ for displacement, ω for velocity and α for acceleration.

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 2 Bearing holders with bearings
- 2 Shaft hangers with bearings
- 2 Shafts 4" x 1/4"
- 2 Collars
- 1 Lever arm 2 in. long with 1/4 in. bore hub
- 1 Lever arm 1 in. long with 1/4 in. bore hub
- 2 Disk dials
- 2 Dial indexes and holders
- *1 Straight link 6 in. long
- 1 Steel rule 6 in. long
- 2 Sheets of linear graph paper

*See Appendix A

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 8-7. Be sure that the bearing plate shaft is 2-3/4 in. above the breadboard surface.

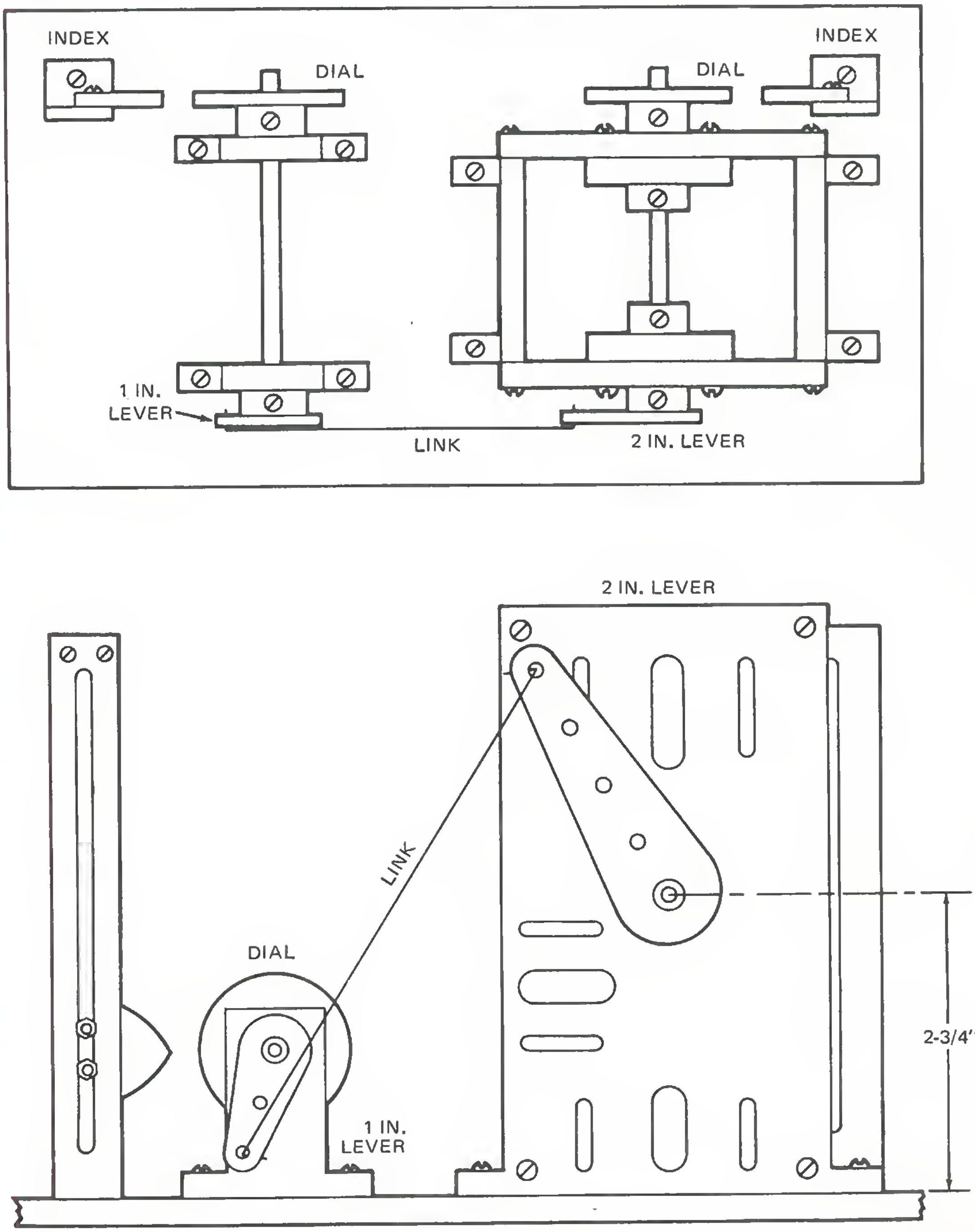


Fig. 8-7 The Experimental Mechanism

3. Adjust the spacing between the shaft hangers and the bearing plates so that both lever arms can point straight up at the same time.
4. Allow the mechanism find its own "rest" position and set the two dials to zero.
5. Measure and record the length of each of the links (ℓ_o, ℓ_1, ℓ_2, and ℓ_c).
6. Rotate the dial attached to the shorter lever arm in 20° increments and record the other dial reading at each increment.

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = ________ ℓ_2 = ________ ℓ_c = ________ ℓ_o = ________

Fig. 8-8 Data Table for the First Trial

7. Assuming that the driver were turning at 100 RPM compute the average angular velocity of the follower in each increment in step 6.
8. Plot a smooth curve of the follower position versus time using the conditions of step 7.
9. Similarly plot follower velocity versus time.
10. Move the link to the *next* hole nearer the shaft and repeat steps 3 through 9.

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = ________ ℓ_2 = ________ ℓ_c = ________ ℓ_o = ________

Fig. 8-9 Data Table for the Second Trial

ANALYSIS GUIDE. Draw a skeleton sketch of your mechanism showing the lengths of the links and the two extreme positions of the rocker. Using the law of cosines, calculate the rocker swing and compare with that observed. Explain any differences noted. Using your measured link lengths, verify that this mechanism satisfies the conditions given in the discussion for a crank-rocker. Do this for both sets of links used.

PROBLEMS

1. Assume that the crank rotation of the mechanism used by you in the experiment was rotating clockwise at 60 RPM. What is the time required for the rocker to move to the left? What is the time needed for its return movement to the right?
2. What must be the rotational speed in RPM of a crank 3.5 inches in length of its linear velocity is 500 feet/minute?
3. The crank described in problem 2 starts at rest with an angular acceleration of 3 radians/second/second. If the acceleration is constant, what is the angular velocity after 7 seconds? What is the velocity of a point 3.5 inches from the center at this time? What was the average velocity of this point?
4. Using equation 8.4, compute one set of angular positions of the crank and rocker used during this experiment. Be sure the angles you use correspond to those of figure 8-4. Compare the computed values with the observed values and comment on the differences.

experiment 9 DRAG-LINK MECHANISM

INTRODUCTION. An important inversion of the four-bar linkage is the one known as the drag link or double crank mechanism. In this type mechanism both of the links pinned to the frame are able to make complete rotations. In this experiment you will investigate the characteristics and features of a drag link mechanism.

DISCUSSION. As you already know, if you have a four-bar linkage mechanism with lengths such that the shortest link can make a full rotation, you have what is known as a type I linkage. If the shortest link is indeed a crank and one of the adjacent links is fixed, then you have what is known as a type I linkage, known as a crank and rocker.

If the shortest link is used as the coupler or connecting rod, you will have what is known as a double-rocker mechanism. And, if you fix the shortest link, that is, make it the frame, the resulting mechanism will be known as a drag-link or double-crank mechanism. These three inversions of type I linkages are illustrated in figure 9-1.

As you can see in figure 9-1, if the longest or next longest link is fixed, one link may rotate and one oscillate. If the next shortest link is fixed, both links may oscillate. And finally, if the shortest link is fixed, the links may both rotate through a full 360 degrees.

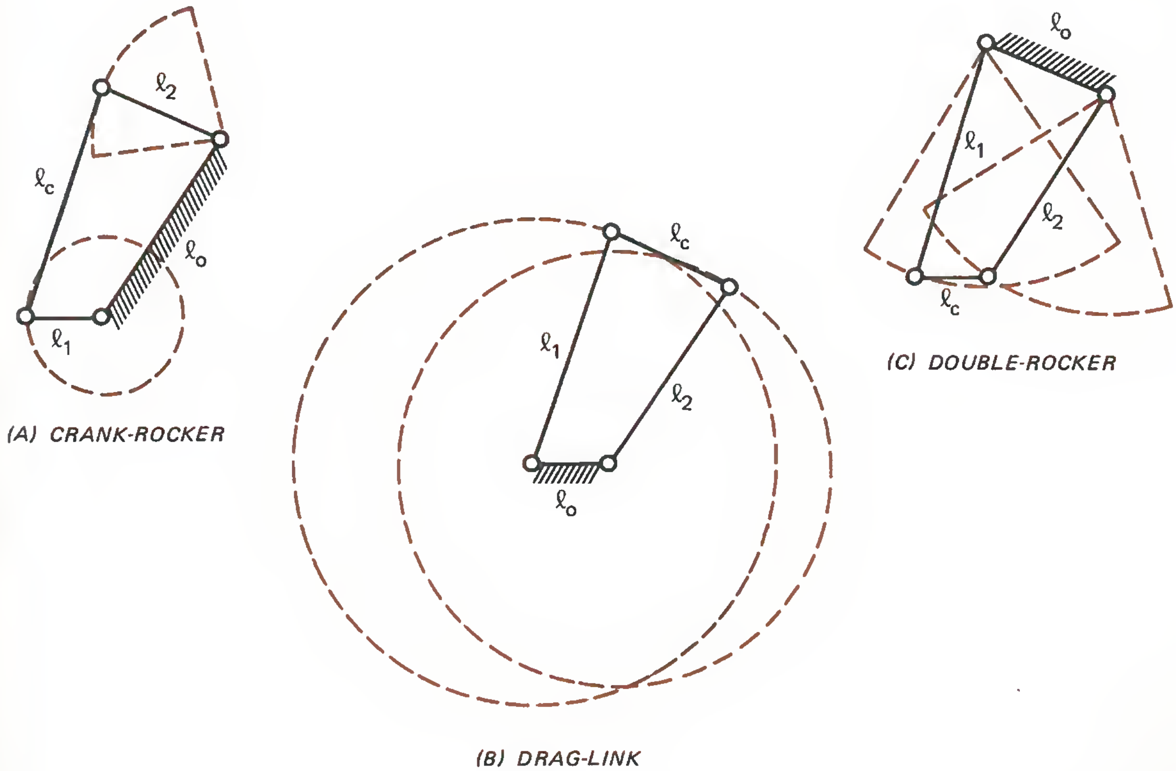


Fig. 9-1 Inversions of a Four-Bar Mechanism

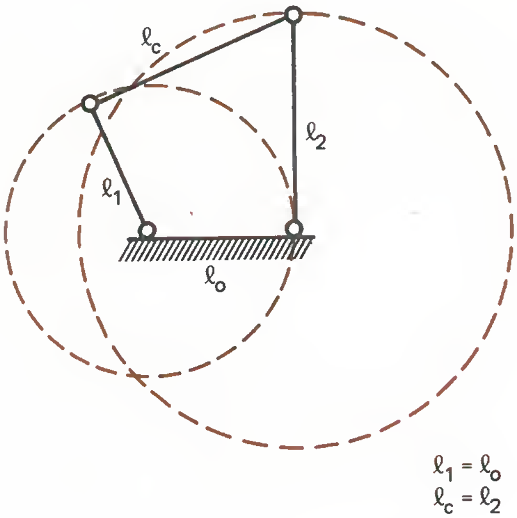


Fig. 9-2 Special Drag-Link Mechanism

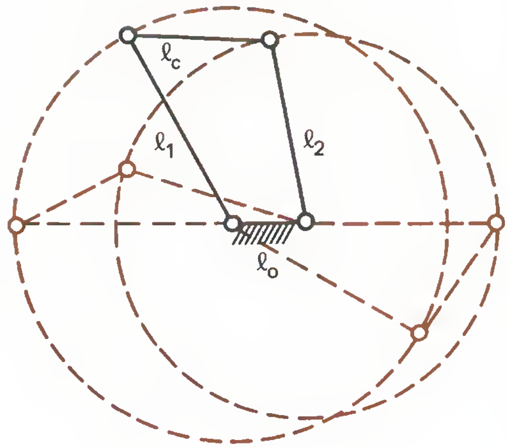


Fig. 9-3 Drag-Link Mechanism: Critical Positions

If the driver link has uniform motion (angular velocity is constant), it will transmit to the follower link a highly variable angular velocity as they both make full rotations about their fixed centers. This variable velocity is used in practical applications of the drag link mechanism, especially in quick-return type mechanisms. For example, if the length of links ℓ_1 and ℓ_o are the same and the lengths of ℓ_c and ℓ_2 are the same, then when link ℓ_2 makes one revolution, the driver link ℓ_1 will have made two revolutions. This is illustrated in figure 9-2.

In this special drag-link mechanism where the driver length equals the frame distance between pivot points, and where the connecting rod length equals the driven lever length, the driving angle and the driven angle are nearly proportional over a considerable portion of the cycle. In the position shown in figure 9-2, a 20-degree movement of crank ℓ_1 will result in approximately a 20-degree movement of crank ℓ_2. Much beyond this 20-degree movement, however, finds crank ℓ_2 remaining almost still as crank ℓ_2 continues to rotate.

The general requirements for a drag-link mechanism may be obtained by looking at what are known as the critical positions of the mechanism. Since both links rotate a full revolution, there will be a time when each of the rotating links will be colinear with the fixed frame (link ℓ_o in our diagram). At this time a triangle will be formed as is shown in figure 9-3.

From the critical positions you can see that the following relationships must be true in order for this mechanism to function:

$$\ell_o + \ell_2 < \ell_1 + \ell_c \tag{9.1}$$

$$\ell_c < \ell_1 + \ell_2 - \ell_o \tag{9.2}$$

$$\ell_1 < \ell_c + \ell_2 - \ell_o \tag{9.3}$$

These three inequalities, coupled with the fact that the shortest link is fixed, are the requirements for a drag-link mechanism. In these inequalities link ℓ_1 and link ℓ_2 are the rotating cranks and link ℓ_c is the coupler or connecting rod.

We will now look at the two in-line positions of crank ℓ_1 which will be the driven crank. Let's assume that these two positions represent a desired output. Crank ℓ_2 will be the driver crank and will be rotating at a constant angular velocity. The problem will be to determine if there is a difference between the time it takes crank ℓ_1 to move to the left and to move to the right and to determine what this time difference will be.

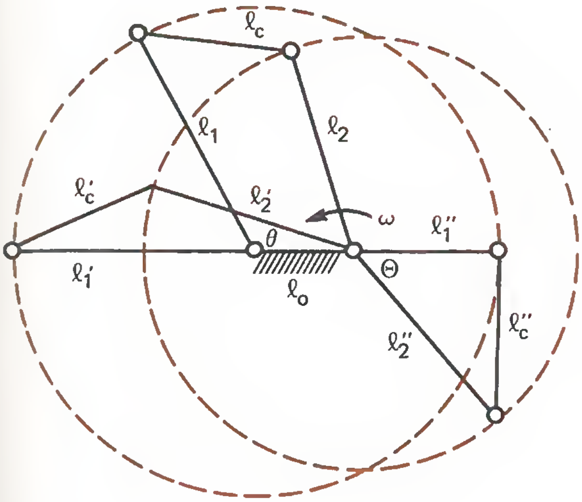


Fig. 9-4 Drag-Link Motion Analysis

You can see by the sketch in figure 9-4 that link ℓ_1 will travel from its extreme right position to its extreme left position when link ℓ_2 moves from position ℓ''_2 to ℓ'_2. Since ℓ_2 travels at a constant rate, the time required for this motion is

$$\boxed{t_1 = T\left[\frac{180^\circ + \theta - \Theta}{360^\circ}\right]} \qquad (9.4)$$

where

t_1 is the time of motion from ℓ''_1 to ℓ'_1
T is the time of one rotation of link ℓ_2

You can see by inspection that this time will be longer than the time required to move from ℓ'_1 to ℓ''_1. The time required for movement from left to right position is

$$\boxed{t_2 = T\left[\frac{180^\circ - \theta + \Theta}{360^\circ}\right]} \qquad (9.5)$$

To check if t_1 plus t_2 equals the time for one revolution we add equation 9.4 to 9.5 and see that

$$t_1 + t_2 = T\left[\frac{180 + \theta - \Theta}{360}\right] + T\left[\frac{180 - \theta + \Theta}{360}\right]$$

$$= T\left[\frac{180 + \theta - \Theta + 180 - \theta + \Theta}{360}\right]$$

$$= T\left[\frac{360}{360}\right] = T$$

and since T equals the time for one revolution, the two expressions will add to equal this time.

The values of angles θ and Θ shown in figure 9-4 can be found by using the cosine law and the lengths of the linkages. It is important to draw a sketch of a proposed drag link mechanism to check the possible motions graphically as well as using analytical techniques. The cosine law gives the following relationships.

$$\theta = \text{arc cos}\ \frac{(\ell_1 - \ell_o)^2 - (\ell_c^2 + \ell_2^2)}{2\ell_2(\ell_1 - \ell_o)} \qquad (9.6)$$

$$\Theta = \text{arc cos}\ \frac{(\ell_1 + \ell_o)^2 - (\ell_c^2 + \ell_2^2)}{2\ell_2(\ell_1 + \ell_o)} \qquad (9.7)$$

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Bearing holders with bearings
2 Shaft hangers 1-1/2 in. with bearings
2 Shafts 4" x 1/4"
2 Collars
1 Lever arm 2 in. long with
  1/4 in. bore hub
1 Lever arm 1 in. long with
  1/4 in. bore hub
1 Steel rule 6 in. long
*1 Reverse link 2 in. long
2 Disk dials
2 Dial indexes and mounts
2 Sheets of linear graph paper

*For link construction details see appendix A.

PROCEDURE

1. Inspect each of your components to be sure they are not damaged.
2. Construct the mechanism shown in figure 9-5. Note that the bearing plate shaft should be 2-3/4 in. above the breadboard.
3. Allow the mechanism to find its own "rest" position and set both dials to zero.
4. Rotate the dial fixed to the shorter lever arm in 20-degree increments and record the angular displacement of the other dial at each increment. Continue in this way until you have completed a whole revolution.
5. Assuming that the driver lever arm was turning at a constant angular velocity of 100 RPM, compute and record the angular velocity of the follower in each of the increments used in step 4.
6. Plot a smooth curve of the follower lever arm's displacement versus time assuming the conditions of step 5.
7. Similarly plot the follower lever arm's velocity versus time.
8. Repeat steps 4 through 7 using the longer lever arm as the driver and the shorter arm as the follower.
9. Measure and record the length of each link in the mechanism.

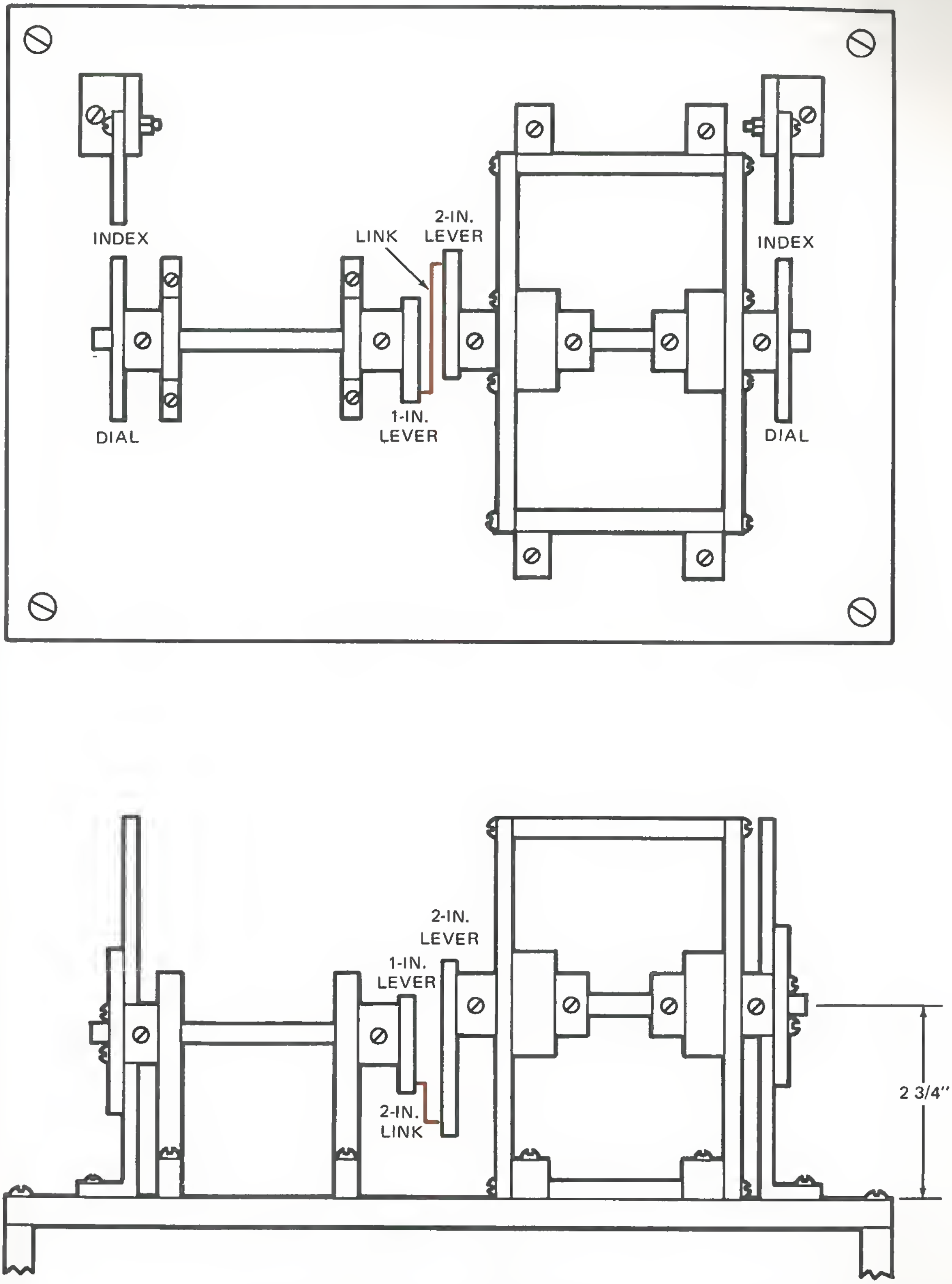


Fig. 9-5 The Experimental Setup

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = ________ ℓ_2 = ________

ℓ_c = ________ ℓ_o = ________

Fig. 9-6 Data Table First Trial

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = ________ ℓ_2 = ________

ℓ_c = ________ ℓ_o = ________

Fig. 9-7 Data Table Second Trial

ANALYSIS GUIDE. Draw a sketch of your experimental setup using figure 9-3 as a guide. Indicate on your sketch the maximum and minimum points of velocity of the follower shaft. Add additional comments of your own to clarify the actions involved in drag-link mechanisms. Using the measured lengths, show that the experimental mechanism satisfies the conditions given in the discussion for a drag-link.

PROBLEMS

1. Figure 9-3 has linkages with the following lengths: ℓ_o = 4 in.; ℓ_1 = 10.5 in.; ℓ_c = 7 in.; and ℓ_2 = 9 in. ℓ_o is a fixed link and ℓ_2 is the driver. How many degrees must crank ℓ_2 rotate to carry crank ℓ_1 from the extended colinear position to the overlapping colinear position?

2. If crank ℓ_2 in problem 1 rotates at 200 RPM, what is the time required for crank ℓ_1 to rotate from the extended colinear position to the overlapping colinear position? What is the time for its return? Assume that crank ℓ_2 rotates in a counterclockwise direction.

3. List and discuss three practical applications for the drag link mechanism.

4. Assume that link ℓ_o and link ℓ_1 shown in figure 9-3 are 6 inches and 10 inches, respectively, and that links ℓ'_c and ℓ_2 are 7 and 8 inches. If link ℓ_1 is rotating at 600 RPM, how long will it take link ℓ_2 to rotate from a vertical position upward to a vertical position downward? Link ℓ_1 is rotating in a counterclockwise direction. How long will it take for link ℓ_2 to rotate from the vertical downward position back to the upright position? Compute the ratio of these two times.

experiment 10 DOUBLE–ROCKER MECHANISM

INTRODUCTION. Many practical four-bar applications do not require any of the links to turn through a complete revolution. In many such applications a type III four-bar mechanism can be used to achieve the desired motion. In this experiment we shall examine the operation of this type of four-bar mechanism.

DISCUSSION. Double rockers, like all other four-bar mechanisms, must be possible mechanisms. That is, the sum of the three shorter links must be longer than the longer link. With this condition satisfied, the easiest way to get a double rocker is to make the coupling link shorter than any other. Figure 10-1 shows this type of double rocker.

In general, any combination such that the sum of the longest and the shortest links is greater than the sum of the other two gives a double rocker, no matter which link is fixed. We can get some insight into the operation of this type of mechanism by considering its limiting positions. The principal limiting positions are shown in figure 10-2.

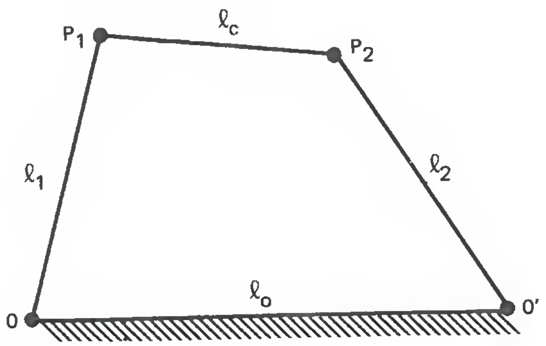


Fig. 10-1 A Double-Rocker Mechanism

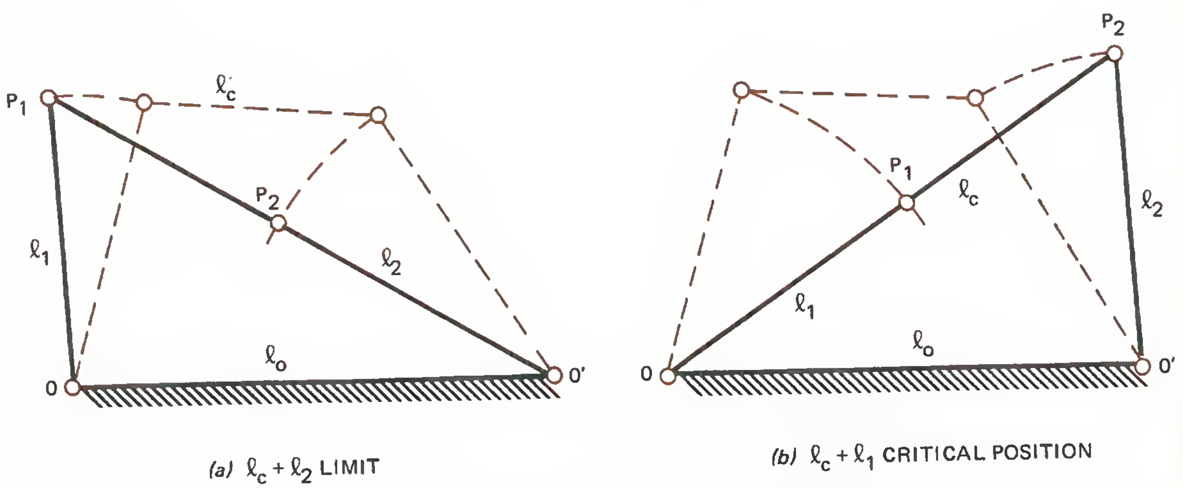


Fig. 10-2 Double-Rocker Critical Positions

If we start with the driver (ℓ_1) in the counterclockwise limiting position as shown in figure 10-2a, we see that the link lengths must satisfy the inequality.

$$\ell_c + \ell_2 < \ell_o + \ell_1 \qquad (10.1)$$

Then, as we rotate the driver clockwise, we can reach the point where the follower reaches its clockwise limit as shown in figure 10-2b. At this point the link lengths must satisfy the inequality

$$\ell_c + \ell_1 < \ell_o + \ell_2 \qquad (10.2)$$

If we add these two inequalities (10.1 and 10.2) we have

$$2\ell_c + \ell_2 + \ell_1 < 2\ell_o + \ell_2 + \ell_1$$

Subtracting (ℓ_2 + ℓ_1) from each side gives us

$$2\ell_c < 2\ell_o$$

or

$$\ell_c < \ell_o \qquad (10.3)$$

or, in other words, ℓ_c must be less than ℓ_o. Similarly, if we subtract the two original inequalities we have

$$\ell_2 - \ell_1 < \ell_1 - \ell_2$$

Adding (ℓ_2 + ℓ_1) to both sides renders

$$2\ell_2 < 2\ell_1$$

or

$$\ell_2 < \ell_1 \qquad (10.4)$$

So, we see that if we wish to build a double-rocker mechanism which has the limiting positions shown in figure 10-2, then ℓ_c must be less than ℓ_o *and* ℓ_2 must be less than ℓ_1.

Now if you examine figure 10-2a again, you will see that while ℓ_1 has reached its counterclockwise limit, ℓ_2 has not. We find ℓ_2 could continue to go counterclockwise until it reaches the position shown in figure 10-3. (Note that this figure is not to scale.)

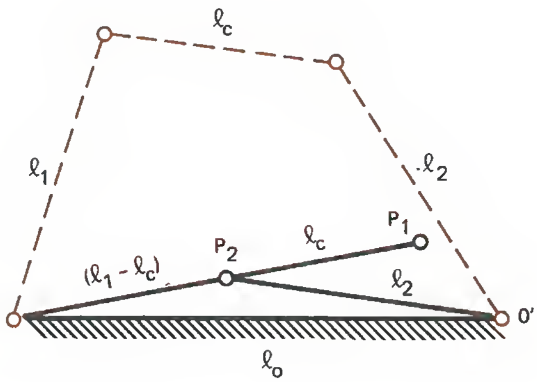


Fig. 10-3 $\ell_1 - \ell_c$ Limiting Position

If this kind of limiting is to occur, then the link lengths must satisfy the inequality

$$\ell_o < \ell_2 + \ell_1 - \ell_c$$

or

$$\boxed{\ell_o + \ell_c < \ell_2 + \ell_1} \qquad (10.5)$$

In other words, if we want the mechanism to have this type of limiting we choose link lengths which satisfy this condition. Conversely, this type of limiting cannot occur if $\ell_o + \ell_c$ is greater than $\ell_2 + \ell_1$.

It is also possible to construct a double-rocker mechanism which passes through the critical position shown in figure 10-4.

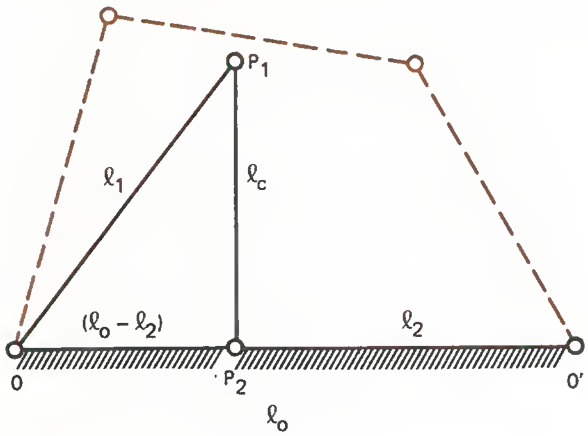


Fig. 10-4 $\ell_o - \ell_2$ *Critical Position*

In this case the link lengths must satisfy the inequality

$$\ell_o - \ell_2 < \ell_c + \ell_1$$

or

$$\boxed{\ell_o - \ell_c < \ell_2 - \ell_1} \quad (10.6)$$

As before, we can produce or prevent this condition by choosing link lengths which do or do not satisfy this condition.

There are other double-rocker limiting and critical conditions; however, they do not introduce new inequalities so we will not consider them at this time.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Bearing holders with bearings
2 Shaft hangers with bearings
2 Shafts 4" X 1/4"
2 Collars
2 Disk dials
2 Dial indexes with mounts
1 Lever arm 2 in. long with 1/4 in. bore hub
1 Lever arm 1 in. long with 1/4 in. bore hub
*1 Reverse link 3/4 in. long
1 Steel rule 6 in. long
2 Sheets of linear graph paper

*For link construction details see Appendix A.

PROCEDURE

1. Inspect each of your components to be sure they are undamaged.
2. Assemble the mechanism shown in figure 10-5.
3. Looking from the right in figure 10-5, move the longer lever arm to its clockwise limit. At this point set both dials to zero.
4. Move the input link (longer lever arm) from zero in 10-degree steps. Record both dial readings at each step. Continue in this way until you reach the counterclockwise limit of the longer lever arm. Take particular note of what occurs if the mechanism passes through a critical position.
5. Starting at the counterclockwise limit of the longer lever arm, slowly move it back toward zero in 10-degree steps. Again record both the input and output dial readings.
6. On a sheet of graph paper plot the input displacement versus output displacement for both sets of data.

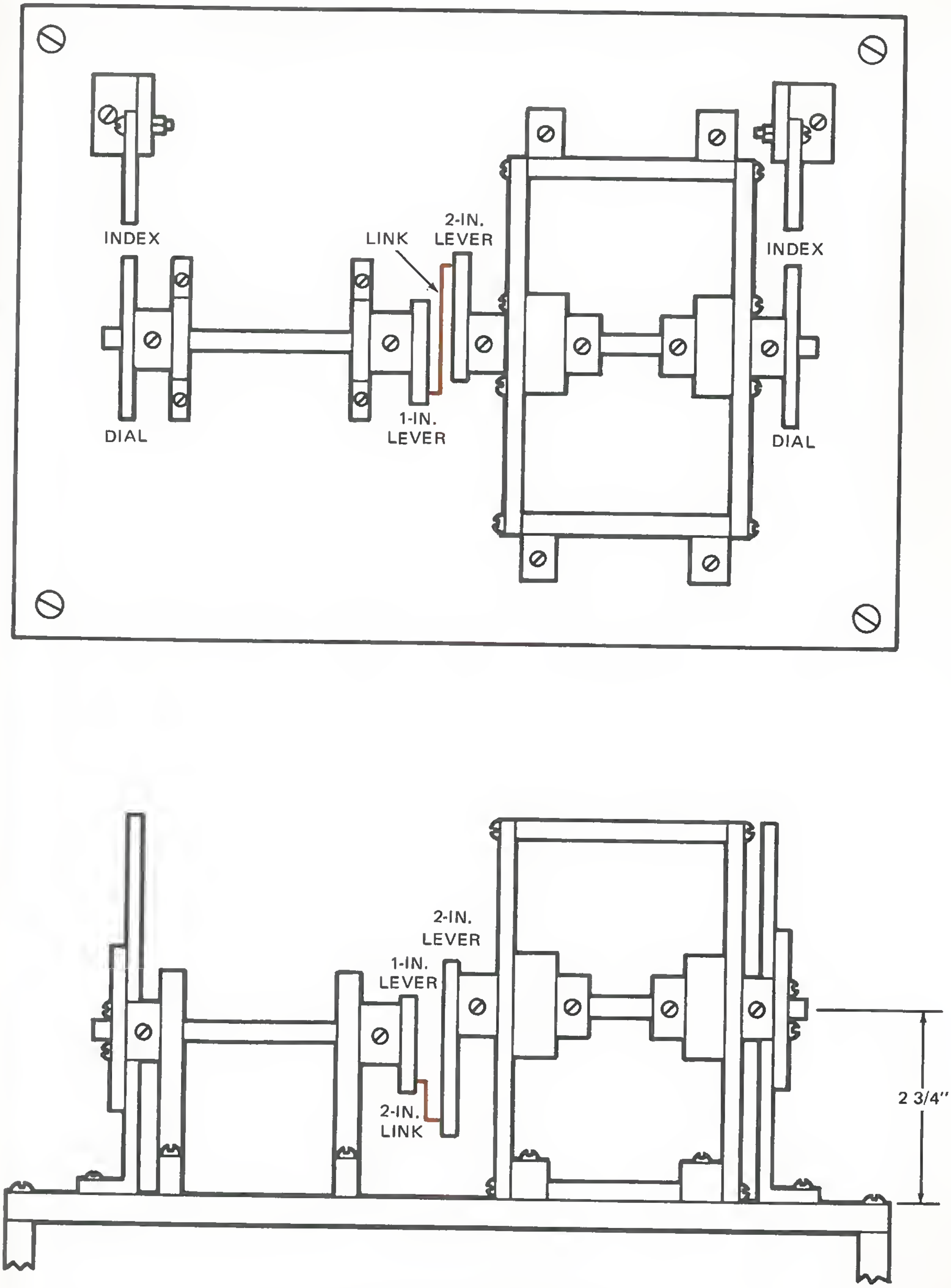


Fig. 10-5 The Experimental Setup

7. Measure and record the length of each linkage in the mechanism (ℓ_o, ℓ_1, ℓ_2, and ℓ_c).
8. Move the bearing plate assembly sideways until the shafts are separated by approximately 2-1/2 in..
9. Repeat steps 3 through 8.

ANALYSIS GUIDE. In analyzing the data from this experiment you should discuss the relative motion between the driver and follower in each case. Consider the effect of changing the shaft spacing. How can you explain this effect? Using your link dimensions you can verify the inequalities given in the discussion. Which ones were satisfied for each case? Make a drawing of each mechanism and discuss the limiting and critical conditions that you encountered.

Counterclockwise

Θ_1	Θ_2

ℓ_o =_____

ℓ_1 =_____

ℓ_2 =_____

ℓ_c =_____

Clockwise

Θ_2	Θ_2

Fig. 10-6 Data Table I

Counterclockwise

Θ_1	Θ_2

ℓ_o = ______

ℓ_1 = ______

ℓ_2 = ______

ℓ_c = ______

Clockwise

Θ_1	Θ_2

Fig. 10-7 Data Table II

PROBLEMS

1. The four-bar mechanism shown in figure 10-8 has the following dimensions: $\ell_1 = \ell_o = 14$ in., $\ell_2 = 10$ in., and $\ell_c = 8$ in.. Will this mechanism work as a double rocker?

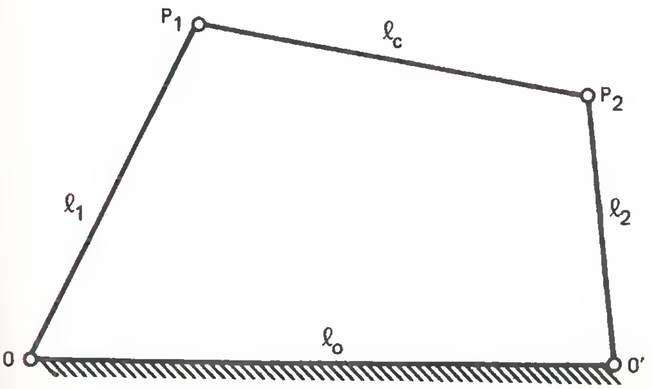


Fig. 10-8 A Four-Bar Mechanism

2. Can the mechanism in figure 10-8 assume an $\ell_1 - \ell_c$ limiting position like that shown in figure 10-3? If so, draw a sketch of it in that position.

3. Can the mechanism in figure 10-8 assume an $\ell_o - \ell_2$ critical position like that shown in figure 10-4? If so, make a sketch of it in that position.

4. Write the inequality that the link lengths in figure 10-9 must satisfy.

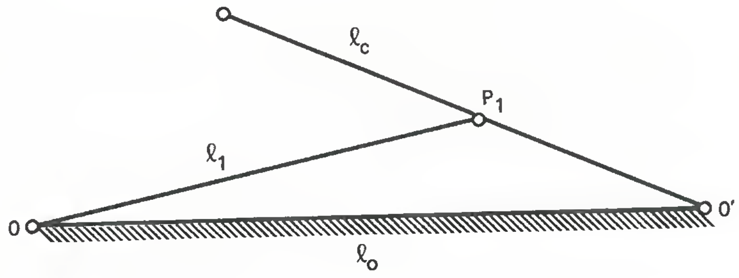


Fig. 10-9 Mechanism for Problem 4.

5. Show that your result in problem 4 is identical to inequality 10-5 in the discussion.

6. Write the inequality that must be satisfied if figure 10-10 is to exist.

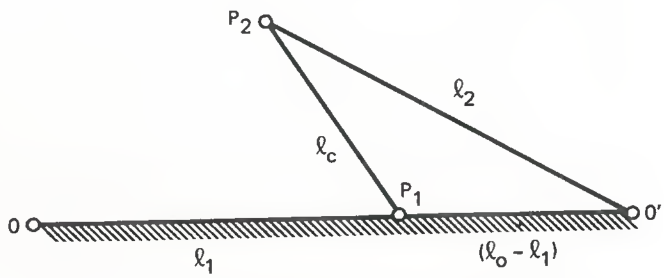


Fig. 10-10 Mechanism for Problem 6

7. Show that your result in problem 6 is identical to inequality 10-6 in the discussion.

experiment 11 FOUR–BAR SUMMARY

INTRODUCTION. The four-bar mechanism is considered by some specialists to be the basic linkage mechanism. In this experiment we shall examine the link length requirements for each of the four-bar mechanism classes.

DISCUSSION. A linkage is an assembly of mechanical components wherein the various members move relative to each other and each component has a prescribed form of motion.

A four-bar *mechanism* is composed of interconnected links, *one of which is fixed.* Figure 11-1 shows a schematic of such a mechanism. In this sketch ℓ_o is the fixed link, ℓ_1 is the input (or driver) link, ℓ_c is the coupling link, and ℓ_2 is the output (or driven) link. The connecting points 0, 0′, P_1, and P_2 are all free to allow relative rotation between the connected links.

If the input link, ℓ_1, can rotate through a full revolution and this causes the output link to rotate through only a part of a revolution, the mechanism is called a crank-rocker or type I mechanism. Figure 11-2 shows the relative motion of the input and output links of a crank-rocker mechanism.

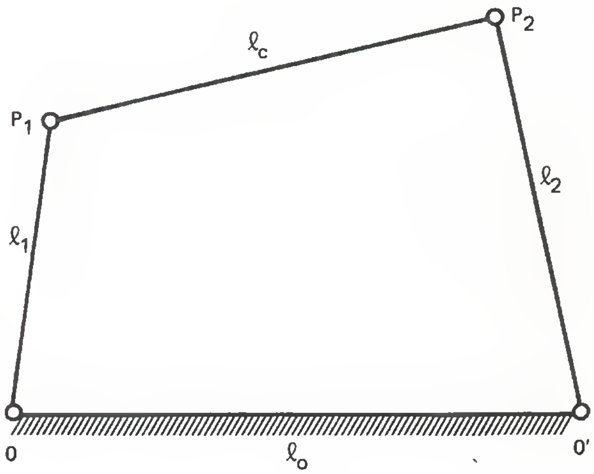


Fig. 11-1 A Four-Bar Mechanism

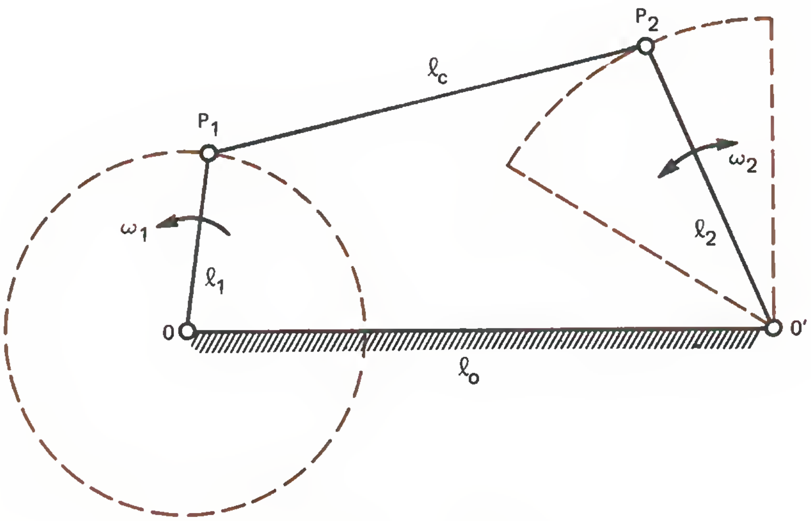


Fig. 11-2 A Crank-Rocker

In analyzing any four-bar mechanism, we first test the relative link length to insure that the mechanism is physically possible to build. We can do this by checking to see if the sum of the three shorter link lengths is greater than the longest link. If they aren't, then they simply won't reach the required connecting points. If the mechanism is possible to build, we then proceed with an analysis of the *critical positions.*

Critical positions are those positions in which two (or more) of the links are colinear. That is, two or more links are lined up with each other. In the crank-rocker mechanism shown in figure 11-2 there are four possible critical positions. Figure 11-3 shows all four of these positions. Notice that as ℓ_1 rotates counterclockwise, it eventually becomes colinear with ℓ_o as shown in figure 11-3a. When this occurs, the output link ℓ_2 is still moving counterclockwise.

As the input link continues its rotation, it soon becomes colinear with ℓ_c as shown in figure 11-3b. At this time ℓ_2 has traveled as far as it can in the counterclockwise direction. In other words, ℓ_2 has reached the limit of its counterclockwise travel. This position could be called the counterclockwise limiting position.

As ℓ_1 continues its rotation counterclockwise, it finally reaches the position shown in figure 11-3c. In this position ℓ_1 is again colinear with ℓ_o, and ℓ_2 is moving clockwise.

Finally, ℓ_1 reaches the position shown in figure 11-3d where it is again colinear with ℓ_c. This time ℓ_2 has reached its clockwise limiting position.

In each of the critical positions we can write mathematical statements about the relative link lengths. For example, in figure 11-3a we see that $\ell_c + \ell_2$ must be greater than $\ell_o + \ell_1$. Otherwise the mechanism could not assume the position shown. Stated symbolically this is

$$\ell_c + \ell_2 > \ell_o + \ell_1$$

For any one of the critical positions we can write three of these statements. The other two for figure 11-3a are

$$\ell_2 < \ell_o + \ell_1 + \ell_c$$

and

$$\ell_c < \ell_o + \ell_1 + \ell_2$$

If we were to write down the three statements relating the lengths of the linkages for all four critical positions, we would have a total of twelve separate inequalities. Among these twelve statements there would be a number of duplications. In fact there would be only six different inequalities. Of these six relationships, three of them would indicate that one link was shorter than the sum of the other three links. This is, of course, the condition that we test for when we consider the physical possibility of a mechanism.

The remaining three inequalities are the only ones that are useful for linkage analysis. These three relationships are:

$$\ell_1 < \ell_c + \ell_2 - \ell_o \tag{11.1}$$

$$\ell_1 < -\ell_c + \ell_2 + \ell_o \tag{11.2}$$

$$\ell_1 < \ell_c - \ell_2 + \ell_o \tag{11.3}$$

We can get some further insight into the relative link length requirements by manipulating these relationships. For example, if we add 11.1 and 11.2 we have

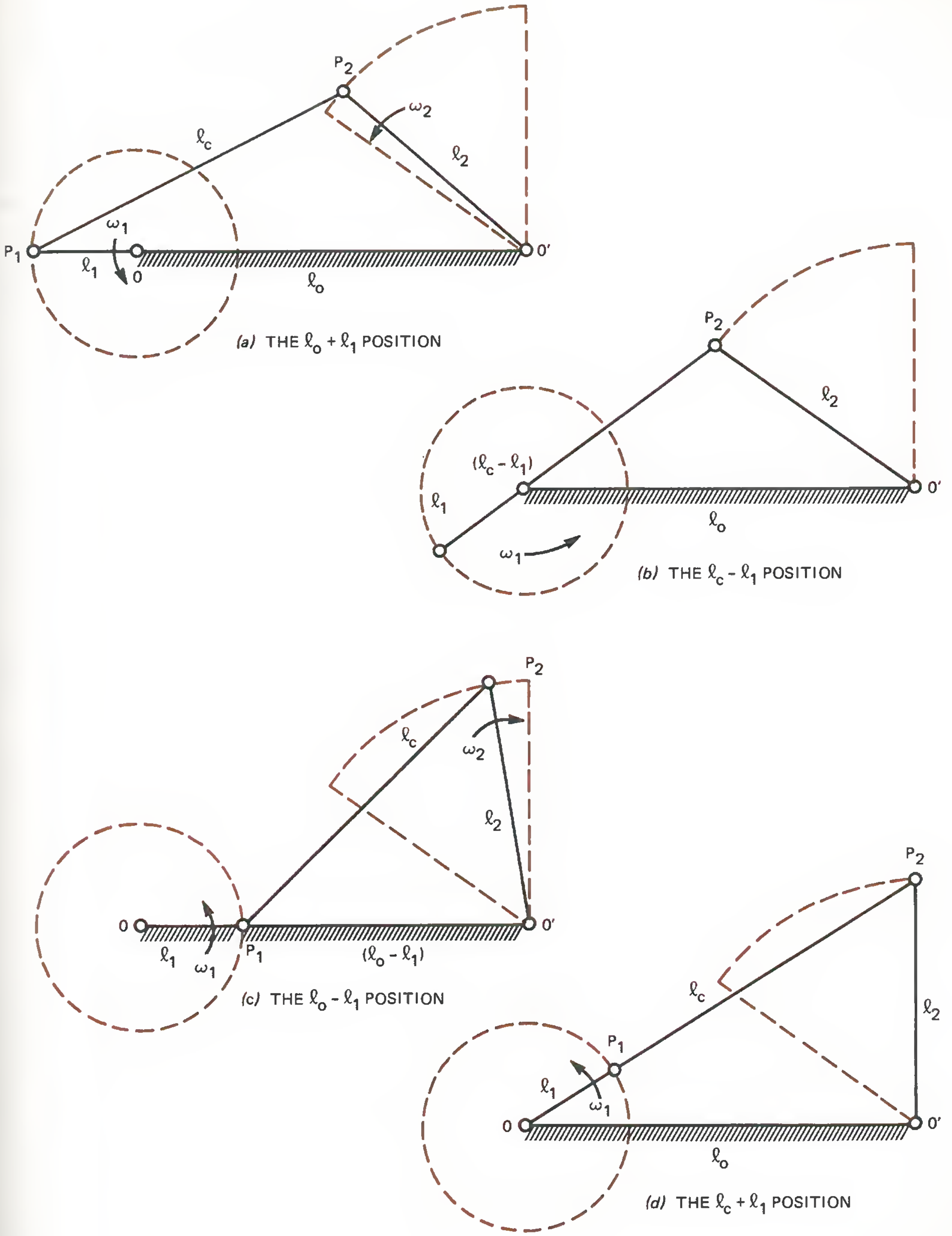


Fig. 11-3 Crank-Rocker Critical Positions

$$2\ell_1 < 2\ell_2 \text{ or } \ell_1 < \ell_2 \tag{11.4a}$$

Similarly, adding 11.1 and 11.3

$$2\ell_1 < 2\ell_c \text{ or } \ell_1 < \ell_c \tag{11.4b}$$

And finally, adding 11.2 and 11.3

$$2\ell_1 < 2\ell_o \text{ or } \ell_1 < \ell_o \tag{11.4c}$$

Comparing these three relationships we observe that ℓ_1 *must* be the shortest link in a crank-rocker mechanism.

We can rearrange inequalities 11.1, 11.2, and 11.3 into the forms

$$\ell_o < \ell_c + \ell_2 - \ell_1$$

$$\ell_c - \ell_2 + \ell_1 < \ell_o$$

$$\ell_2 - \ell_c + \ell_1 < \ell_o$$

The first of these indicates that ℓ_o is shorter than the sum of ℓ_c and ℓ_2 less ℓ_1.

On the other hand, the second and third inequalities can be taken together to mean that ℓ_o is longer than ℓ_1 plus the difference between ℓ_c and ℓ_2. We can state both these conditions as

$$\boxed{|\ell_c - \ell_2| + \ell_1 < \ell_o < \ell_c + \ell_2 - \ell_1} \tag{11.5}$$

This relationship, together with the shortest link being ℓ_1, make up the conditions necessary for a crank-rocker mechanism.

The mechanism shown in figure 11-4 is a type II or drag link mechanism. When the input link (ℓ_1) of this mechanism makes a revolution, the output link (ℓ_2) also goes through a complete revolution.

Analyzing the drag-link mechanism using the critical positions, we find that the fixed

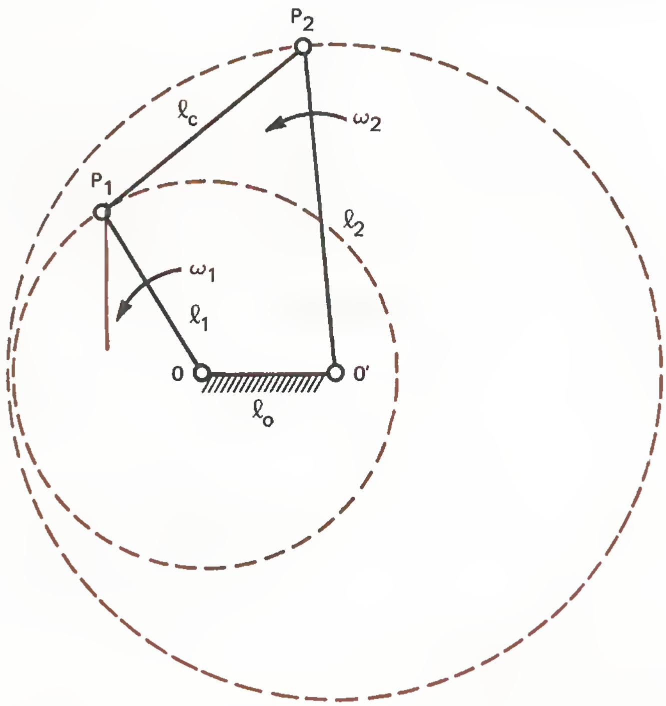


Fig. 11-4 A Drag-Link Mechanism

link (ℓ_o) must be shortest and that the inequality

$$\boxed{|\ell_c - \ell_1| + \ell_o < \ell_2 < \ell_c + \ell_1 - \ell_o} \quad (11.6)$$

must be satisfied.

Notice that in both of the mechanisms considered so far, we get a good hint about operation simply by identifying the shortest link.

The type III four-bar mechanism shown in figure 11-5 is sometimes called a double-rocker mechanism. Critical position analysis reveals that there are three alternate ways to build one of these mechanisms.

If the connecting link (ℓ_c) is the shortest we will have a double-rocker. On the other hand, if the input link (ℓ_1) is the shortest but inequality 11.5 is *not* satisfied, then we will have a double-rocker instead of a drag-link.

It should be noted that either a crank-rocker or drag link can be used as a double-rocker simply by confining the input link in some way.

A type IV mechanism having an indefinite motion relationship is produced whenever an equal sign appears in either inequality 11.5 or 11.6 instead of an inequality sign. Such a mechanism may operate in either type I, type II or type III depending on outside reconstraints and load conditions.

In conslusion, we see that we can classify four-bar mechanisms by identifying the shortest link and testing inequalities 11.5 and 11.6.

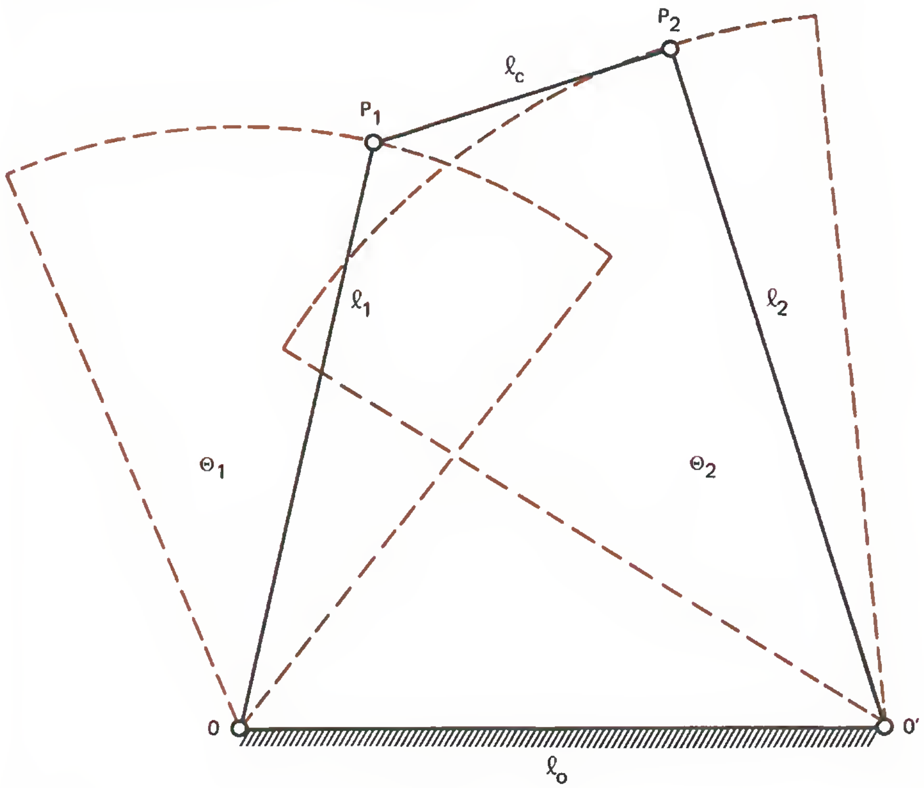


Fig. 11-5 A Double-Rocker Mechanism

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 2 Bearing holders with bearings
- 2 Shaft hangers 1-1/2 in. with bearings
- 2 Shafts 4" X 1/4"
- 4 Collars
- 1 Lever arm 2 in. long with 1/4 in. bore hub
- 1 Lever arm 1 in. long with 1/4 in. bore hub
- *1 Reverse link 2 in. long
- *1 Reverse link 3/4 in. long
- 1 Steel rule 6 in. long

*For link construction details see appendix A.

PROCEDURE

1. Consider a mechanism having link lengths of:

$$\ell_1 = 1.0 \text{ in.} \qquad \ell_2 = 2.0 \text{ in.}$$

$$\ell_o = .75 \text{ in.} \qquad \ell_c = 2.0 \text{ in.}$$

2. Test this mechanism to insure that it is physically possible. Show your work.
3. Using the methods described in the discussion, classify the mechanism by type. Show your work and your conclusion.
4. Using components from the materials list, construct a mechanism having the link lengths specified in step 1.
5. Examine the operation of the mechanism. Does it agree with your classification?
6. Slowly go through one complete input motion cycle. Stop at each critical position and make a sketch of the mechanism.
7. Write three valid inequalities for each critical position.
8. Repeat steps 1 through 7 for a mechanism having linkages of:

$$\ell_1 = 1.0 \text{ in.} \qquad \ell_2 = 2.0 \text{ in.}$$

$$\ell_o = 1.5 \text{ in.} \qquad \ell_c = .75 \text{ in.}$$

9. Repeat steps 1 through 7 for a mechanism having linkages of:

$$\ell_1 = 1.0 \text{ in.} \qquad \ell_2 = 2.0 \text{ in.}$$

$$\ell_o = 2.5 \text{ in.} \qquad \ell_c = 2.0 \text{ in.}$$

ANALYSIS GUIDE. In analyzing your results from this exercise you should consider which of the inequalities you wrote would be useful in analyzing the corresponding mechanism. Discuss any instability you observed in the mechanisms at the critical positions.

PROBLEMS

1. Write the twelve inequalities for the critical positions shown in figure 11-3.
2. Which of the inequalities in problem 1 simply state that one link must be shorter than the sum of the remaining links?
3. Draw sketches showing four critical positions of a type II four-bar mechanism.
4. Repeat problem 1 for the mechanism in problem 3.
5. Repeat problem 2 for the mechanism in problem 3.
6. Which inequalities in problem 3 are the most useful in analyzing the mechanism?
7. Using your inequalities from problem 6, show that ℓ_0 must be the shortest link in a drag-link.

experiment 12 FOUR-BAR PROBLEM

INTRODUCTION. Mechanisms are almost always designed to have specific input/output characteristics. In this experiment we shall examine one design approach which can be used with double-rocker mechanisms.

DISCUSSION. A common problem in the design of mechanisms is that of converting one oscillatory motion into another. Any type of four-bar mechanisms will accomplish this purpose and this approach is usually the simplest and most logical. Let's assume the two levers that are to rock are 8 inches apart and the driving lever is 12 inches long. When the driving lever swings 20 degrees, we desire the driven lever to swing 30 degrees.

The problem is to find the length of the driven rocker and the length of the coupler link. The given elements of the problem are shown in figure 12-1. The driver link is identified as link ℓ_1 and the distance between the two rocker pivots is labeled link ℓ_o and is shown as the frame. We will assume that the initial position of the driven link, ℓ_2, is parallel to link ℓ_1.

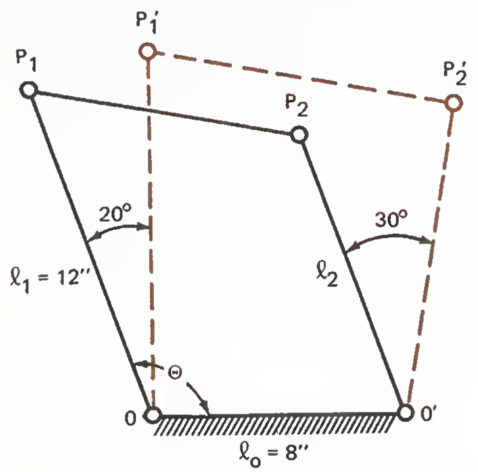


Fig. 12-1 Double-Rocker Design Problems

To solve this problem we will invert the mechanism by assuming that link ℓ_2 remains fixed and rotate the frame (link ℓ_o) 30 degrees counterclockwise.

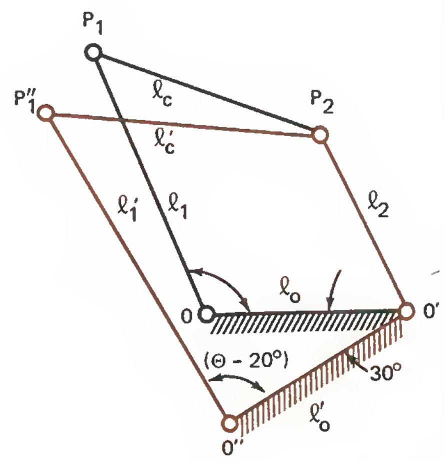


Fig. 12-2 Double-Rocker Problem Solution

The position of point 0″ will be in its proper relationship to link ℓ_2 as if link ℓ_2 had rotated 30 degrees clockwise. From this point 0″, link ℓ_1 is drawn in its second position (angle Θ – 20 degrees).

Point P_1 in this "inverted" position of link ℓ_1 is labeled P''_1 as shown in figure 12-2. The next step is to connect point P_1 to point P''_1. You might note that point P_1 is in the proper relative position to link ℓ_2 after both links have rotated through their designated number of degrees.

The perpendicular bisector of line P_1-P''_1 is next drawn. On this line will be found the

center of any circle passing through the two relative positions of P_1. Therefore, the intersection of the line and link ℓ_2 gives us point P_2, solving the linkage problem. The completed mechanism shown in its two desired positions is illustrated in figure 12-3. In a practical situation the next step would be to check the solution by assembling and trying out the design.

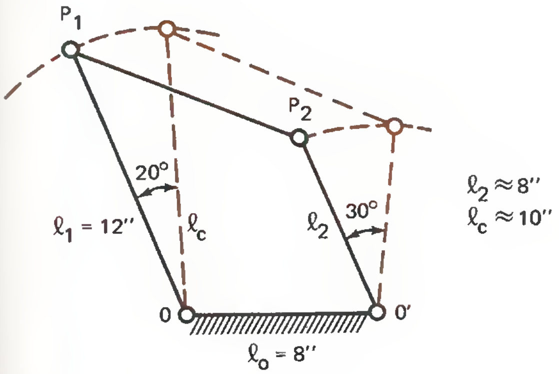


Fig. 12-3 Completed Double-Rocker Design

One of the most useful analytical equations for coordinating the motions of two levers is the Freudenstein equation — named for the engineer who derived it. The previous problem was solved using geometrical layout techniques, and with two positions of the levers, this approach is fairly straightforward. Although you finish with only close approximations, the geometrical approach is usually accurate enough for most purposes. Increasing demands for precision require that a more analytical approach be taken.

Figure 12-4 shows a general layout for any four-bar mechanism. The angular positions that we are usually interested in are those labeled Θ and θ. Angle α is labeled in the figure and is used in the derivation of the Freudenstein equation.

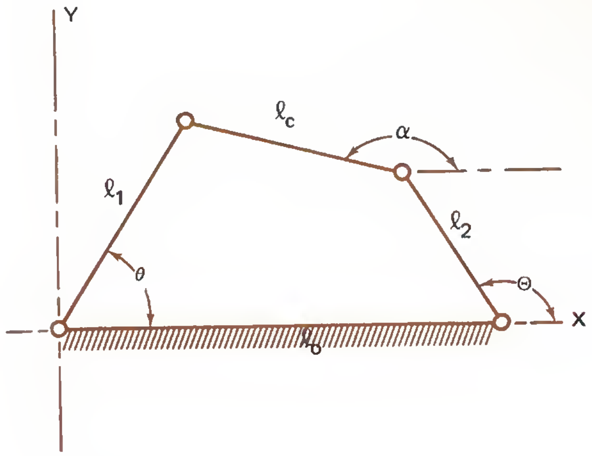


Fig. 12-4 Reference Diagram for Freudenstein's Equation

If we consider each of the links as a vector, we know that the sum of the X components must equal zero:

$$\ell_c \cos\alpha - \ell_1 \cos\theta + \ell_o + \ell_2 \cos\Theta = 0 \tag{12.1}$$

Also, the sum of the Y components must equal zero:

$$\ell_c \sin\alpha - \ell_1 \sin\theta + \ell_2 \sin\Theta = 0 \tag{12.2}$$

Squaring and rearranging equations 12.1 and 12.2 gives the following expressions:

$$\ell_c^2 \cos^2\alpha = (\ell_1 \cos\theta - \ell_o - \ell_2 \cos\Theta)^2 \tag{12.3}$$

$$\ell_c^2 \sin^2\alpha = (\ell_1 \sin\theta - \ell_2 \sin\Theta)^2 \tag{12.4}$$

Expanding both sides of equations 12.3 and 12.4, then adding gives

$$\ell_c^2 = \ell_1^2 + \ell_o^2 + \ell_2^2 - 2\ell_1\ell_o \cos\theta$$
$$- 2\ell_2\ell_1 \cos\theta \cos\Theta - 2\ell_2\ell_1$$
$$\sin\theta \sin\Theta - 2\ell_2\ell_o \cos\Theta$$

By rewriting this equation, we derive the Freudenstein equation

$$\boxed{K_1 \cos \Theta - K_2 \cos \theta + K_3 = \cos(\Theta - \theta)} \quad (12.5)$$

where:

$$K_1 = \ell_o/\ell_1$$

$$K_2 = \ell_o/\ell_2$$

$$K_3 = (\ell_o^2 + \ell_2^2 - \ell_c^2 + \ell_1^2) / (2\ell_2\ell_1)$$

This equation may be used to solve linkage mechanisms when you desire three different positions of both of the rocker arms. Three different angular positions inserted into equation 12.5 would give you three equations in three unknowns. You can then solve for K_1, K_2, and K_3. Knowing, or assuming, the value of one length of a link permits the complete solution.

Let's use this equation to solve the problem previously solved by geometric means. Figure 12-5 restates the problem using the linkage nomenclature used in Freudenstein's equation.

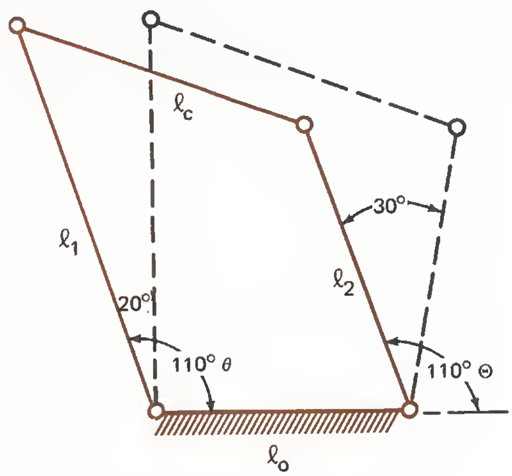


Fig. 12-5 Double-Rocker Problem Relabeled

In our problem we assumed a position for the driver link, now called link ℓ_1, and we assume the driven link, now called link ℓ_2 to be parallel to link ℓ_1. The initial angles are: $\theta = 100°$ and $\Theta = 110°$. The movement of link ℓ_1 is 20 degrees clockwise and we desired a corresponding movement of link ℓ_2 through 30 degrees. The second set of angular positions is: $\theta = 90°$ and $\Theta = 80°$.

Substituting the first set of angles into the Freudenstein equation (12.5) gives

$$\cos 110° = -.34202$$

$$\therefore K_1(\cos 110) - K_2(\cos 110) + K_3$$
$$= \cos(110 - 110)$$
$$= -0.34202K_1$$
$$+ 0.34202K_2 + K_3 = 1$$

And since $K_1 = 0.66667$, $(K_1 = \ell_o/\ell_1 = 8/12)$, then

$$0.34202K_2 + K_3 = 1.22801 \quad (12.6)$$

Using the second set of angular positions gives

$$(0.66667)(\cos 80) - K_2(\cos 90) + K_3$$

$$= (80 - 90)$$

From which we find that

$$K_3 = 0.86904 \quad (12.7)$$

Substituting equation 12.7 into 21.6 gives

$$K_2 = 1.04955$$

Since $K_2 = \ell_o/\ell_2$

$$\ell_2 = \frac{8}{1.04955} = 7.6223$$

Using the expression for K_3 given with equation 12.5 determines that

$$\ell_c = 10.358$$

You can see that these values compare rather closely with those determined by graphical means shown in figure 12-3. You now have a technique of determining quite accurately the lengths of linkages when angular positions are known. This technique may also be used to determine the mechanism if three sets of angular relationships are given. Inserting the corresponding values of θ and Θ will give you three separate equations in the three unknowns, K_1, K_2, and K_3. Applying these values to a known or to an assumed linkage length will give the rest of the mechanism dimensions.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
4 Bearing holders with bearings
2 Shafts 4" X 1/4"
2 Disk dials
2 Dial indexes with mounts
1 Lever arm 1 in. long with 1/4 in. bore hub
1 Lever arm 2 in. long with 1/4 in. bore hub
1 Wire link (length determined by student)
1 Steel rule 6 in. long

PROCEDURE

1. A certain double-rocker application has an input link (ℓ_1) of 2 in. and an output link (ℓ_2) of 1 in. The two links are parallel when they are vertical. When the input link rotates 45° clockwise, the output link rotates 90°. Using the Freudenstein equation, determine ℓ_o and ℓ_c. Record your results in the data table.
2. Verify graphically that your values are correct. Turn in your graphical verification with your data.
3. Construct a straight wire link to use for ℓ_c. Construction details may be found in Appendix A.
4. Assemble the mechanism.
5. Attach dials to the input and output shafts. Set them to indicate 90° when both lever arms are pointing vertically upward.
6. Rotate the input dial clockwise in 5° steps. Record both input and output angle at each point until you have covered the range specified in step 1.
7. Plot a curve of input versus output angular displacement.
8. Reset the dials and indexes so that they read 90° when the levers are pointing straight down.
9. Repeat steps 6, and 7.

ANALYSIS GUIDE. In your analysis of these data you should discuss the extent to which the mechanism satisfies the original requirements. Was the input to output relationship linear? Did the two data runs agree with each other?

$\ell_o =$		$\ell_c =$	
First Trial		Second Trial	
θ	Θ	θ	Θ

Fig. 12-6 The Data Table

PROBLEMS

1. What type of mechanism was used in the experiment?
2. Give three different practical uses for the double-rocker mechanism.
3. Show how equation 12.5 is obtained from the previous equation given in the discussion of this experiment.
4. Using either graphical or analytical techniques, design a four-bar mechanism that will place the follower lever through 15 degrees as the driver lever goes through 26 degrees; then, as the driver lever goes another 33 degrees, the follower moves through 11 degrees. In all cases the angular movement is in the clockwise direction.

experiment 13 SLIDER CRANK MECHANISMS

INTRODUCTION. Another type of four link mechanism commonly used in engines, pumps, and compressors is the slider-crank mechanism. One application of this type mechanism that you are familiar with is that of the piston and the crank shaft in your automobile engine. In this experiment we will examine the mechanical details of this mechanism.

DISCUSSION. Figure 13-1 shows four possible versions of a slider crank linkage. Sketch (a) illustrates the common mechanism used in engines and pumps where the guide or frame is fixed and the block moves along it. The other three alternatives have the block sliding along a link but one of the other links is fixed. These alternatives will not be analyzed in this experiment but you will occasionally see them used in different applications.

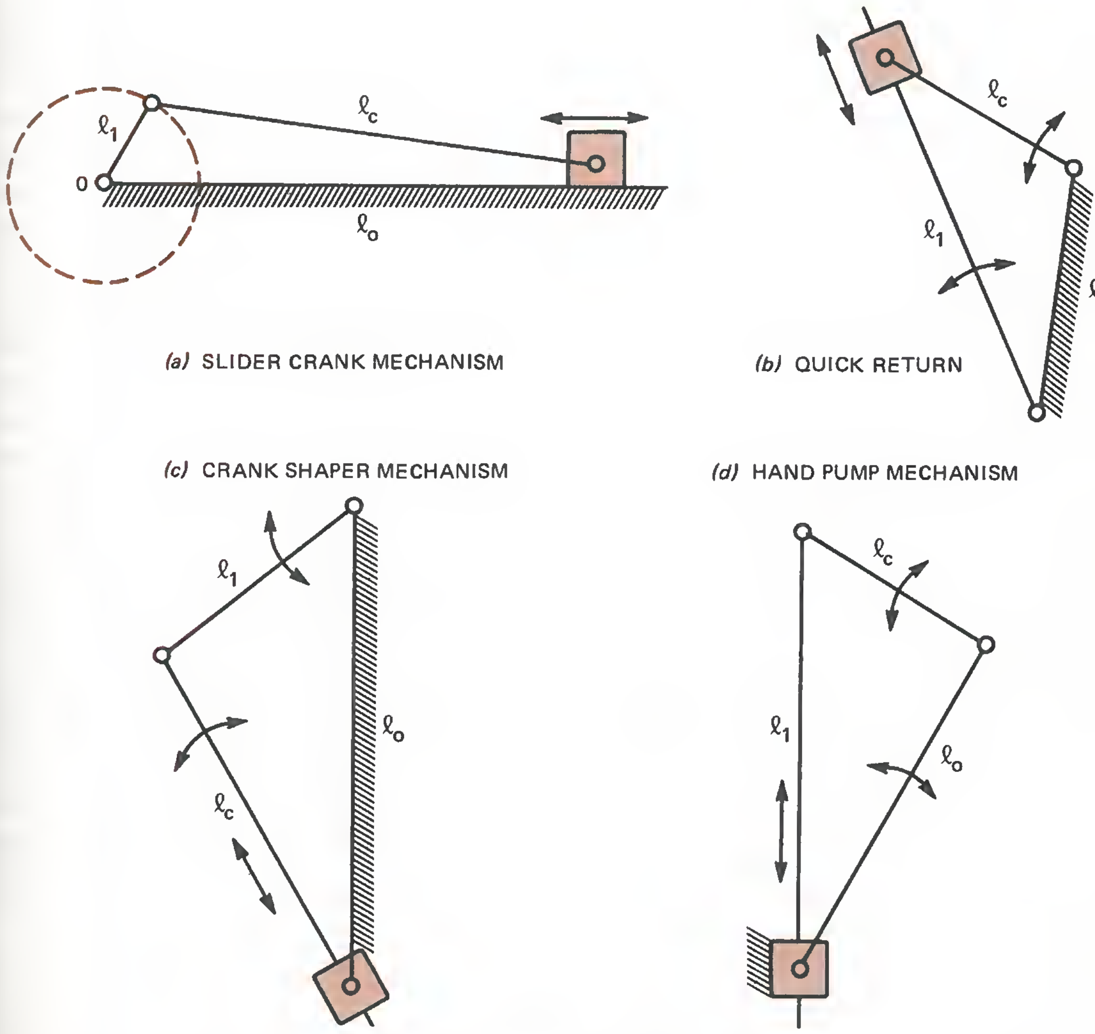


Fig. 13-1 Different Versions of a Slider-Crank Linkage

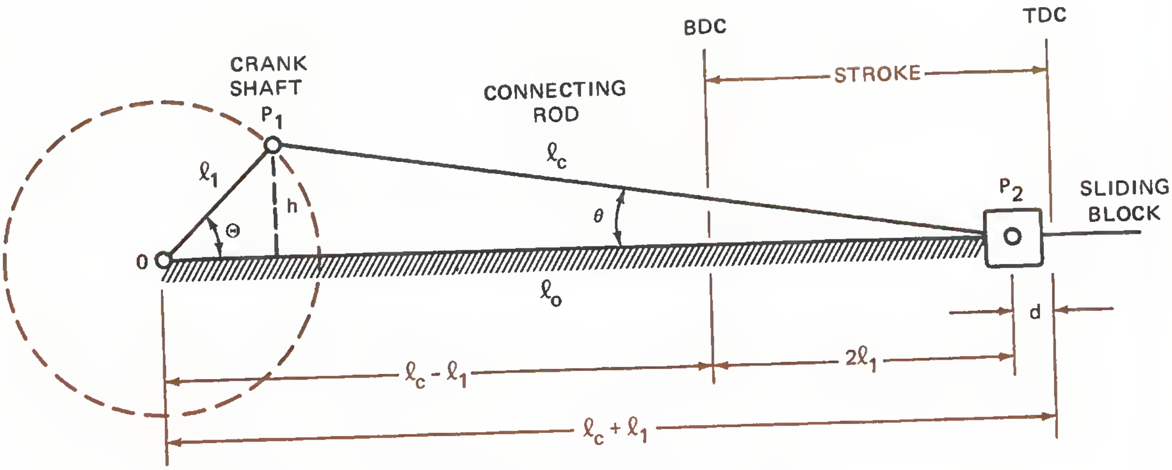


Fig. 13-2 Slider Crank Notation

Figure 13-2 illustrates the mechanism to be discussed in this experiment. The connection to the sliding block is known as the *wristpin.* The crank in figure 13-2 is labeled ℓ_1 and its center of rotation is labeled 0. When the wristpin is at its travel farthest away from the crank center, it is said to be at *top dead center.* When it is closest to the crank center, it is said to be at *bottom dead center.* The difference between top dead center (TDC) and bottom dead center (BDC) is known as the *stroke.* When the crank center is on the same line of motion as the slider, the TDC position from the crank center equals

$$TDC = \ell_1 + \ell_c \qquad (13.1)$$

The position of the wristpin at BDC equals

$$BDC = \ell_c - \ell_1 \qquad (13.2)$$

The stroke equals the difference between TDC and BDC as expressed in equations 13.1 and 13.2

$$\text{Stroke} = TDC - BDC = (\ell_1 + \ell_c) - (\ell_c - \ell_1) = 2\ell_1 \qquad (13.3)$$

The slider crank mechanism, like most four link mechanisms, can be analyzed for most practical purposes by using graphical techniques. This usually consists of a scale drawing showing the extreme positions as well as known intermediate dimensions. The appearance of the computer upon the industrial scene has made fairly complicated equations much easier to solve. For example, if you have a complicated equation expressing the motion of the slider crank involving several trigonometric functions, the computer can readily be programmed to solve this equation for many small increments of angular displacement of the crank. In just a few minutes after it is programmed, the computer can give you a tabulation of displacements, velocities, and accelerations. Let's look at a way of analyzing the motion of the slider block shown in figure 13-2.

As shown, we will let the ratio of the connecting rod, ℓ_c, to the crank, ℓ_1, be equal to k; that is,

$$k = \frac{\ell_c}{\ell_1}$$

and since

$$h = \ell_1 \sin \Theta = \ell_c \sin \theta$$

then

$$\sin\theta = \frac{\sin\Theta}{k} \tag{13.4}$$

The movement of the slider, d, is

$$\boxed{\begin{aligned} d &= (\ell_c + \ell_1) - \ell_1 \cos\Theta - \ell_c \cos\theta \\ &= \ell_1 (1 - \cos\Theta) + \ell_c (1 - \cos\theta) \end{aligned}} \tag{13.5}$$

From trigonometry we know that $\cos\theta = \sqrt{1 - \sin^2\theta}$. Substituting the value of $\sin\theta$ from equation 13.4 into this gives

$$\cos\theta = \sqrt{(1 - \frac{\sin^2\Theta}{k^2})}$$

Therefore, substituting into equation 13.5

$$d = \ell_1 (1 - \cos\Theta) + \ell_c \left(1 - \sqrt{1 - \frac{\sin^2\Theta}{k^2}}\right) \tag{13.6}$$

Although equation 13.6 does give an exact expression for the slider displacement, it is difficult to use this for many values of the crank angle, Θ, without the use of a computer. A close approximation to equation 13.6 is more commonly used and the error is quite small. It should be noted that the slider displacement, d, is measured from TDC and is considered positive in these equations.

$$\boxed{d \cong \ell_1 (1 - \cos\Theta) + \left(\frac{\ell_1^2}{2\ell_c} \sin^2\Theta\right)} \tag{13.7}$$

You can see that you can quite readily solve equation 13.7 for given values of crank angular displacement.

Since the error is quite small, we shall also use equation 13.7 to determine approximate solutions to the slider velocity and acceleration for given crank angle positions.

The velocity of the slider equals the first derivative of 13.7, or

$$\boxed{V_P = \ell_1 \omega \left(\sin\Theta + \frac{\sin^2\Theta}{2k}\right)} \tag{13.8}$$

and acceleration equals the first derivative of equation 13.8, or the second derivation of equation 13.7 which equals

$$\boxed{a_P = \ell_1 \omega^2 \left(\cos\Theta + \frac{\cos 2\Theta}{k}\right)} \tag{13.9}$$

In equation 13.8 and 13.9, ω is the angular velocity of the crank link, ℓ_1, and positive values indicate counterclockwise rotation of link ℓ_1. Corresponding positive values of V_P and a_P indicate that the velocity or acceleration is toward the crank center 0.

As the crank rotates, the connecting rod will oscillate around point P_2, first in a clockwise and then in a counterclockwise direction. Normally the linkage (ℓ_c) will have an angular velocity and an angular acceleration. Keeping in mind that both velocity and acceleration are vector quantities, the relative motion is the difference in motion between two points. Velocity at point P_1 and point P_1 with respect to point P_2 is found by

$$V_{P_1P_2} = V_{P_10} - V_{P_20}$$

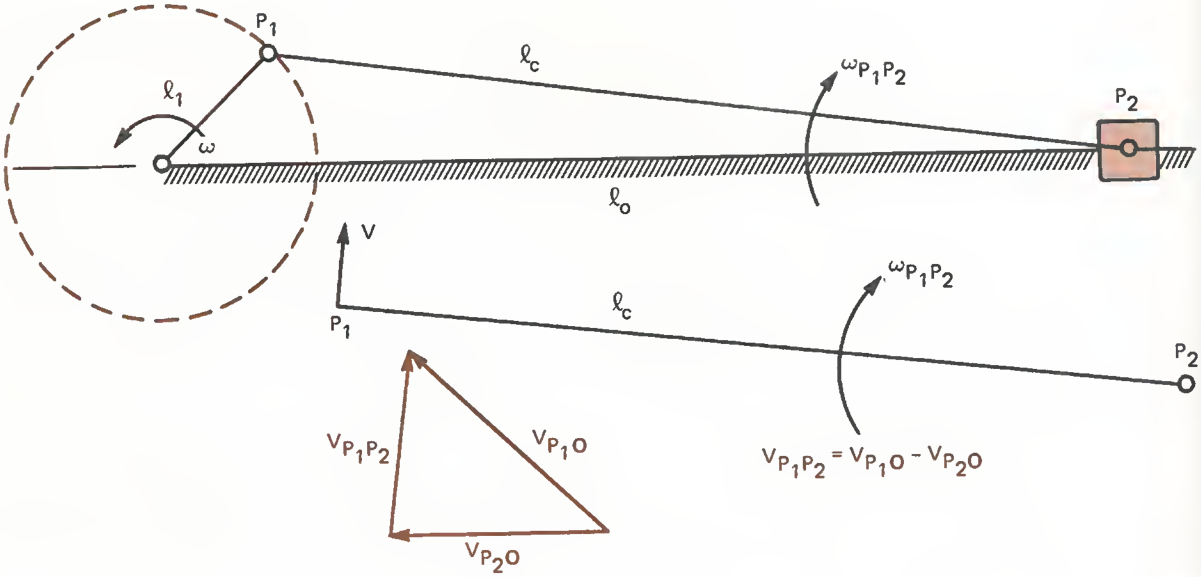


Fig. 13-3 Slider Block Velocities

This relationship is shown in figure 13-3.

Knowing the directions and two of the values, you can use graphical techniques as shown in the preceding figure to solve the various velocities. However, there are more accurate analytical methods. The following formulas are exact equations for calculating the angular velocity and acceleration of the connecting rod.

$$\boxed{\omega_{P_1P_2} = 1 \frac{\omega \cos \Theta}{(k^2 - \sin^2 \Theta)^{1/2}}} \tag{13.10}$$

$$\boxed{\alpha_{P_1P_2} = \frac{\omega^2 (k^2 - 1) \sin \Theta}{(k^2 - \sin^2 \Theta)^{2/3}}} \tag{13.11}$$

When you solve the angular velocity of the connecting rod (link ℓ_c), you can then determine the velocity of the rod at point P_1 by

$$V_{P_1P_2} = \ell_c \times \omega_{P_1P_2}$$

remembering that angles are measured in radians and not degrees.

Another item that is of frequent interest is the maximum velocity of the slider and just where this maximum velocity occurs. A simple formula has been developed which possesses accuracy sufficient for most purposes so long as the ratio of lengths between ℓ_c and ℓ_1 ($\ell_c/\ell_1 = k$) is greater than 1.5. This formula gives the crank angle Θ present when the slider reaches maximum velocity:

$$\boxed{\Theta = \arc\cos \frac{1}{(k^2 + 3)^{1/2}}} \tag{13.12}$$

This equation is accurate to within one minute of the correct angle when the ratio ℓ_c/ℓ_1 is 4.0.

Formulas 13.1 through 13.12 will enable you to analyze nearly all aspects of existing slider crank mechanisms. It might be wise to briefly discuss the offset slider crank mechanism before concluding this discussion. This situation is illustrated in figure 13-4. In this figure the offset distance is indicated by the letter "y". Right triangle solutions give the expressions indicated for the distances shown in the diagram.

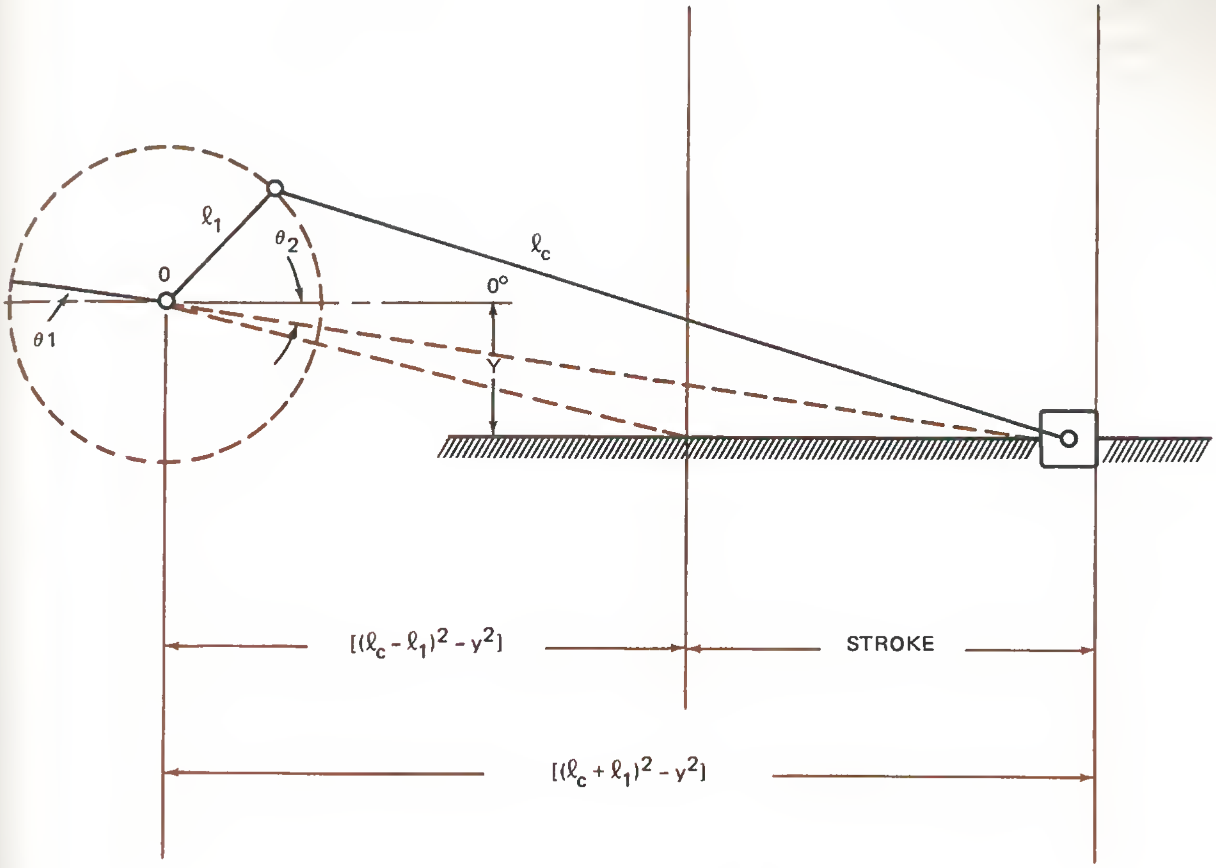


Fig. 13-4 Offset Slider Crank Mechanism

In figure 13-4 the angle of the crank when the slider is at TDC is designated as θ_2 while at BDC it is θ_1. You can see that the motion of the slider does not occur simply as the crank rotates 180 degrees. Motion of the slider as it moves to the right begins when link ℓ_1 is located at the upper arm of angle θ, and ends when the crank is at the lower arm of the angle θ_2. If we call this angular displacement of the crank Θ_R and the angular displacement of the crank for the left motion of the slider Θ_L, then we see

$$\Theta_R = 180^\circ + \theta_1 - \theta_2$$

and

$$\Theta_L = 180^\circ - \theta_1 + \theta_2$$

You can see from the figure also that the following relationships are true;

$$\theta_1 = \text{arc sin} \frac{y}{\ell_c - \ell_1}$$

$$\theta_2 = \text{arc sin} \frac{y}{\ell_c + \ell_1}$$

It is obvious that the offset distance, y, must be less than the distance $\ell_c - \ell_1$ for this mechanism to function. It bears repeating that graphical drawings to scale of mechanisms will often be sufficient for practical purposes. As with all engineering type problems, the sketch can serve as a check on your analytical computations.

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 2 Bearing mounts with bearings
- 2 Shafts 4" X 1/4" in.
- 4 Collars
- 2 Shaft hangers with bearings
- 1 Lever arm 1 in. long with 1/4 in. bore hub
- 1 Rigid shaft coupling
- 1 Disk dial
- 1 Dial index with mount
- *1 Wire loop link 3 in. long
- 1 Steel rule 6 in. long
- 1 Spacer No. 6 X 1/8 in. long X 1/32 in. wall thickness
- 1 Screw 6-32 X 1/4 in. round head

*For details of wire link construction see appendix A.

PROCEDURE

1. Inspect each of your components to be sure they are undamaged.
2. Assemble the mechanism shown in figure 13-5.
3. Turn the lever shaft several times to insure that the slider moves freely. It may be desirable to lubricate the slider shaft.
4. Adjust the bearing plate assembly so that the lever shaft and the slider shaft are the same height above the breadboard.
5. Set the lever arm so that it is pointing directly toward the slider, then set the disk dial to read zero.
6. Lay the steel rule across the shaft hangers so that its zero end lines up with the end of the slider shaft. Tape the rule in place if necessary.
7. Starting at zero on the dial, record the lever angle (Θ) and the slider displacement (X) every 20° for a complete revolution of the lever.
8. Measure and record the lengths of the lever arm (ℓ_1) and coupling link (ℓ_c).
9. For each data point (Θ, X) compute and record the distance (d) that the slider has moved from TDC.
10. Using the lengths of the lever and wire link, compute and record the value of K for this mechanism.
11. Use equation 13.6 and your values of Θ, ℓ_1, ℓ_c, and K to calculate d for each data point.

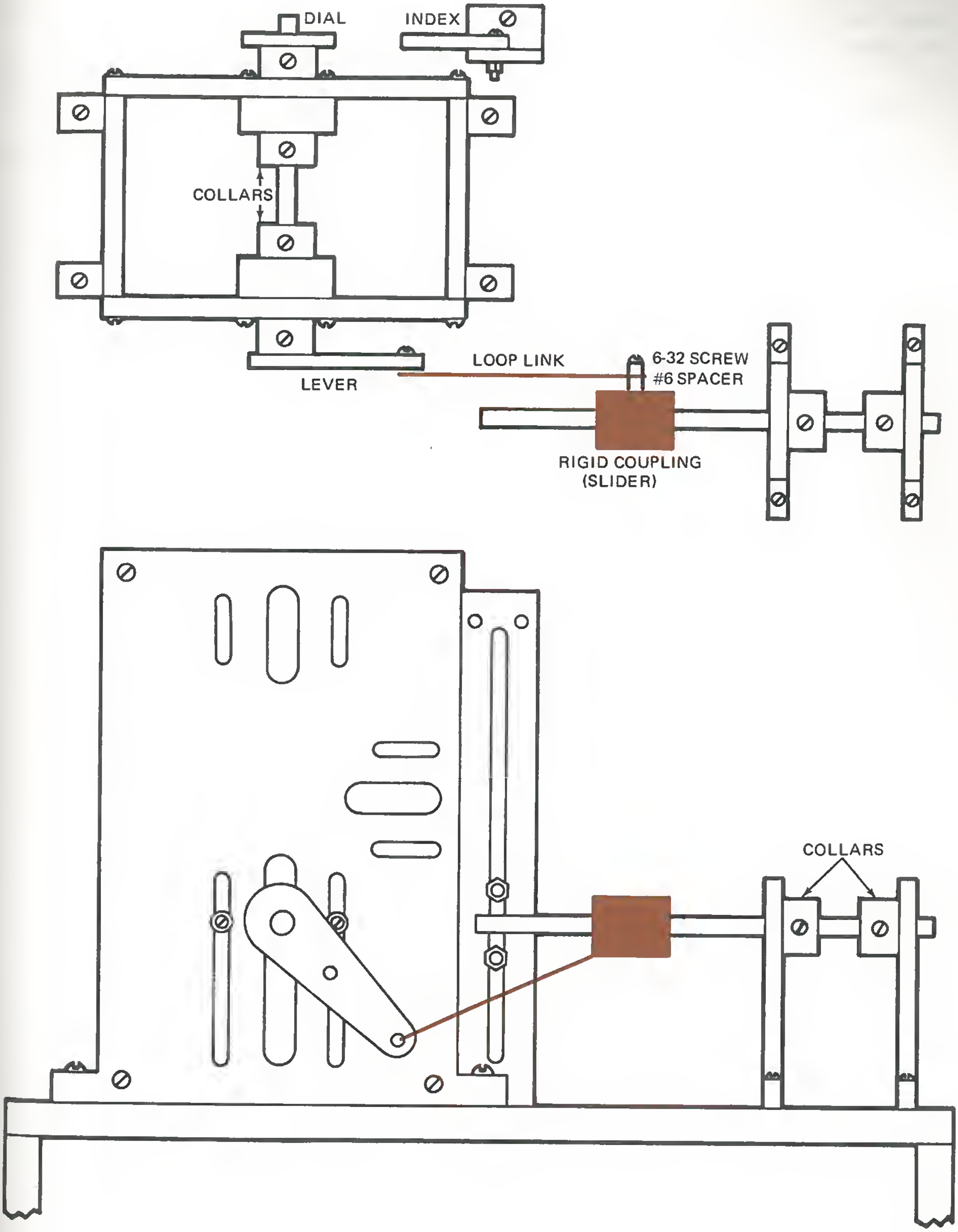


Fig. 13-5 The Experimental Mechanism

ANALYSIS GUIDE. In examining the results from this experiment you should make a detailed comparison of the measured and computed values of the slider displacement (d).

Also discuss how the mechanism acted at the critical positions encountered in the experiment.

Did you observe any tendency of the slider to rotate on its shaft? If so, why did it do it? Would rotation affect the results?

ℓ_1	ℓ_c	K

Θ	X	d (Meas.)	d (Comp.)

Fig. 13-6 The Data Tables

PROBLEMS

1. List at least one practical application of each of the four inversions of the slider block mechanism shown in figure 13-1.

2. An in-line slider crank mechanism has a crank length of 2 inches and a connecting rod length of 8 inches. The crank turns counterclockwise at 480 RPM. Find the following values when the crank is at 60 degrees (zero degrees is at the horizontal position when the slider is at TDC.)
   a. Slider displacement in inches
   b. Slider velocity in ft/sec.
   c. Slider acceleration in ft/sec.^{2}
   d. Connecting rod angular velocity in rad/sec.
   e. Connecting rod angular acceleration in rad/sec.^{2}
   f. The crank angle giving maximum slider velocity in radians and in degrees.

3. A sliding block mechanism has a 2-inch crank and a 6-inch connecting rod. The stroke line of the slider is horizontal and located 2 inches above the center of the crank. Make a neat, scale drawing of this mechanism and determine the length of the stroke in inches. Then, assume the crank rotates constant at 120 RPM and determine the time in seconds for the forward and the return stroke. The crank is rotating in a clockwise direction.

experiment 14 QUICK RETURN MECHANISM I

INTRODUCTION. In various types of machines we often desire that there be a definite difference between the time of a movement in one direction and the return movement. When this is the case, the mechanism we use is called a quick-return mechanism. In this experiment we will examine some methods of obtaining a quick-return motion.

DISCUSSION. When only a small time difference is required in a forward and return stroke, an offset slider crank mechanism can be employed. You can see by the mechanism illustrated in figure 14-1, there is a difference in the two strokes of the sliding block. With the crank link ℓ_1 rotating counterclockwise, the slider will move from TDC to BDC while the crank goes through an angle, Θ. The return stroke will be through a larger angle, θ. If the crank angular velocity ω is constant, then the motion from BDC to TDC takes a longer interval of time. However, the time difference in this mechanism is always relatively small. We frequently want a greater time differential than is possible with this particular type of mechanism. The ratio Θ/θ is known as the *ratio of time of advance to time of return*. We can determine this ratio if we know the offset distance and the lengths of links ℓ_1 and ℓ_c by using the trigonometric relationship between them.

A mechanism that will give us a larger time difference between advance and return motions is the drag link slider shown in figure 14-2. This type mechanism is better known as the *Whitworth quick-return mechanism.*

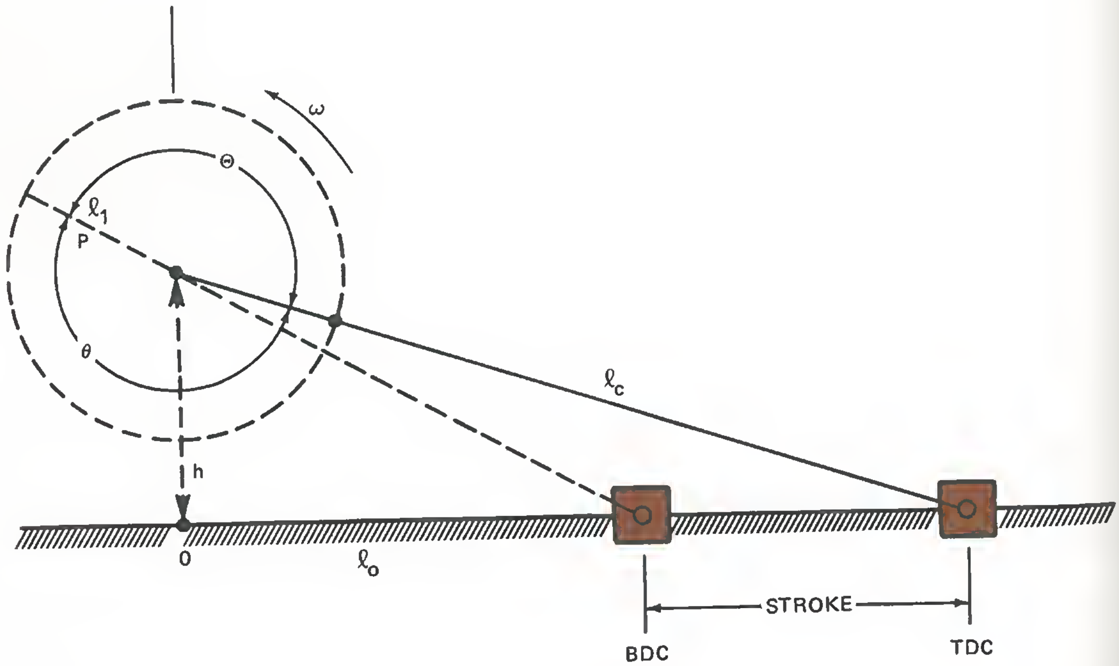


Fig. 14-1 Offset Sliding Block Quick-Return

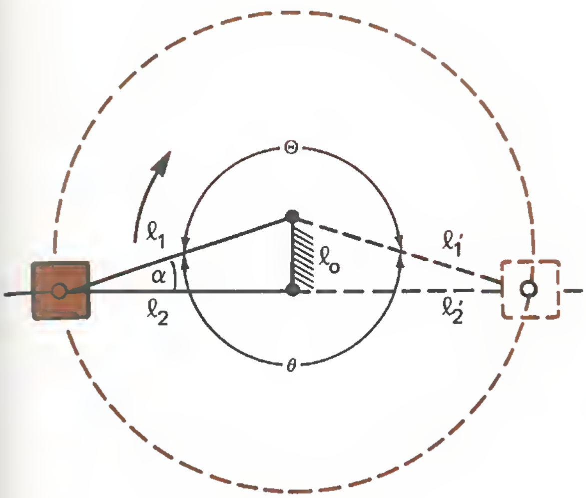


Fig. 14-2 Whitworth Quick-Return Mechanism

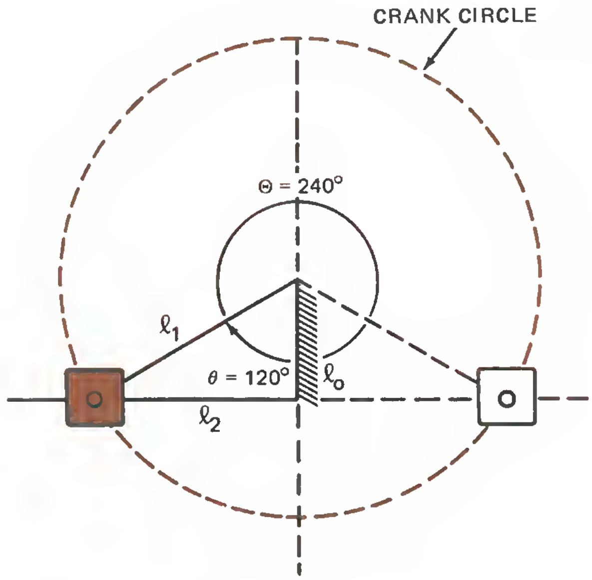


Fig. 14-3 Whitworth Quick Return with 2:1 Ratio

In the Whitworth quick-return, the small link, ℓ_o, is fixed. A slider is attached to the driver link, ℓ_1. The follower, ℓ_2, provides the output motion. The motion from the left to the right occurs as link ℓ_1 travels through angle Θ and the return motion is through angle θ. As you can see in figure 14-2, angles Θ and θ are made by crank ℓ_1 when ℓ_2 is in the horizontal position. You can see that angle θ is equal to

$$\theta = 180^\circ + 2\alpha \tag{14.1}$$

where α is arc sin ℓ_o/ℓ_1. In other words, if we know the distance between the crank center and the driven link center (ℓ_o), and the length of the crank (ℓ_1), we can find the time of travel in one direction by using equation 14.1 and by knowing the angular velocity of crank ℓ_1.

The end of the follower, ℓ_2, can be connected to whatever mechanism you wish to drive. The ratio of Θ to θ will give the time-of-advance to time-of-return relationship.

Let's assume that you wish to have a mechanism take twice as long to return as to advance. This means that you want to have angle Θ be double the value of angle θ. Since $\Theta + \theta$ equals 360 degrees, then Θ would be equal to 240 degrees and θ would be equal to 120 degrees. Knowing this, plus either the length of the crank ℓ_1 or of the distance ℓ_o, we can construct the desired mechanism. Let's assume that we know the length of the crank. Figure 14-3 shows the layout necessary. We arbitrarily select a rotation point for the crank on the frame. Then we draw a vertical line through this point. Next an angle of 240 degrees is drawn symmetrical to this vertical line. This locates the positions of crank ℓ_1 on the circumference of the crank circle. Location of the pivot for the driven crank ℓ_2 is found by the intersection of a line between these two circumference positions and the vertical line through ℓ_2's pivot. We now have a mechanism giving a Θ/θ ratio of 2 to 1.

In the mechanism shown in figure 14-2, if the crank ℓ_1 takes three seconds to make

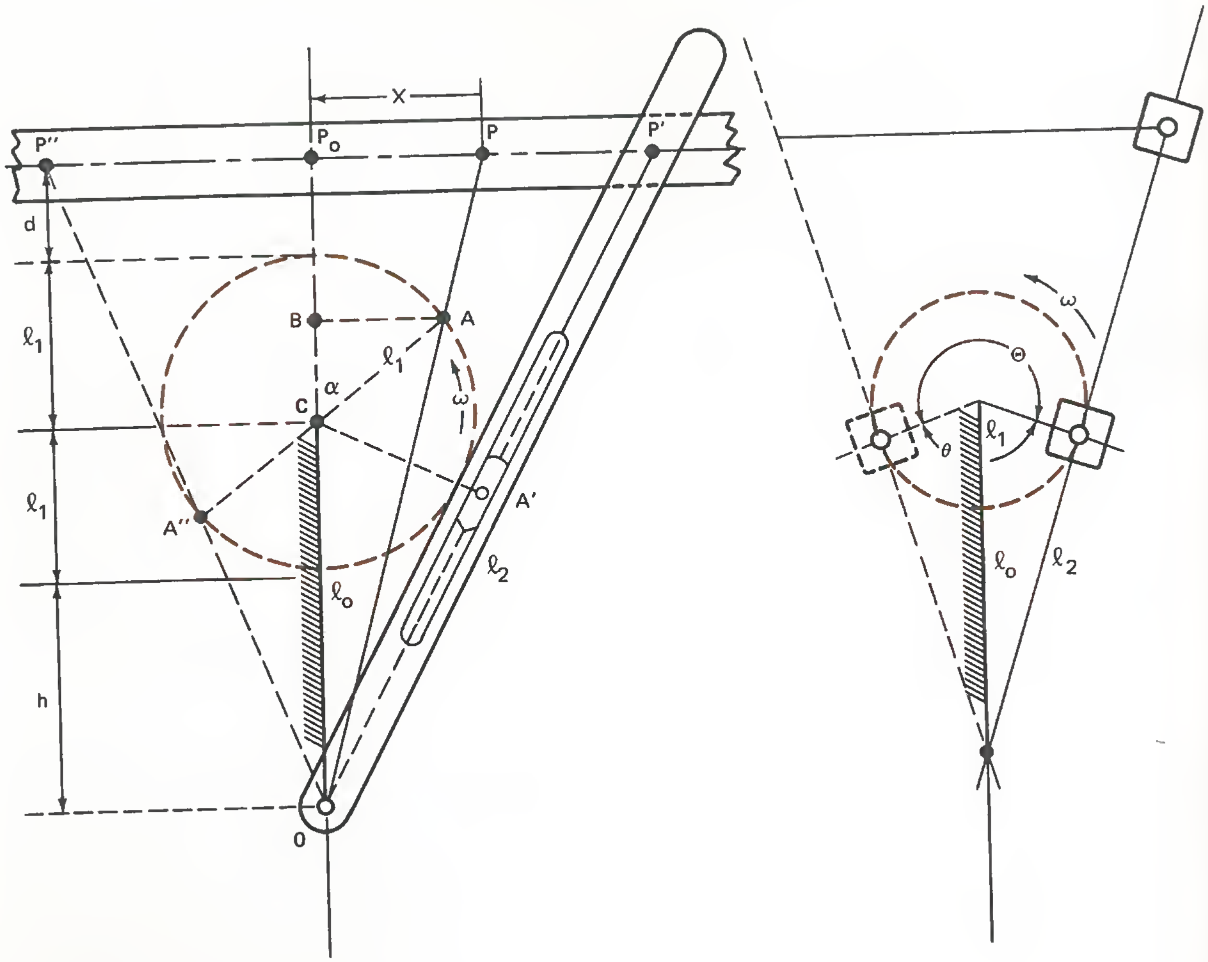


Fig. 14-4 Crank Shaper Quick Return Mechanism

one complete revolution and is turning at a constant speed, then the output link ℓ_2 will take two seconds to rotate from its left horizontal position to its right horizontal position. And since the crank rotates only 120 degrees for the return to its starting position, the output crank ℓ_2 will take one second to return to its left horizontal position. You can see that a Whitworth quick return mechanism can give you almost any time difference you want.

The device shown in figure 14-4 is a commonly used quick-return mechanism. You will see this technique used to move the cutting tool of a shaper that cuts in one direction only and is idle in the return direction. Since the cutting tool is idle in the reverse or non-cutting direction, this operation is made as quick as possible. We will assume that the crank will rotate in a counterclockwise direction with a constant angular velocity. Crank ℓ_1 will make a complete revolution and the follower arm will oscillate. We will also assume that the zero position of both the follower arm and the crank is with the crank pointing downward and, of course, the follower arm in its center position. You can see by the skeleton drawing that our angles Θ and θ can be determined by drawing tangents to the crank circle from the pivot point of the follower arm. Then we can draw a line from the crank pivot to these points of tangency.

The follower moves left through angle Θ and back to the right through angle θ. Since the crank is rotating at a constant speed, the follower takes longer to move to the left because angle Θ is larger than angle θ. The relative time of motion in the two directions is again Θ/θ.

The sketch on the left in figure 14-4 is typical of the application of this type mechanism. The crank ℓ_1 rotates and the slider in arm OP is caused to translate and oscillate about pivot point O. Point P moves from extreme right to extreme left as crank ℓ_1 moves from A′ to A″. The return movement is much faster. Let's see if we can calculate the displacement of point P.

We will let P_o be the zero position with movement to the right as positive. We will measure the angular displacement of ℓ_1 from the bottom position as indicated by the position of α and crank position CA. Further, we will let the distance from the line of movement of P to the upper point of the crank circle be d. The distance from the crank circle to O will be h. P_oP, we will call x.

Take an arbitrary position of the crank ℓ_1 and draw a perpendicular to the vertical centerline BA. Then we know that

$$AB = \ell_1 \sin\alpha$$

$$OB = OC + CB = (h + \ell_1) + (-\ell_1 \cos\alpha)$$

You can see in figure 14-4 that triangles OAB and OPP_o are similar; thus

$$\frac{P_oP}{P_oO} = \frac{AB}{OB}$$

or

$$\frac{x}{2\ell_1 + h + d} = \frac{\ell_1 \sin\alpha}{(h + \ell_1) - \ell_1 \cos\alpha}$$

Thus:

$$\boxed{x = \frac{(2\ell_1 + h + d)(\ell_1 \sin\alpha)}{(h + \ell_1) - \ell_1 \cos\alpha}} \qquad (14.2)$$

You will notice that equation 14.2 gives x as a function of α. Also note that ℓ_1, h, and d in the right member of this equation are constants. Equation 14.2 is the equation for the displacement of point P from its mid-position.

To find the velocity of point P you can differentiate the displacement equation. The second derivative of this equation will give the equation for the acceleration of point P. Both the velocity and the acceleration are useful parameters when analyzing a practical device such as a shaper cutting tool.

Once values are inserted for α, d and h, the derivatives of equation 14.2 are easy to obtain. The first derivative of equation 14.2 will be $dx/d\alpha$. But the velocity we wish to obtain is dx/dt. If we remember the following relationship, we have little difficulty obtaining dx/dt:

$$\boxed{V_P = \frac{dx}{dt} = \frac{dx\,d\alpha}{d\alpha\,dt} = \omega\frac{dx}{d\alpha}} \qquad (14.3)$$

Using the relationships in equation 14.3 we simply multiply the first derivative of equation 14.2 by the angular velocity of crank ℓ_1 to obtain the velocity of point P.

Another type of quick return mechanism employs a more conventional four-bar arrangement. Figure 14-5 shows an example of such an arrangement. The four-bar mechanism is a drag-link assembly and the stroke of the block is limited as shown in figure 14-6. The actual length of the stroke and the time ratio Θ/θ can be found by making a scale drawing of the mechanism. It is not uncommon for the ratio Θ/θ to be as small as 1:10.

In some applications two or more quick-return mechanisms are used in tandem to produce a compound Θ/θ ratio.

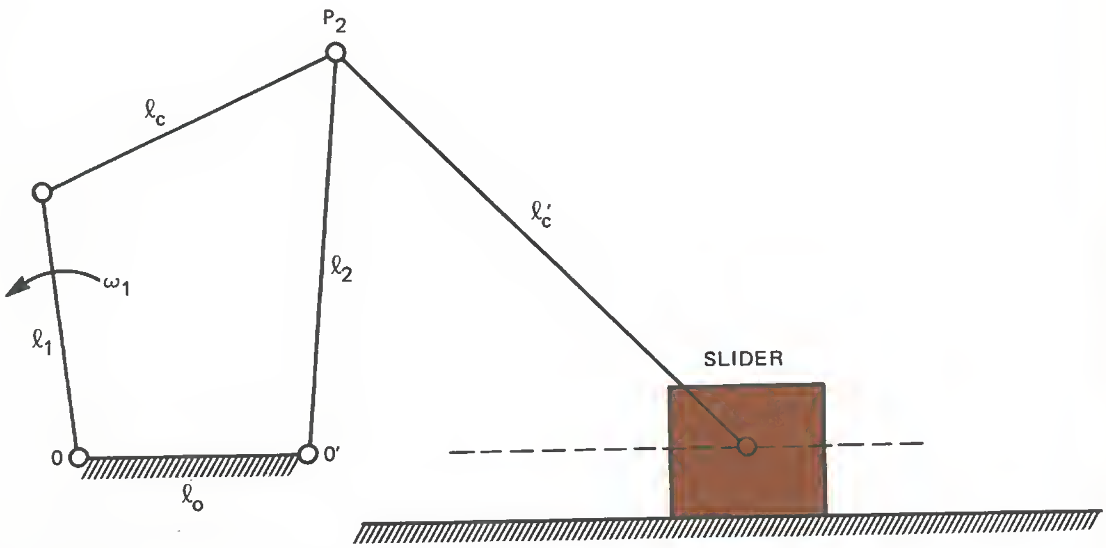


Fig. 14-5 A Four-Bar Quick Return Mechanism

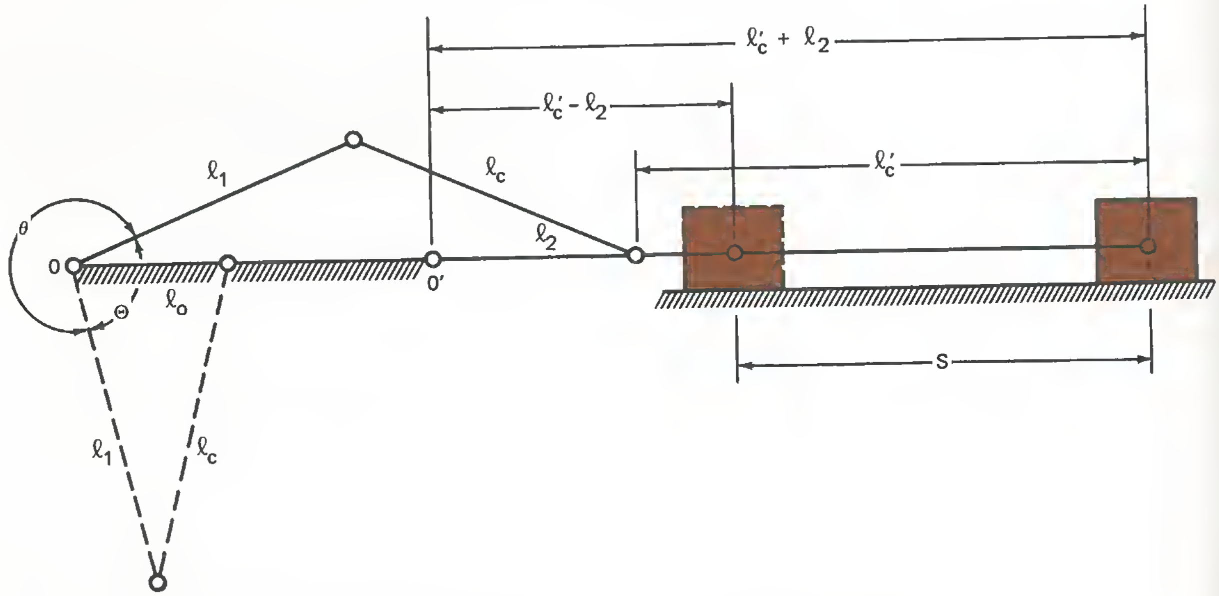


Fig. 14-6 Stroke of Four-Bar Quick-Return

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Shaft hangers with bearings
4 Bearing holders with bearings
2 Shafts 2″ x 1/4″
1 Shaft 4″ x 1/4″
4 Collars
2 Lever arms 1 in. long with 1/4 in. bore hubs
1 Lever arm 2 in. long with 1/4 in. bore hub
1 Disk dial
1 Dial index and mount
1 Rigid coupling
1 Screw 6-32 x 1/4 roundhead
1 Spacer #6 x 1/8 in. long x 1/32 in. wall thickness
1 Steel rule 6 in. long
*1 Wire reverse link 2 in. long
*1 Wire loop link 3 in. long

*For details of wire link construction refer to appendix A.

PROCEDURE

1. Inspect your components to be sure they are undamaged.
2. Construct the bearing plate assembly shown in figure 14-7.

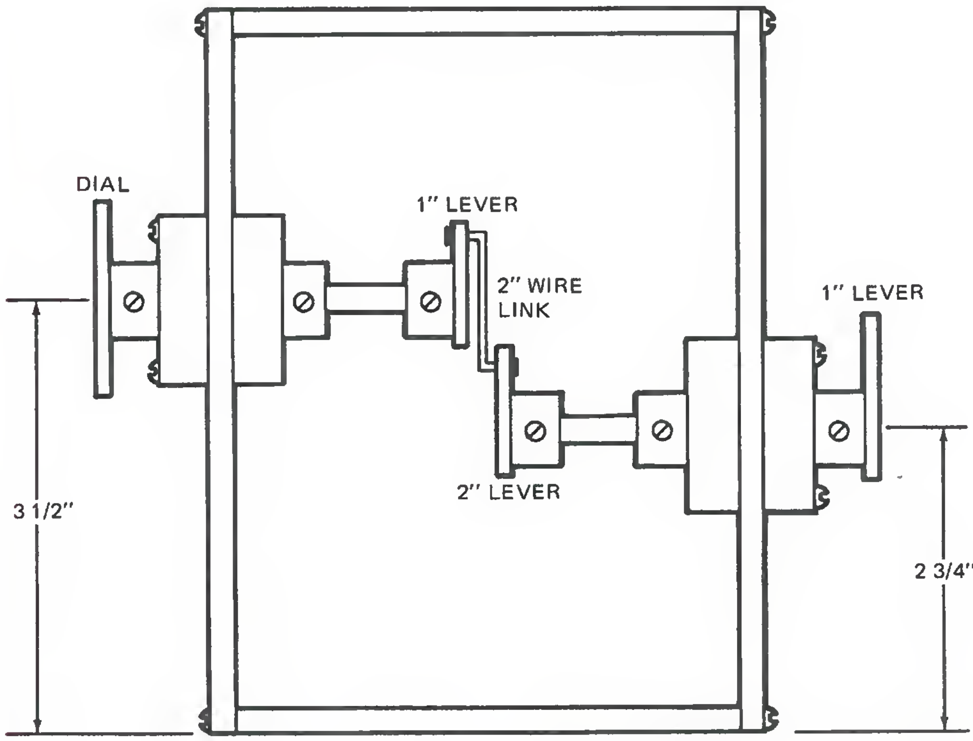


Fig. 14-7 The Bearing Plate Assembly

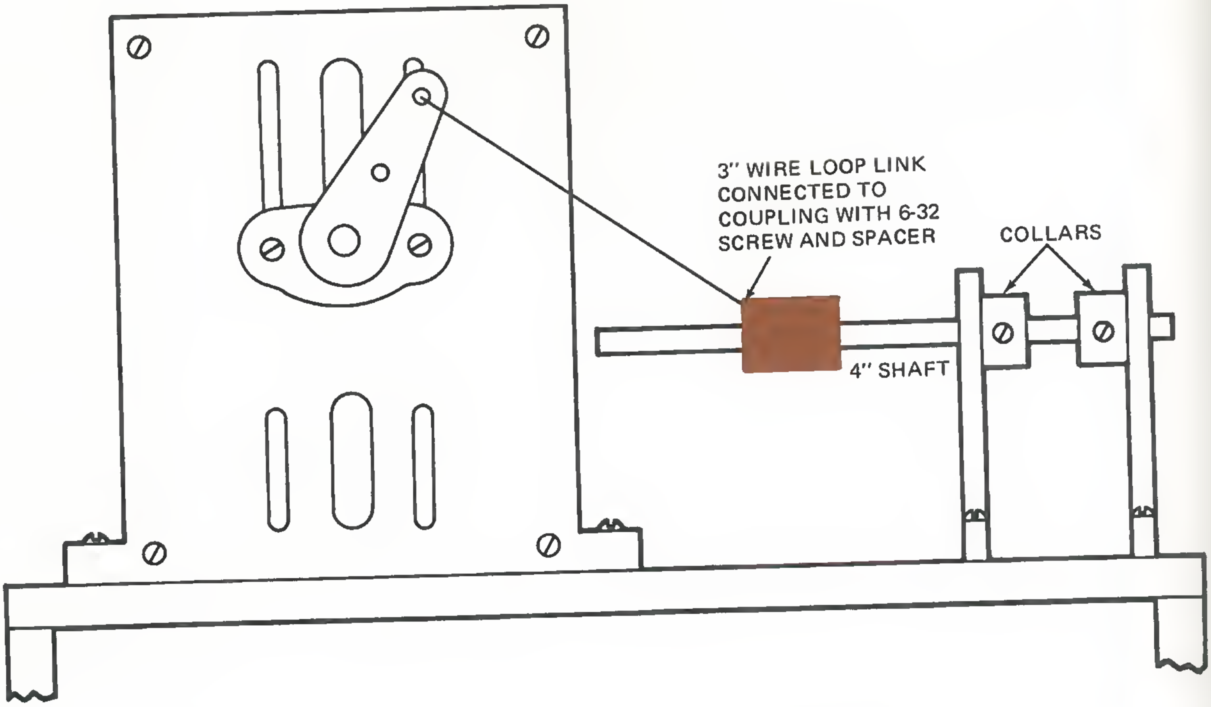


Fig. 14-8 The Experimental Mechanism

3. Mount the bearing plate assembly on the breadboard as shown in figure 14-8. Also mount the dial index assembly.
4. Rotate the dial several times to insure that the mechanism operates freely. Lubricate the 4-in. shaft if necessary.
5. Adjust the 1-in. lever arm that is outside the bearing plate so that it points just opposite to the 2-in. lever arm. These two arms should be approximately horizontal when the 1-in. input lever points vertically downward.
6. Set the mechanism so that the slider is at top dead center. In this position, adjust the dial and index to read zero.
7. Lay the 6-in. steel rule across the slider shaft hangers so that its zero end lines up with the end of the shaft. Tape it in position if necessary.
8. Starting with zero degrees on the dial, measure and record the dial angle (Θ) and the slider displacement (X) every 20 degrees for one full dial revolution in the clockwise direction.
9. Repeat step 8 for one full dial revolution in the counterclockwise direction.
10. Measure and record the length of each link in the four-bar mechanism (ℓ_1, ℓ_o, ℓ_c, and ℓ_2).
11. Measure and record the lengths (ℓ'_1, ℓ'_c, and ℓ'_o) of the links in the slider-crank mechanism.
12. Measure and record the stroke (S) of the slider-crank mechanism.

13. Adjust the 1-in. lever that drives the slider so that it is pointing in the same direction as the 2-in. lever.

14. Repeat steps 8 and 9. Record the data as Θ' and X'.

ℓ_1	ℓ_o	ℓ_c	ℓ_2	ℓ'_1	ℓ'_c	ℓ'_o	S
Clockwise		Counterclockwise		Clockwise		Counterclockwise	
Θ	X	Θ	X	Θ'	X'	Θ'	X'

Fig. 14-9 The Data Table

ANALYSIS GUIDE. In your analysis of these data you should plot a curve for each set of Θ and X. On the curve identify the regions of slider travel from TDC to BDC and from BDC to TDC. Determine the ratio of time of advance to time of return for the four modes of operation.

PROBLEMS

1. Make a sketch showing why equation 14.1 is true.
2. Using figure 14-4 as a guide and using equation 14.2, make a scale drawing of the mechanism having the following dimensions. Then, assuming a crank angular velocity of 4π radians per second, compute the velocity of point P and the acceleration of point P when α is 330 degrees.

$$\text{Crank } \ell_1 = 1.25 \text{ in.}$$
$$d = 0.25 \text{ in.}$$
$$h = 0.75 \text{ in.}$$

3. In problem 2 what is the Θ/θ ratio? How much time does it take for P to move in each direction?
4. In problem 2 what is the maximum velocity reached by P? What is the maximum acceleration? Where, with respect to α, do these occur?
5. List five practical applications of quick-return mechanisms.

experiment 15 *TRANSLATIONAL CAMS*

INTRODUCTION. Changes in movements are common place happenings in mechanisms and machinery. One method of changing movement, such as changing from rotary to up-and-down, is through the use of a cam. In this experiment we shall investigate some basic features of cam action with concentration on translational-type cams.

DISCUSSION. A cam is usually a plate or cylinder which transfers motion to a follower by means of its edge or by a groove cut in its surface. A cam can be a projection on a revolving shaft or a projection on a revolving wheel. It may be a sliding piece or a groove which imparts on oscillating motion to the follower. Or, in some cases, the cam does not move at all but rather imparts a change in motion to a contacting part that is moving.

Cams seldom transmit power in the sense that gear trains do. They are most often utilized to modify a mechanical motion. Serving this purpose, cams have been said to be the brains of the automatic machinery in use today. They are responsible for the various motions of the many individual machine parts.

All cam mechanisms can be separated into three distinctive parts: the driving link or *cam;* the driven link or *follower;* and the fixed link providing support or *frame.* As you might suspect, there are many ways of calssifying and categorizing cams and cam followers. Figure 15-1 illustrates cams classified as plate or disk; cylindrical; translational; and face cams.

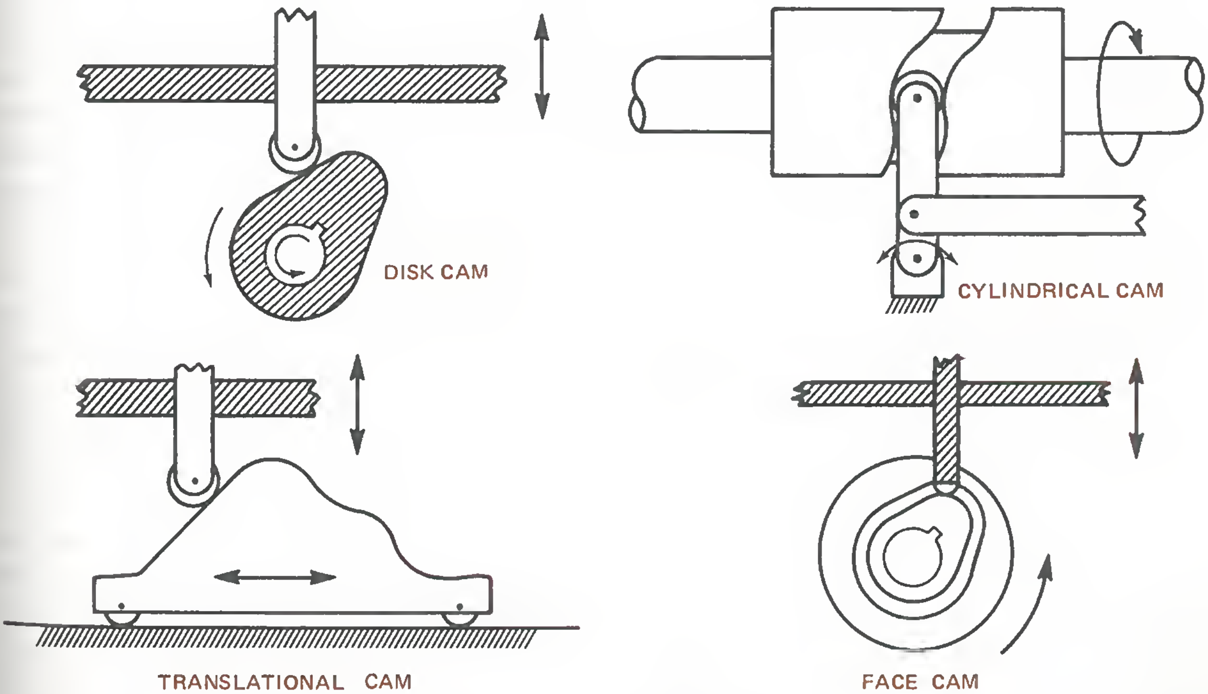


Fig. 15-1 Cam Classifications

By examining these illustrations you should notice that for one complete revolution of most cams, the follower makes one complete trip out and back over its path. The position of the follower at any instant depends upon the shape of the cam. In the practical design of cams an angular velocity ratio is not directly involved, but the follower must be in a definite series of positions while the cam occupies a corresponding series of positions.

The study of cam mechanisms is usually done graphically because it is the path of the follower and the amount of its motion that we are interested in. The most common forms of motion desired in the follower are uniform, harmonic, and uniform acceleration and deceleration. In the planning of a cam, the initial position, length of stroke, character of motion, and direction of motion of the follower are usually known. The angular motion of the cam and the location of the cam axis with regard to the location of the follower are also known. The problem remains to determine the shape of the cam profile that will produce the desired follower motion.

In this experiment we will use a translational cam like the one shown in the lower left of figure 15-1. In this case the cam moves back and forth horizontally and the follower moves up and down.

In planning a particular translational cam, a displacement graph is very useful. The horizontal axis is usually related to cam motion starting at some "zero" position on the left and proceeding to the final position on the right. In our first attempt we will arbitrarily mark the horizontal axis to indicate relative positions of the slide. For a constant speed of operation, this horizontal axis represents time.

The vertical axis represents the position of the follower corresponding to the times marked on the horizontal axis. In other words, we are plotting follower displacement (vertically) against cam positions (horizontally). If we call the vertical distance, y, and the horizontal position, x, then we have the familiar mathematical expression: $y = f(x)$. The value of y depends upon the value of x, or, the position of the follower depends upon the cam position.

To illustrate this technique, let's suppose that we have a machine that includes a pinion-driven rack. Moreover, suppose that the rack is 6 inches long. In working with this machine we find that we need a motion that starts after the rack has moved 2 inches. This motion is to be at 1/2 the rack rate and must stop after the rack has traveled a total of 5 inches. A translational cam can be used.

To lay out this cam, we first mark the start and end points at 2 and 5 inches respectively on a piece of cam material (plastic, steel, etc.) as shown in figure 15-2. Then we choose a reasonable margin at the bottom of the cam so that it can be mounted to the rack. Let's say about 1 inch will be sufficient. We mark off this margin as shown in the figure.

Now, the follower must be a point follower and will ride along the top edge of the cam. And we don't want it to move-up-and-down before the rack reaches the 2-inch mark so we make the cam flat from zero to the 2-inch mark.

From 2 inches to 5 inches we want the follower to move up at half the rate that the rack moves. Since the rack will move 3 inches, the follower must move half that much or 1-1/2 inches upward. We mark this 1-1/2 inches above the margin at 5 inches horizon-

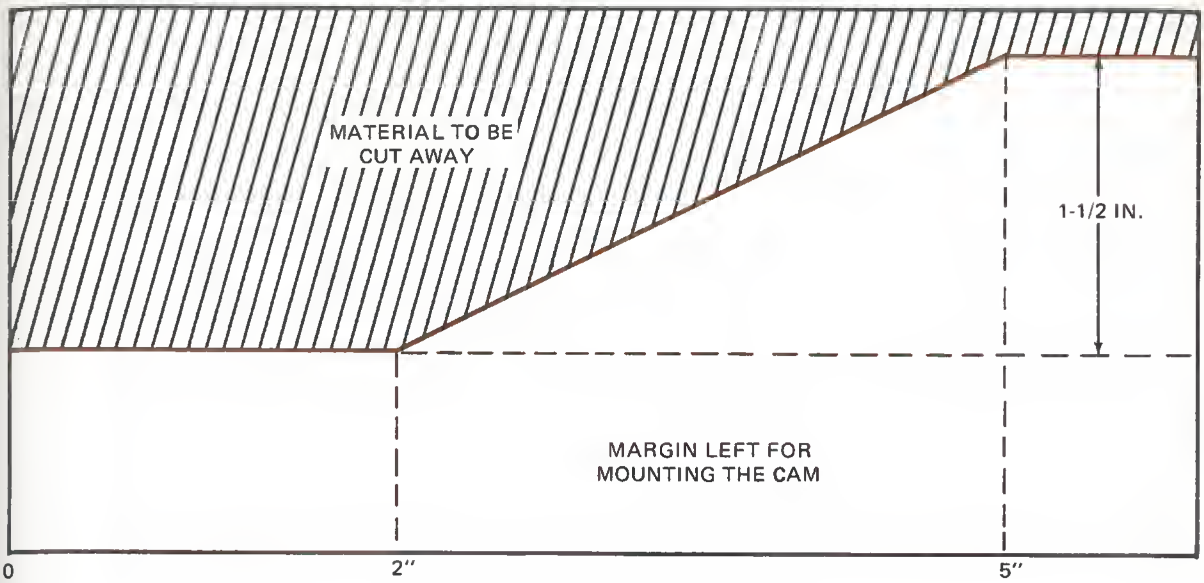


Fig. 15-2 Laying out the Cam

tally. Then, we just connect the 2 in. and 5 in. points with a straight line.

Finally, since the follower is not to move in a vertical direction from 5 to 6 inches, we complete the cam profile with a flat line in this region.

We can describe this cam analytically with a set of three conditional equations. To get these equations we divide the cam into the three straight line regions of rack (or cam) displacement. If we call the cam displacement x, then the regions are:

1. From $x = 0$ to $x = 2$ $(0 < x < 2)$
2. From $x = 2$ to $x = 5$ $(2 < x < 5)$
3. From $x = 5$ to $x = 6$ $(5 < x < 6)$

In the first region the cam height y (this is also the follower position) is constant at the reference level which we shall call zero. So, in this region we can use the conditional equation,

$$y = 0 \qquad 0 < x < 2$$

In the second interval that follows a straight line sloping upward, the equation for such a line is

$$y = Mx + b$$

where M is the slope and b is the y-axis intercept. Since the line rises 1-1/2 inches over a run of 3 inches, the slope is

$$\boxed{M = \frac{\text{rise}}{\text{run}} = \frac{1\frac{1}{2}}{3} = \frac{1}{2}}$$

Then, since at $x = 2$, y must equal zero, we have

$$\begin{aligned} y &= Mx + b \\ 0 &= (1/2)2 + b \\ 0 &= 1 + b \\ -1 &= b \end{aligned}$$

Consequently, the conditional equation for this region is

$$y = \frac{1}{2}x - 1 \qquad 2 < x < 5$$

Finally, in the third region y is a constant

1-1/2 inches above the reference level, so

$$y = 1\text{-}1/2 \quad 5 < x < 6$$

is the conditional equation for this region.

Using these methods we can get equations which describe any linear cam profile.

In some cases the follower is used to produce angular displacements by allowing the follower to operate through a lever arm about a pivot. In such a case the relationship between the angular displacement (Θ) of lever arm and the cam profile displacement (y) is

$$\Theta = 2 \sin^{-1} \frac{y}{2r} \qquad (15.1)$$

where r is the length of the lever arm.

On the other hand, if we know Θ and r but y is unknown, then we can find it with

$$y = 2r \sin \frac{\Theta}{2} \qquad (15.2)$$

which is, of course, equation 15.1 solved for y.

In this experiment a translational cam will be used to produce angular motion by allowing the follower to move around a pivot point. We will let gravity hold the cam follower in position on the cam. With slow-moving machinery this will work satisfactorily; however, sudden changes in cam profile would cause problems at higher speeds. The sudden changes in profile would impart an inertial force to the follower (F = ma) and probably cause it to "bounce" and lose contact with the cam. Various methods are used to avoid this situation in high-speed cams. The primary technique used is to avoid sudden changes in the cam profile. Other methods include the use of cam-follower springs and constrained motion such as that employed by a face cam.

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 3 Shafts 1/4 X 4 in. long
- 6 Bearing holders with bearings
- 8 Collars
- 1 Lever arm 2 in. long with 1/4 in. bore hub
- 1 Roller type cam follower approx. 1/4 in. OD
- 1 Disk dial
- 1 Dial index with mount
- 1 Dial caliper (0 - 4 in.)
- 1 1-1/2 in. X 9 in. piece of sheet metal approx. 0.05 in. thick
- 1 Pair sheet metal shears
- 1 Steel rule 6 in. long

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. On a piece of sheet metal lay out the translational cam shown in figure 15-3.
3. Carefully cut out the cam.
4. Construct the bearing plate assembly shown in figure 15-4. Mount the cam support shafts about 3-1/2 in. apart somewhat to the right of the bearing plate center. Mount the follower shaft as high as possible. Measure and record the length of the follower lever arm.

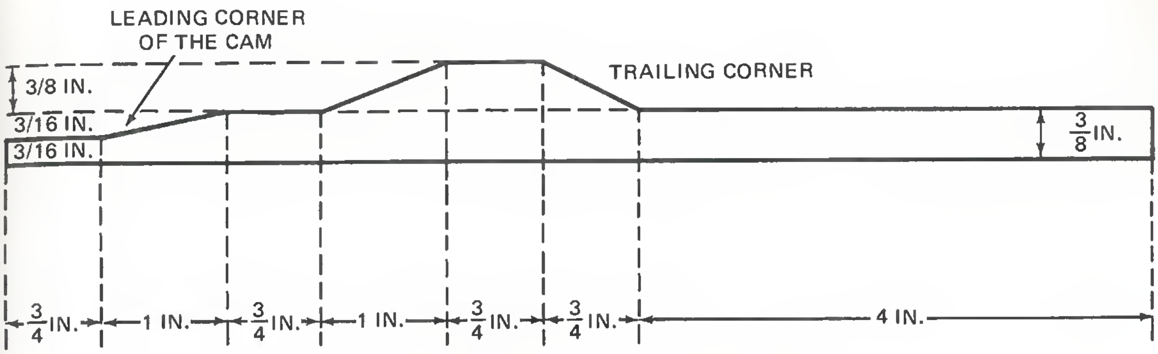


Fig. 15-3 The Experimental Cam

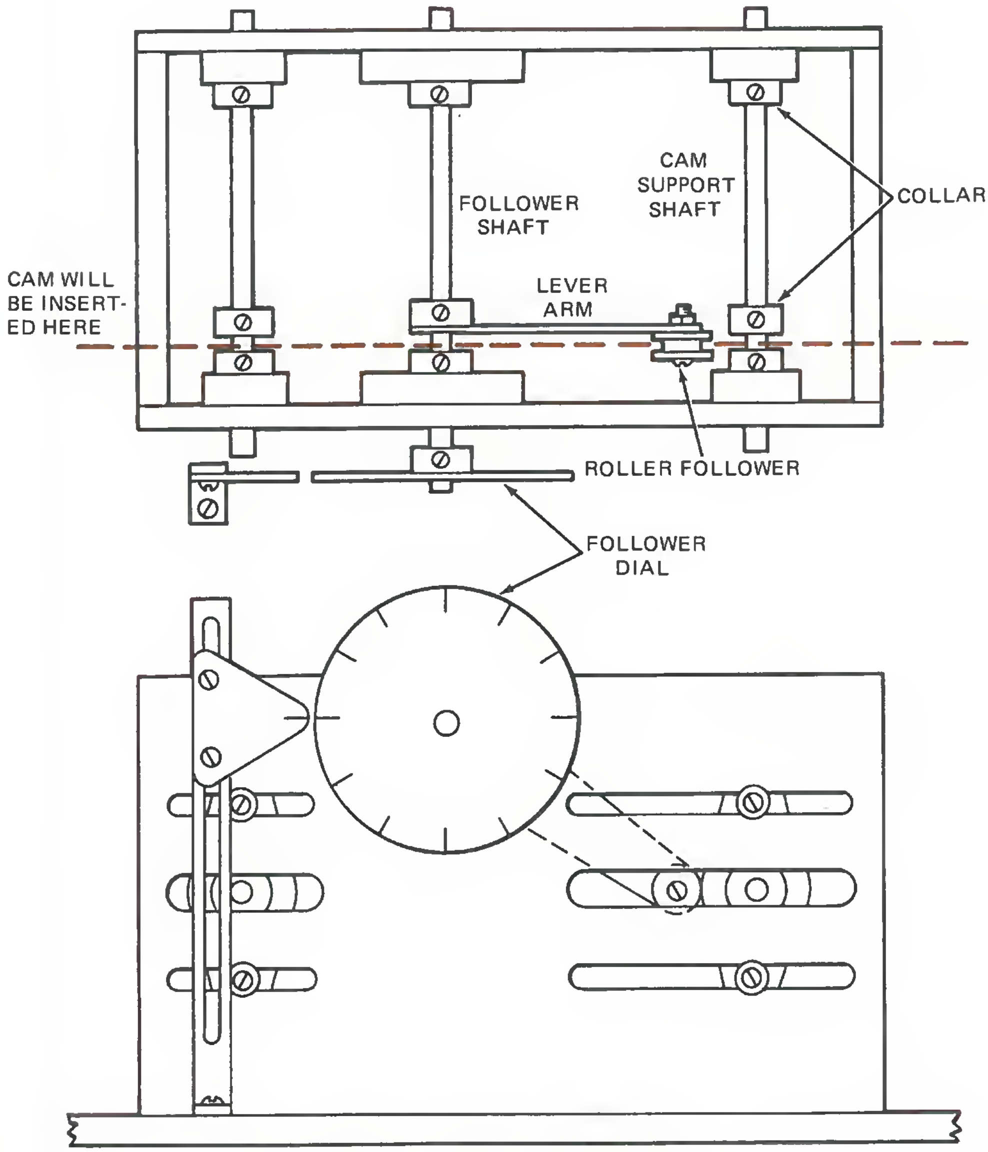


Fig. 15-4 The Bearing Plate Assembly

5. Mount the bearing plate assembly on the spring balance stand.
6. Insert the cam, leading corner first, from the left side of the bearing plate assembly so that it rests on the cam support shafts between the pairs of collars.
7. Adjust the collar pairs so that they will hold the cam upright.
8. Position the cam so that the center of the roller follower is resting against the leading corner of the cam.
9. Adjust the follower dial for zero reading. Put a reference mark on the side of the cam directly below the center of the roller follower.
10. Move the cam approximately 1/4 in. Measure and record both the cam displacement (x) and the angle through which the cam follower dial has rotated (Θ).
11. Repeat step ten in increments of approximately 1/4 in. until you reach the trailing corner of the cam.
12. Return the cam to its original starting position (as in step 8) and repeat steps 9, 10, and 11 two more times.
13. Using the data from the three cam passes, compute and record the average values of x and Θ for each set of data.

First Pass		Second Pass		Third Pass		Average Values	
x	Θ	x	Θ	x	Θ	x	Θ

Length of follower lever arm r = ________

Fig. 15-5 The Data Table

ANALYSIS GUIDE. From the data obtained, plot a graph using the transverse cam position as the abscissa and the cam-follower angle as the ordinate. Compare your results with the physical layout of the cam. From your graph, compute the change in angle for each change in horizontal distance. For each of these compute the ratio "delta y/delta x". Write the equations for the lines represented by your graph. Summarize the function of a transverse cam and give at least three practical uses of this type cam action.

PROBLEMS

1. If the mathematical equation for the motion of a transverse cam is $y = 3x + 4$, how many units will the cam follower move vertically for each unit of motion of the horizontal cam? (*Hint: Use the first derivative of the equation, or make a quick plot of its graph.*)

2. In the discussion three common forms of cam motion are mentioned: uniform, harmonic, and uniformly accelerated and decelerated motions. Draw a sketch of each type of motion. If necessary, look this up in a mechanical engineer's handbook.

3. What is meant by the expression "cam-follower pressure angle"? What is the maximum pressure angle that you would expect to be used in practical cam designs? Was this point illustrated in this experiment? Explain and discuss.

4. The graph used in this experiment is known as a displacement graph. By what other name is the ratio dy/dx (change in displacement with respect to *time*) known?

5. Assume that the cam you used moved from the leading corner to the trailing corner in two seconds at a uniform velocity. What is the cam-follower's velocity vertically when the cam has moved 2 inches? Express your answer in both mm/sec and in ft/sec.

6. Layout a 6-inch cam profile that will satisfy the following conditional equations:

$$\begin{array}{ll} y = 1/4\,x & 0 < x < 2 \\ y = 1/2 & 2 < x < 3 \\ y = 1/3\,x - 1/2 & 3 < x < 5 \\ y = 1\text{-}1/6 & 5 < x < 6 \end{array}$$

7. Describe in your own words the cam follower motion that would result from the cam profile in problem 6.

experiment 16 DISK CAMS

INTRODUCTION. The most popular type of cam is the disk cam which is also sometimes called the plate cam. With this type of cam, rotary motion is translated into reciprocating or oscillating motion. In this experiment a basic disk cam will be constructed and its resulting motions will be examined.

DISCUSSION. The cam shown in figure 16-1 is commonly referred to as a disk or plate type cam. Its outer edge is in contact with a cam follower. In this case the cam follower has a roller which makes contact with the cam profile. Further, the cam follower is limited by the frame so it can move only vertically.

The follower moves upward as the cam rotates from its initial or *zero position* (defined as position closest to the cam center) to its maximum or total *displacement position* and downward as the cam returns to the initial position. The follower of a disk cam is "pushed" upward by the cam; it is said to be *constrained* by the cam. However, its downward motion must be furnished by gravity, a spring, or another mechanism.

Although disk cams are frequently used in relatively slow mechanisms, they are also often used on rapidly rotating shafts. Any sudden change in motion requires a sudden application of force. This results in a violent take-up of bearing-slack with consequent noise, wear, and vibration. A better approach would be to gradually ease into and out of the extreme cam follower positions rather than attempting to abruptly change its direction or movement. An illustration of this is found in the shape of a sinewave as it approaches its maximum excursion and as it leaves it. With this type of motion, a cam follower would gradually approach its total displacement position and slowly begin travel in the opposite direction. At low speeds this leads to the type of motion that is commonly used with disk cams - *simple harmonic motion.*

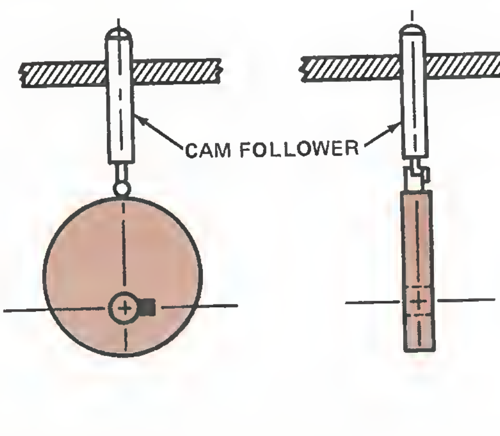


Fig. 16-1 Disk Cam

When a cam follower has simple harmonic motion, its velocity slowly increases from zero in its initial position; obtains maximum velocity midway between zero and its maximum displacement, then slowly decreases to zero. A displacement graph of this type motion is shown in figure 16-2.

The motion illustrated in figure 16-2 is the same motion described by various portions of a sine (or a cosine) curve from its lowest to highest point. You may remember that a sine wave's ordinate value (y) is equal to the vertical distance for a specific number of degrees as measured on the diameter of a unit circle. This basic principle will be used to generate a displacement graph having harmonic motion.

Let's assume that you want a cam follower to move with harmonic motion from its initial position through a distance of 1-1/4 in. Further, to be practical, let's plot the position of the cam follower corresponding to each 30° rotation of the cam. Since the same motion will be followed going down as in going up, it is only necessary to determine what the displacement will be for one-half of the cam rotation.

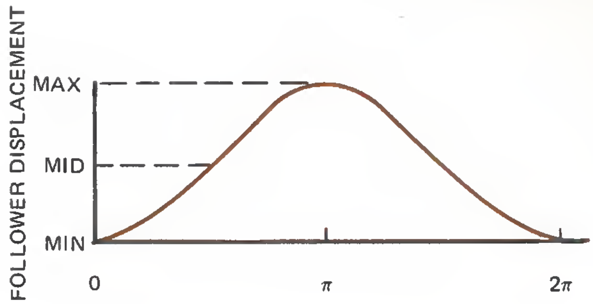


Fig. 16-2 Simple Harmonic Motion-Displacement Graph

The first step in laying out a cam is to draw the displacement diagram. This diagram can be drawn to full scale or to appropriate scale if full scale is impractical. The horizontal axis will represent degrees of rotation of the cam. The vertical axis will represent the displacement of the follower. As shown in figure 16-3, the vertical axis is drawn to full scale - the maximum displacement desired is 1-1/4 in. The horizontal scale was *arbitrarily* selected. Next, a circle is drawn with a diameter equal to the maximum displacement: in this case, a diameter of 1-1/4 in.

The circle is divided into 30° angles as shown. The intersection of the 30° angle radius with the circle circumference is located. A line from that point is projected to the vertical diameter. This gives the distance the cam-follower is to move after each 30° of rotation of the cam. At the appropriate point along the horizontal axis, this vertical distance is marked. For example, in figure 16-3, the vertical distance from 0 to 1 on the circle diameter is the distance indicated on the displacement graph for the 30° position. And, the distance from 0 to 2 is the distance for the 60° position. In a similar fashion, the displacement graph is marked with a series of points corresponding to each 30 degrees of cam rotation. The final step is to draw a smooth curve through these points (fig. 16-3).

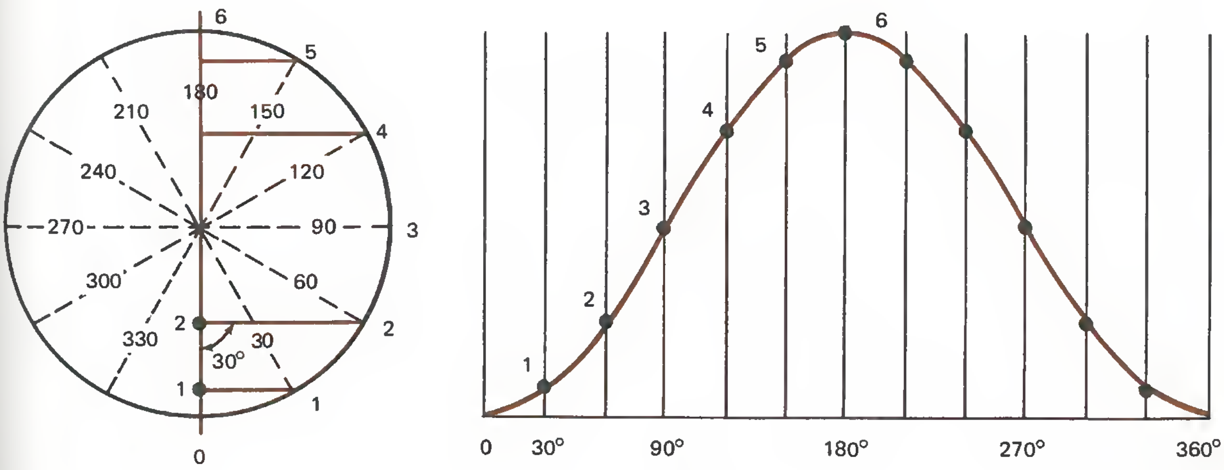


Fig. 16-3 Harmonic Motion Cam Displacement

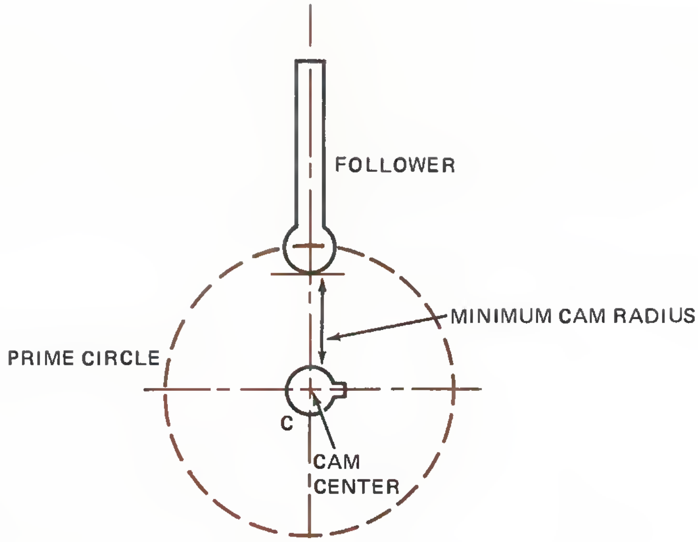


Fig. 16-4 First Steps in Cam Layout

Now the question is how to get this diagram of the desired motion onto an actual cam layout. The first step is to select the center point of our cam layout and to draw a vertical line through this point. Then, at a distance which equals the *smallest* radius of the cam, the cam follower is drawn. These first steps are shown in figure 16-4. The next step in the cam layout is to draw the *prime circle.* This is a circle with a radius from the cam center to the center of the cam follower. The prime circle is shown as a dotted line in figure 16-4.

As shown in figure 16-5, the prime circle is divided into 30° segments, corresponding to the 30° points of the displacement graph illustrated in figure 16-3.

Next, using the roller center as the "0" position, mark the displacements indicated on the displacement graph in figure 16-3 along the center line of the follower. For convenience we usually label these distances. The 30° divisions of the cam are labeled. Assuming that the cam rotation is to be in the clockwise direction, the degrees of rotation *are marked in the counterclockwise direction.*

Still referring to figure 16-5, use the center of the cam, C, as a center and mark on the radial lines the distance from the cam center as indicated by the mark on the cam follower for each 30 degrees of rotation. The arcs drawn on the radial lines give us the *center position* of the follower. Draw the follower circles. You can think of this procedure as holding the cam still and rotating the follower around it. The cam profile is a smooth curve drawn tangent to these roller positions.

Another method of determining the roller center on a particular radial line (for example, on the 150° radial line) would be to draw the prime circle outward on the 150° radial.

Complications can occur with roller followers if there is a rapid change in the cam profile. In many cases the point of contact of the roller is not on the center line of the follower. Figure 16-6 illustrates a typical roller follower in contact with a cam. The force felt by the roller is perpendicular to the surface of contact and acts along the normal to that sur-

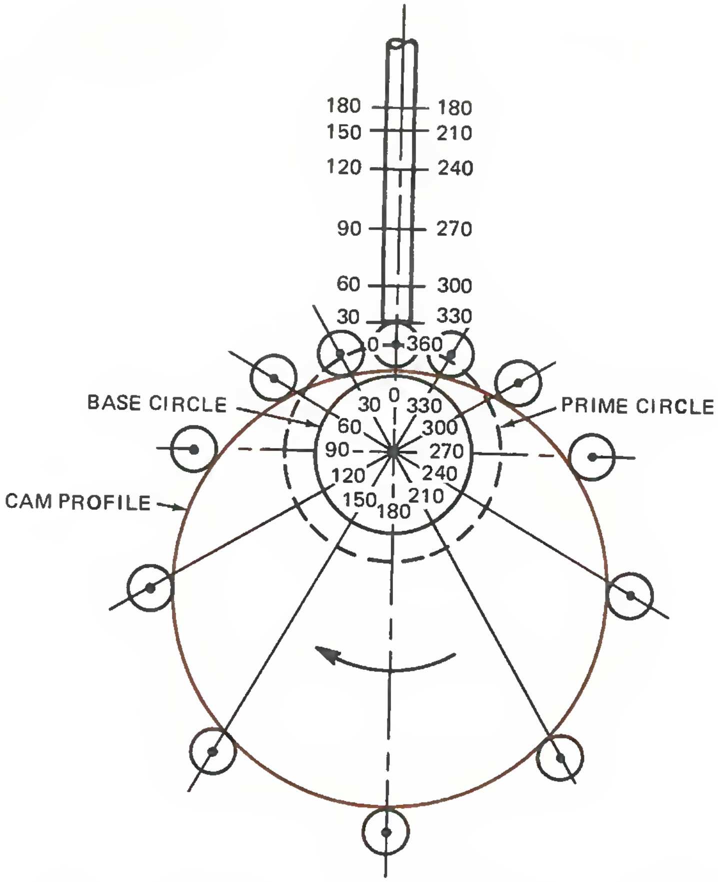


Fig. 16-5 Cam Layout, Roller Follower, Harmonic Motion

face. Naturally, there will be a component of this force along the center line of the cam-follower. The rest of the normal force is felt perpendicular to the cam follower center line as shown.

The term pressure angle is illustrated in figure 16-6. It is the angle between the follower center line and the normal force line. The importance of this angle is its relationship to the lateral force component. If this lateral force becomes too large, the roller will jam. Looking at figure 16-6, you can see that the lateral force component is

$$f(x) = F \sin \theta$$

Thus, the lateral force varies directly with the size of the pressure angle. We try to keep this

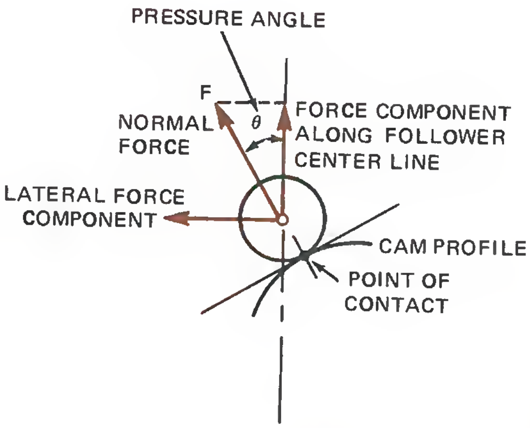


Fig. 16-6 Cam Roller Forces and Pressure Angle

pressure angle as small as possible, but this means making the roller large in diameter to increase the distance between cam and roller centers. Or, the cam diameter can be increased with the same result. However, the size of the cam and cam roller are limited by practical considerations and a compromise must be reached. In most of today's applications, followers will generally handle pressure angles up to about 30 degrees. This means that the lateral force could be as much as one-half the normal force imposed upon the cam roller, ($f(x) = F_n \sin \theta$).

Again referring to figure 16-5, you can observe that the point of contact between the roller and the cam profile is not always the same point lying on a cam radius line. From this diagram the pressure angle of the roller can be estimated. This is one major reason for using the center of the roller as the basis for determining the cam profile. If you were to draw a line through the centers of the roller positions, it would be parallel to the cam profile and is called the *prime curve.*

In some applications the cam follower is located at the end of the lever arm that is pivoted about a point as shown in figure 16-7. In such a case the angular displacement of the follower arm (Θ) is related to the vertical displacement of the follower by

$$\Theta = 2 \sin^{-1}\left(\frac{y}{2r}\right)$$

or

$$y = 2r \sin \frac{\Theta}{2}$$

where r is the length of the follower arm.

Disk type cams are also frequently used to trip a microswitch at a given angular displacement. Cams for this purpose are usually layed out in two concentric circles as seen in figure 16-8. Such electromechanical switches are widely used in automatic controls.

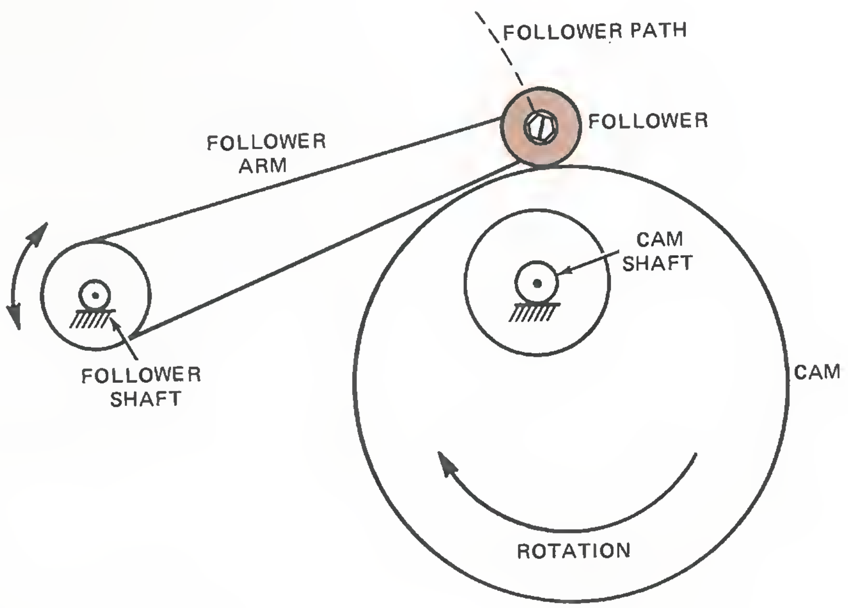


Fig. 16-7 A Pivoted Cam Follower

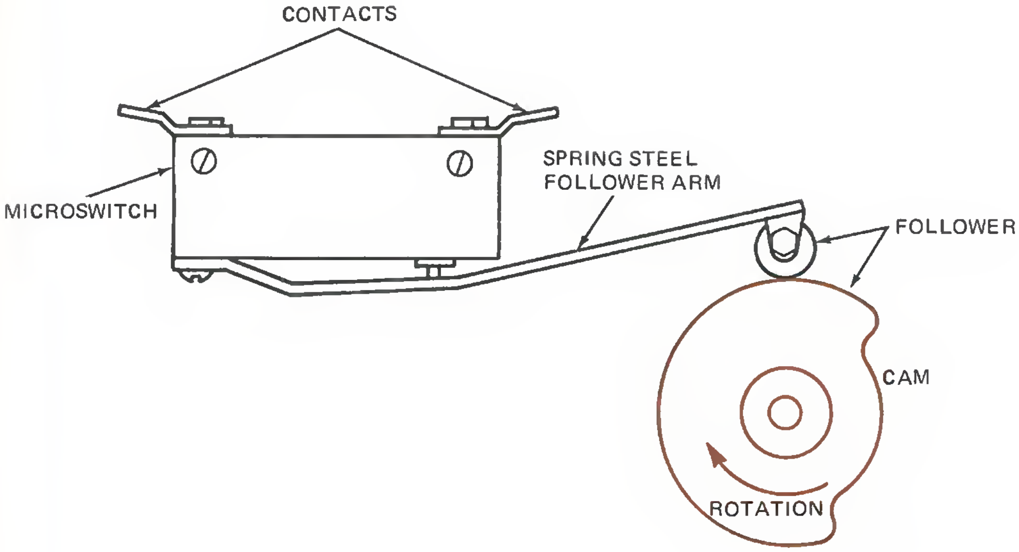


Fig. 16-8 A Cam-Operated Microswitch

MATERIALS

- 2 Bearing plates with spacers
- 1 Breadboard with legs and clamps
- 1 Dial caliper (0 – 4 in.)
- 1 Cam follower roller approx. 1/4″ OD
- 1 Protractor
- 1 Piece of sheet metal 4″ X 4″ X approx. 0.05 in. thick
- 1 Flat file
- 1 Hand drill and twist bit (3/8 in.)
- 1 Universal pin hub (1/4 in. bore)
- 6 Bearing holders with bearings
- 3 Collars
- 1 Lever arm approximately 1 in. long with 1/4 in. bore hub
- 1 Spur gear approximately 1-1/2 in. OD with 1/4 in. bore hub
- 1 Spur pinion approximately 3/4 in. OD with 1/4 in. bore hub
- 2 Disk dials
- 2 Dial indices with mounts
- 3 Shafts 4″ X 1/4″
- 1 Lever arm approximately 2 in. long with 1/4 in. bore hub
- 1 Extension type spring approximately 1-1/2 in. long

PROCEDURE

1. Inspect each of your components to insure that they are undamaged. Count the number of teeth on the two gear wheels.

2. Measure and record the diameter of the cam follower roller (d).

3. Using the method presented in the discussion, lay out a simple harmonic cam on a piece of 4″ X 4″ sheet metal. The follower should have travel of 1-1/2 inches from minimum to maximum displacement.

4. Carefully cut out the cam and file any rough spots in the profile smooth. Put a 3/8-inch hole in the cam center.
5. Mount the cam on the universal pin hub.
6. Construct the bearing plate assembly shown in figure 16-9.
7. Mount the bearing plate on the breadboard.
8. Rotate the cam until the follower is at its minimum displacement location.

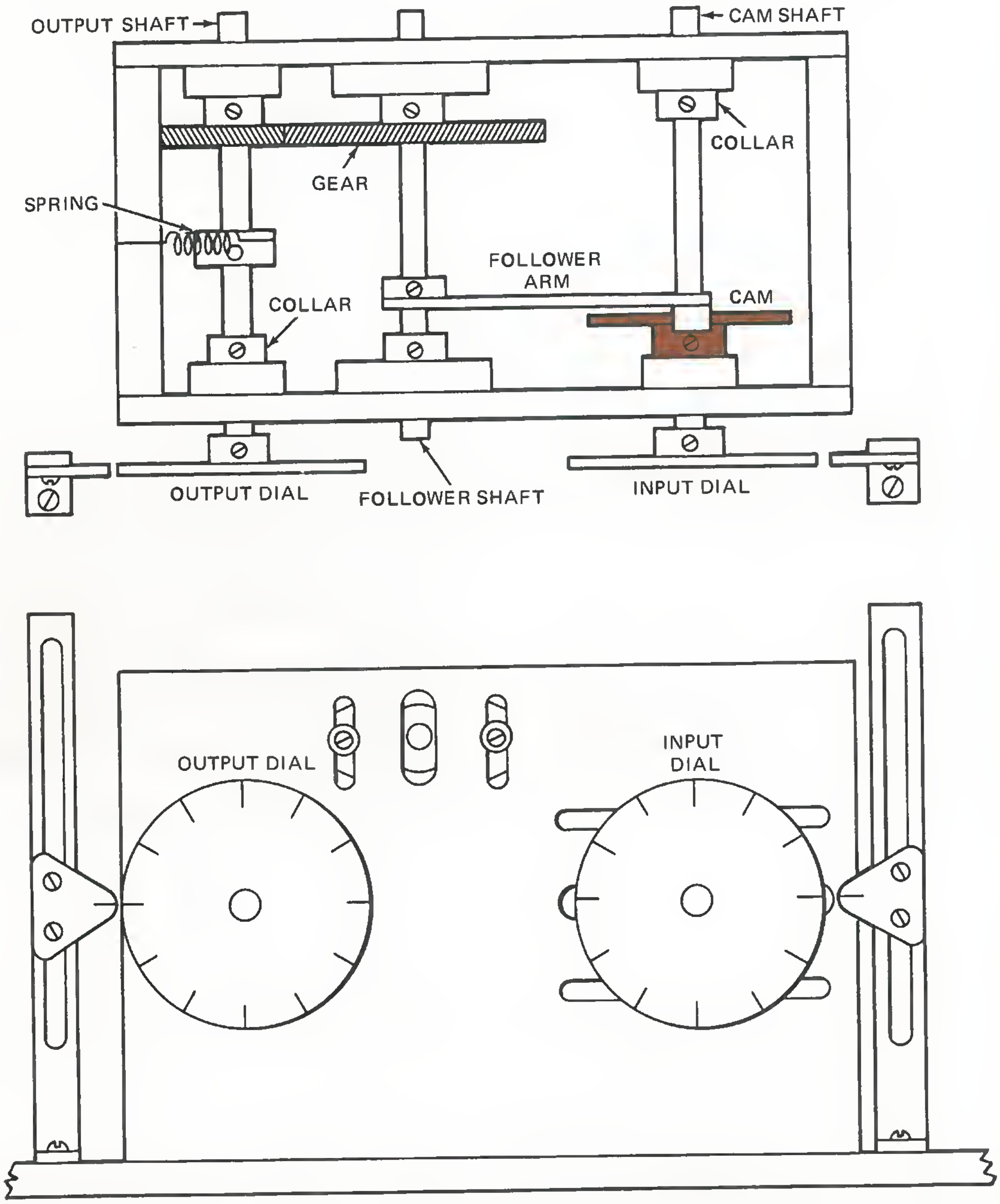
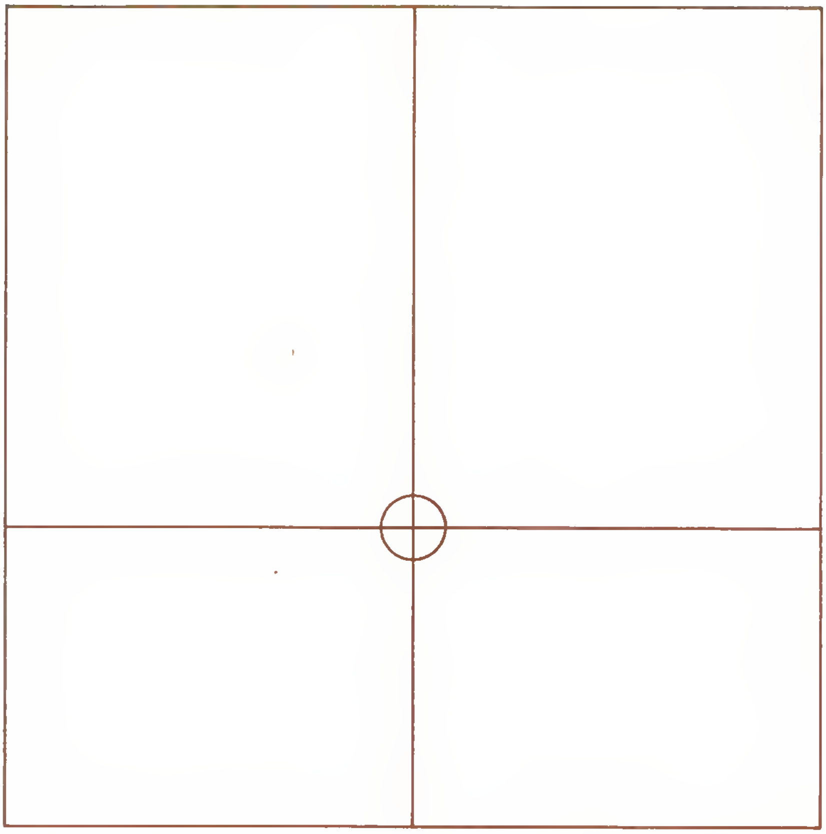


Fig. 16-9 The Bearing Plate Assembly

Cam Profile



N_p	N_g	d

Gear & Follower Data

Θ_i	Θ_o	y

Fig. 16-10 The Data Tables

9. Adjust the tension arm and spring on the output shaft so that it holds the follower against the cam.
10. Set both dials to zero.
11. Rotate the cam dial to the 30° position and record both dial readings (Θ_i & Θ_o).
12. Repeat step 11 for cam dial positions of 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, and 360 degrees.
13. Carefully return the cam dial to the zero position.
14. For each data point, compute the vertical displacement (y) of the follower.
15. Plot a curve of follower displacement (y) versus cam displacement (Θ_i).
16. Remove the cam and trace its profile in the space provided in the data table.

ANALYSIS GUIDE. In your analysis discuss the differences between harmonic motion and linear motion. From your observations during this experiment, discuss the importance of accurate machining operations when manufacturing cams. Discuss sources of errors possible when laying out a cam profile and methods of minimizing these errors. Discuss the follower pressure angle as a function of cam displacement. If you felt more resistance to rotation at the maximum pressure angle position, explain why and estimate how much more force was required at that point. If you did not feel an increase in rotational resistance explain this. Add any other comments you feel to be applicable regarding disk cams.

PROBLEMS

1. Write the mathematical expression (equation) for figure 16-3. Express this equation both as $y = f(\sin\Theta)$ and $y = f(\cos\Theta)$.
2. If the cam whose follower motion is represented by figure 16-3 rotates at a speed of 1800 RPM, what is the velocity of the follower at the 30 degree position; the 90 degree position; the 180 degree position; and the 360 degree position?
3. With the cam used in this experiment, assume that the spring tension is three pounds when the pressure angle is maximum. Compute the normal force and the lateral force felt by the roller.
4. For a given cam it is found that the maximum pressure angle is 30 degrees. It is decided to rebuild the cam using a minimum radius twice the original. Is the new pressure angle increased or decreased? Explain.
5. Draw the displacement graph for a medium speed cam whose follower must rise one inch during the first 90 degrees of camshaft rotation, dwell for the next 30 degrees, return to the initial point during the next 50 degrees and dwell for the remaining 190 degrees.

experiment 17 PIVOTED FOLLOWERS

INTRODUCTION. It is frequently advantageous to use the properties of levers in conjunction with cam operation. In this experiment we will investigate cams having followers that are pivoted. Parabolic motion is used, instead of simple harmonic motion, in the cam layout and observations will be made about the graphs of displacement, velocity, and acceleration.

DISCUSSION. Cam followers often move in a straight line; that is, they have rectilinear motion. However, you will find that levers are very frequently used as cam followers. This is done to take advantage of the properties of the lever, such as motion change or force change. Two representative ways of doing this are shown in figure 17-1.

You can see that the roller in contact with the cam will not move in a straight line. Since the roller is "constrained" by the fulcrum or pivot point, it will move along a circular path having a radius equal to the length of the lever arm as indicated by the letter X in figure 17-1. To show how this affects a cam layout, we will examine a translational cam and then a disk cam.

A type of motion that is even smoother than simple harmonic motion is called *parabolic* motion. Sometimes this motion is called uniformly accelerated motion and it is similar to harmonic motion in that sudden changes in displacement are avoided.

A cam follower having parabolic motion will have a constant acceleration during the first half of its motion and a constant deceleration during the second half of its motion when deceleration and acceleration have equal times. The equation giving displacement as a function of acceleration and of time (cam position increments) is

$$s = 1/2\ at^2 \qquad (17.1)$$

where s is the distance, a is the acceleration, and t is time. Since the acceleration is constant during the first half of the follower rise, the distance given by equation 17.1 will equal the square of the time multiplied by a constant. This means that the follower will travel three times as far during the third time inter-

Fig. 17-1 Cams with Pivoted Followers

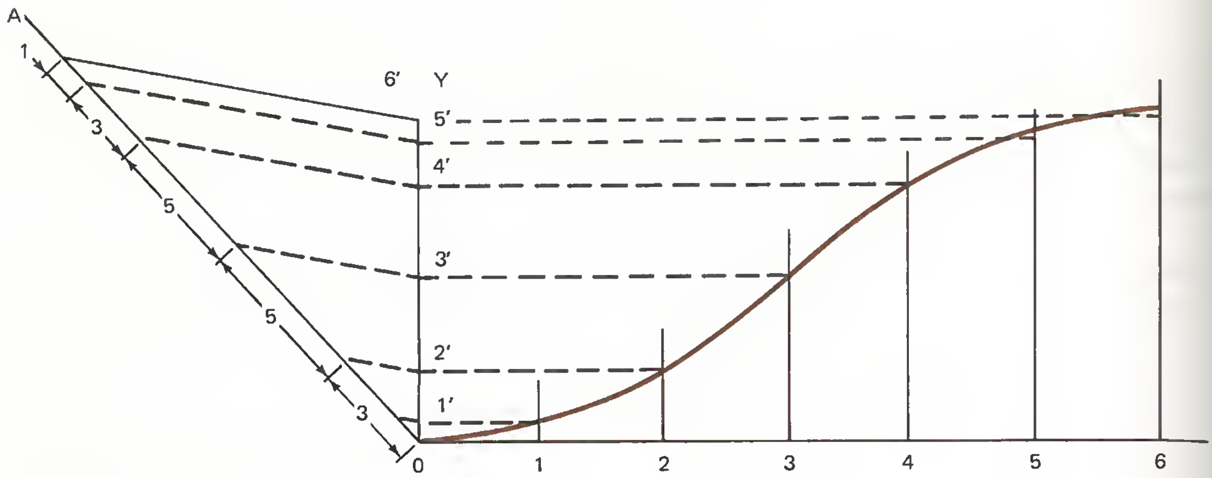


Fig. 17-2 Parabolic Motion Displacement Graph

val as it did during the first; and so forth. The differences in travel, if the first time interval is "1", are 1, 3, 5, 7, 9 . . . Then when a constant deceleration (negative acceleration) is applied during the second half of travel, the reverse of this sequence is appropriate: for example, 9, 7, 5, 3, 1. This type of motion is illustrated in figure 17-2. The horizontal axis represents time or cam position increments and the vertical axis represents the follower displacement. The maximum displacement is shown as the distance OY in figure 17-2.

To construct the displacement graph shown in figure 17-2, a line AO is drawn at any convenient angle. Along this line, mark a distance equal to 1 unit. Then the next interval is three times this long, the next 5, and so forth as is shown. You will notice that this gives six divisions (1, 3, 5, 5, 3, 1). If you desire more divisions, use 1, 3, 5, 9, 9, 5, 3, 1 which will give eight divisions. In fact, any even number of divisions can be obtained by using longer series of subdivisions.

The next step is to draw a line connecting the end of OA back to OY (the actual follower displacement distance). Then draw lines parallel to AY connecting the divisions of line OA to line OY. This procedure will divide line OY into similar line segments. In other words, the distance 0-1′ is one-third the distance 1′-2′.

Then, connect the heights indicated on line OY across the graph to the corresponding cam position indicated on the horizontal axis. The points thus obtained are then connected with a smooth curve as is shown. It may be of interest to you to compare this curve with that of simple harmonic motion. From this displacement graph it is possible to construct the desired cam. In this particular case, only the rise of the follower is graphed.

Now let's see what difference a pivoted follower will make. Instead of the vertical lines drawn in figure 17-2, the cam follower will follow an arc. Let's assume the same

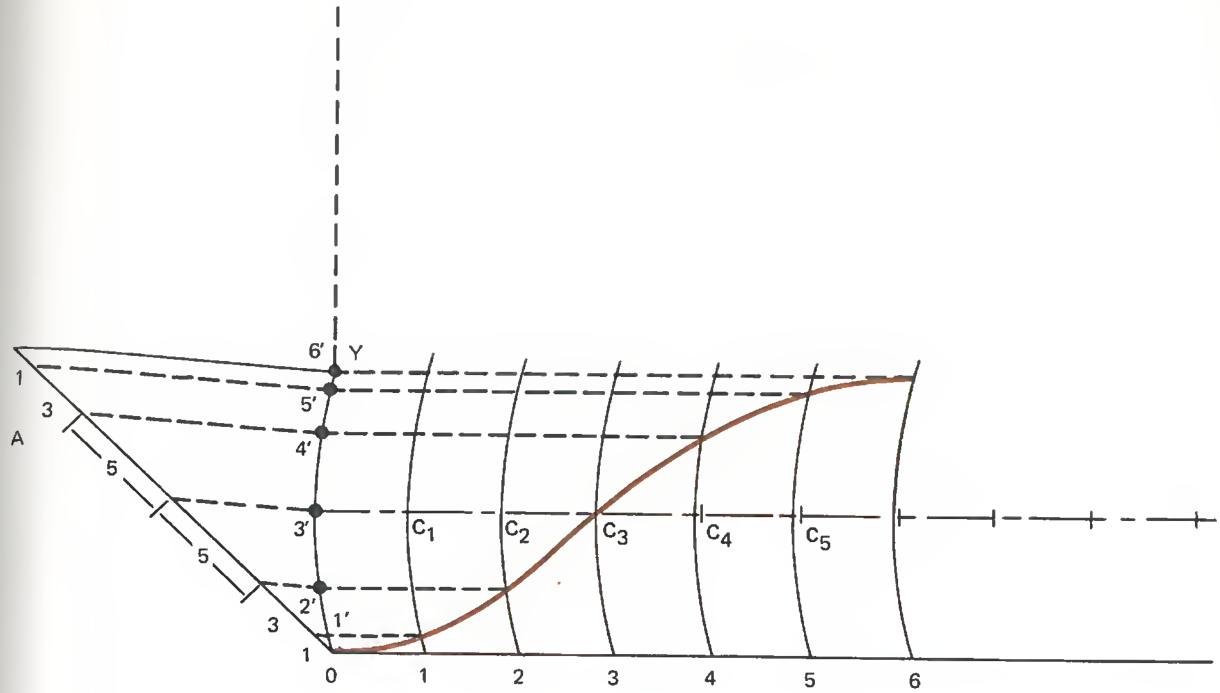


Fig. 17-3 Displacement Graph - Cam with Parabolic Motion and Pivoted Follower

type of motion is desired over these first six cam positions, but that we have a pivoted cam follower with a two-inch lever arm. Also, the pivot point is located midway between minimum and maximum travel of the follower. The basic change this causes in our displacement graph is that the ordinates are no longer straight lines; they are arcs. These arcs are drawn from the assumed pivot position using the correct length of lever arm. The various pivot points are labeled C_1, C_2, etc. Line OA has been drawn to illustrate that the same parabolic motion will be formed from these line segments. If the downward travel is also parabolic, then the mirror image of figure 17-3 would be used.

How can we transfer this type of motion into the profile of a disk cam? Let's look at an entire design problem. The following features are desired:

Follower Type - Pivoted Roller
Roller Diameter - 3/16 inch
Follwer Arm - 2 inches long
Minimum Radius of Cam - 7/32 inch
Follower Center on-line with Roller Minimum Travel
Motion Desired - Parabolic rise for 120 degrees, dwell for 120 degrees, parabolic fall for 120 degrees. Follower to rise through 45 degrees.

First, although not absolutely essential, let's draw the displacement diagram that has these characteristics. This is shown in figure 17-4.

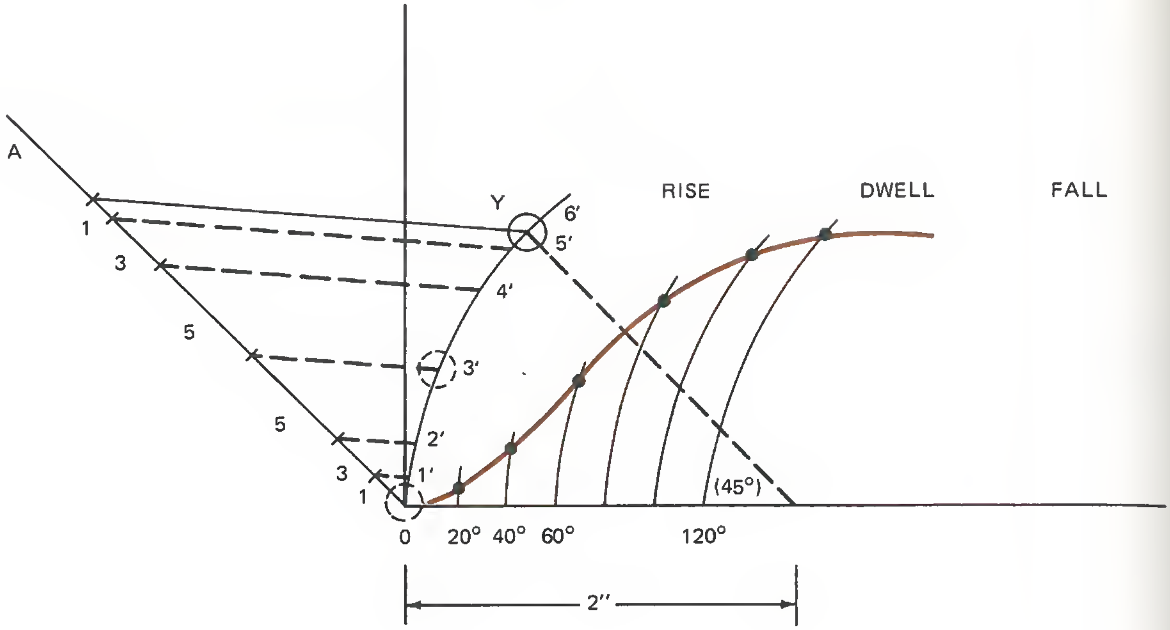


Fig. 17-4 Displacement Graph - Disk Cam with Parabolic Motion and Pivoted Follower

In figure 17-4, the lever movements are given and the curve of the displacement is drawn for the rise only. The fall will follow these same distances. The arc OY is drawn with a 2-inch radius. The parabolic motion described on line OA is transferred to this arc and labeled 1′ through 6′. Since this motion is desired over a cam rotation of 120 degrees, then each movement will occur during 20 degrees of cam motion.

The first steps in the layout of the cam profile are to locate the cam center, its minimum diameter, the roller position, and the initial position of the follower pivot point.

The next step is to draw the desired 45 degree rotation of the follower and transfer the distances from the displacement graph to this arc. These are labeled 1′ through 6′ in figure 17-5. Notice that the lever arm will be tangent to a circle drawn through the center of the roller. As the cam rotates, the lever arm will move off this position. Instead of rotating the cam on paper, we will move the lever. Since the cam motion will be clockwise, the lever will appear to rotate counterclockwise.

The prime circle is divided into 20 degree segments. Locate C_1 by moving the lever center 20 degrees counterclockwise, then draw a tangent at this point to the prime circle. In a similar manner the other centers and the appropriate lengths are transferred to these arcs. This gives the locations of the roller center.

Next, the roller is drawn in these positions. A smooth curve tangent to the roller positions gives the desired cam profile as is shown in figure 17-5. It should be noted that the roller dwells at maximum position from positions 6 through 12; that is, from 120 degrees through 240 degrees of cam rotation. A careful examination of figure 17-5 will reveal that this profile has been obtained.

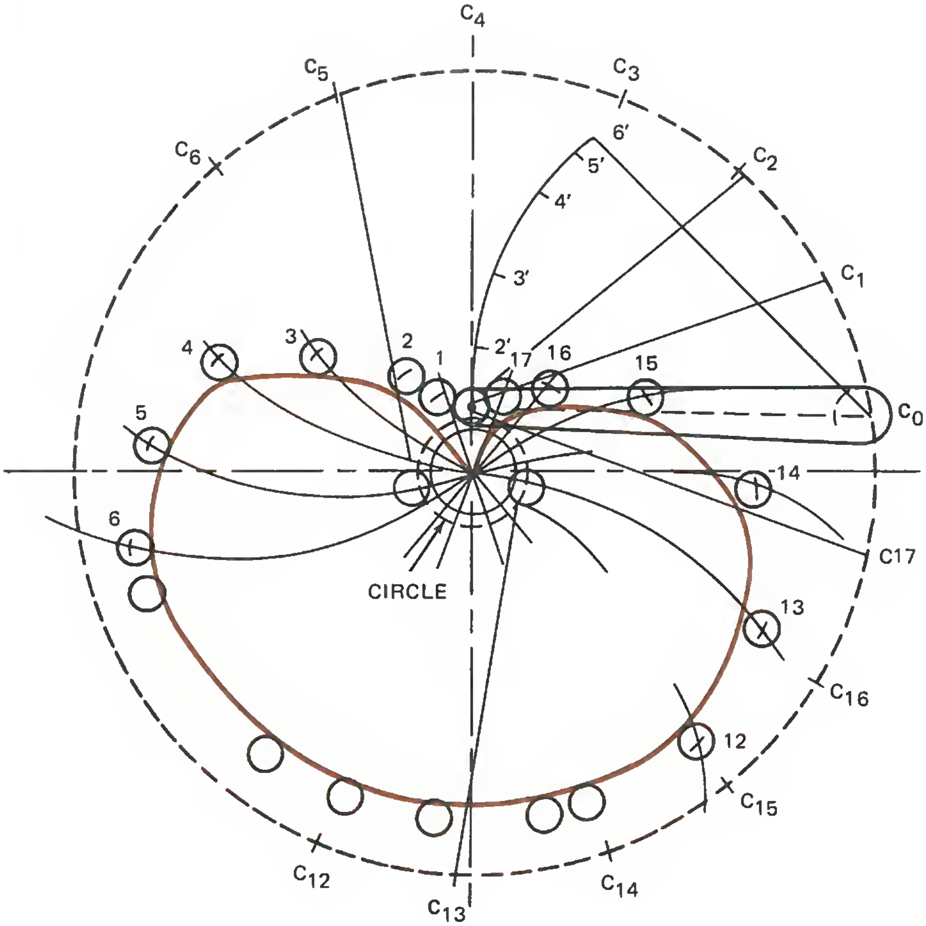


Fig. 17-5 Disk Cam Profile for Pivoted Follower

There are times when a technician must analyze not only the displacement versus time characteristics of cams, but also the velocity and the acceleration of the cam action. As you know, if you have the mathematical equation for displacements, the derivative of this equation (ds/dt) will give you the velocity. The derivative of the velocity equation (or the second derivative of the displacement equation) will give the equation for acceleration. Frequently, we do not have a basic equation and must depend upon graphical techniques. These approaches give you accuracies sufficient for most purposes.

Remember that acceleration is velocity change per unit of time (dv/dt). You can be moving 200 ft/sec and have zero acceleration. Negative acceleration will give a decrease in velocity.

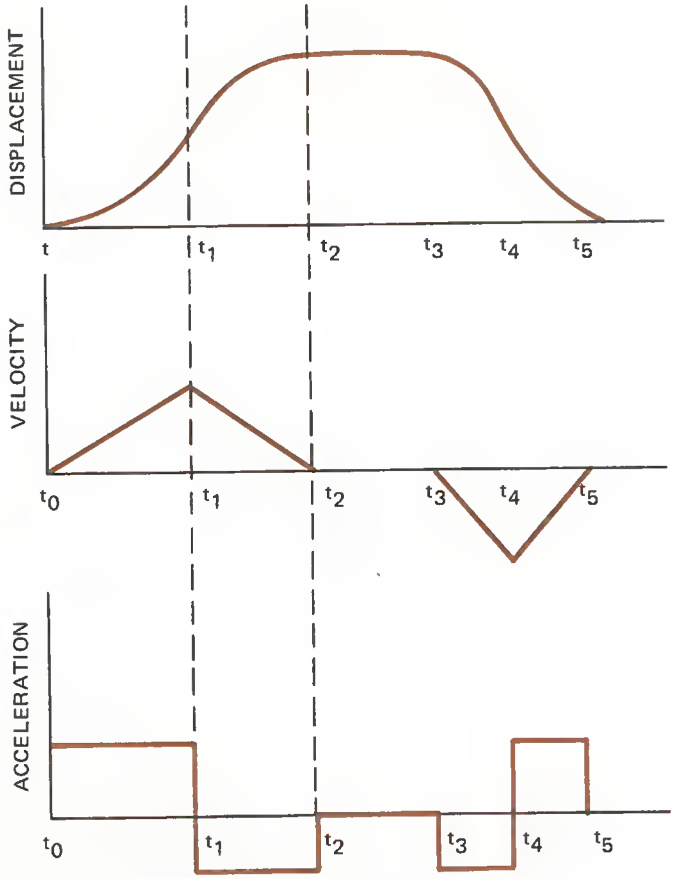


Fig. 17-6 Displacement, Velocity, Acceleration Graphs

Figure 17-6 is the graphical representation of parabolic motion. Remember that parabolic motion occurs when the acceleration is constant during the first half of follower motion and the deceleration (negative acceleration) is constant during the second half. The bottom graph in this figure is representative of acceleration. Acceleration is the change in velocity with respect to time (dv/dt), so at any point in time, the height of the acceleration at the point equals the slope of the velocity line.

Since acceleration is constant from t_0 to t_1, the velocity must be linear. For example, if the acceleration is a constant 2 ft/sec^2, the velocity must be changing 2 ft/sec every second. This same type of relationship holds between velocity and displacement.

Between t_0 and t_1 the velocity increases from zero to some finite value. The slope of the displacement curve must, accordingly, begin at zero and increase continually. At t_1, the velocity begins to decrease; thus, the slope of the displacement curve is high at t_1 and begins to tilt toward the horizontal until time t_2. At this time velocity is zero and the displacement curve must be horizontal (zero slope).

Between t_1 and t_2 the negative acceleration indicates a negative slope in the velocity curve. This is evident because the velocity curve angles from upper left to lower right.

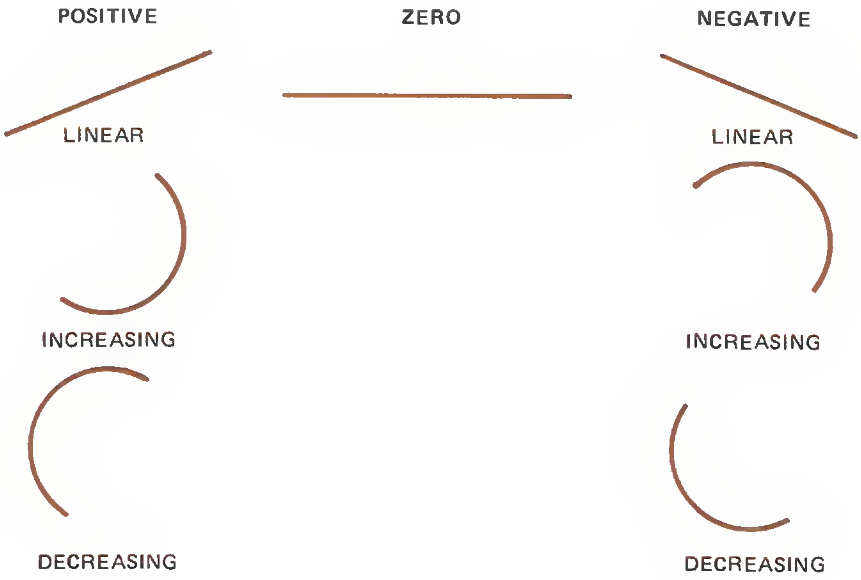


Fig. 17-7 Slope Representations of Curves of Motion

From this brief discussion you should be able to form some generalizations regarding these three related graphs. If these graphs are arranged in vertical order, then:

> The slope of the curve at any point on any diagram *equals*
>
> the height of the ordinate at that point on the next lower diagram.

There are seven different types of slopes a curve may have. Zero slope indicates a horizontal line. A positive slope moves upward to the right and can be "increasingly positive" (as in the displacement from t_0 to t_1), or can be "decreasingly positive" (displacement from t_1 to t_2), or linearly positive as in the velocity graph between t_0 to t_1. A negative slope can take on three different forms also: linearly negative (velocity from t_1 to t_2); increasingly negative (displacement from t_3 to t_4); or decreasingly negative (displacement from t_4 to t_5). Graphical representatives of these seven different slopes are shown in figure 17-7.

MATERIALS

- 2 Bearing plates with spacers
- 1 Breadboard with legs and clamps
- 1 Dial caliper (0 - 4 in.)
- 1 Cam follower roller approx. 1/4 in. OD
- 1 Protractor
- 1 Piece of sheet metal 4 x 4 x approx. 0.05 in. thick
- 1 Flat file
- 1 Hand drill and twist bit (3/8 in.)
- 6 Bearing holders with bearings
- 3 Collars
- 2 Disk dials
- 1 Lever arm approx. 4 in. long with 1/4-in. bore hub
- 1 Spur gear approx. 1-1/2 in. OD with 1/4-in. bore hub
- 1 Spur pinion approx. 3/4 in. OD with 1/4-in. bore hub
- 2 Dial indices with mounts
- 3 Shafts 4" x 1/4"
- 1 Lever arm approx. 2 in. long with 1/4-in. bore hub
- 1 Extension type spring approx. 1-1/2 in. long

PROCEDURE

1. Measure and record the diameter of the follower roller.

2. Using a sheet of white paper, draw the cam profile for a cam and a cam follower having the following specifications:

   Cam Rotation: Counterclockwise
   Type Follower: Pivoted or oscillating roller
   Diameter of Follower: As measured by you
   Length of Follower Arm: 2 in.
   Minimum Cam Radius: Left to discretion
   Follower Pivot Center: On same line with cam center
   Follower Movement: Vertical distance of 1 in. maximum
   Type Motion: Parabolic motion from initial point to maximum displacement during 180 degrees of cam rotation. Return motion during the next 180 degrees rotation using parabolic motion.

3. Transfer your cam profile to the piece of sheet metal and carefully cut it out.

4. File smooth any irregularities in the cam profile.

5. Put a 3/8-in. hole in the cam center and mount it on a universal pin hub.

6. Assemble the bearing plate assembly shown in figure 17-8.

7. Mount the bearing plate on the spring balance stand.

8. Rotate the cam until the follower is at its minimum displacement location.

9. Adjust the tension arm and spring on the output shaft so that it holds the follower against the cam

10. Set both dials to zero.

11. Rotate the cam dial to the 30° position and record both dial readings (Θ_i & Θ_o).

12. Repeat step 11 for cam dial positions of 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, and 360 degrees.

13. Carefully return the cam dial to the zero position.

14. For each data point compute the vertical displacement (y) of the follower.

15. Plot a curve of follower displacement (y) versus cam displacement (Θ_i).

16. Remove the cam and trace its profile in the space provided in the data table.

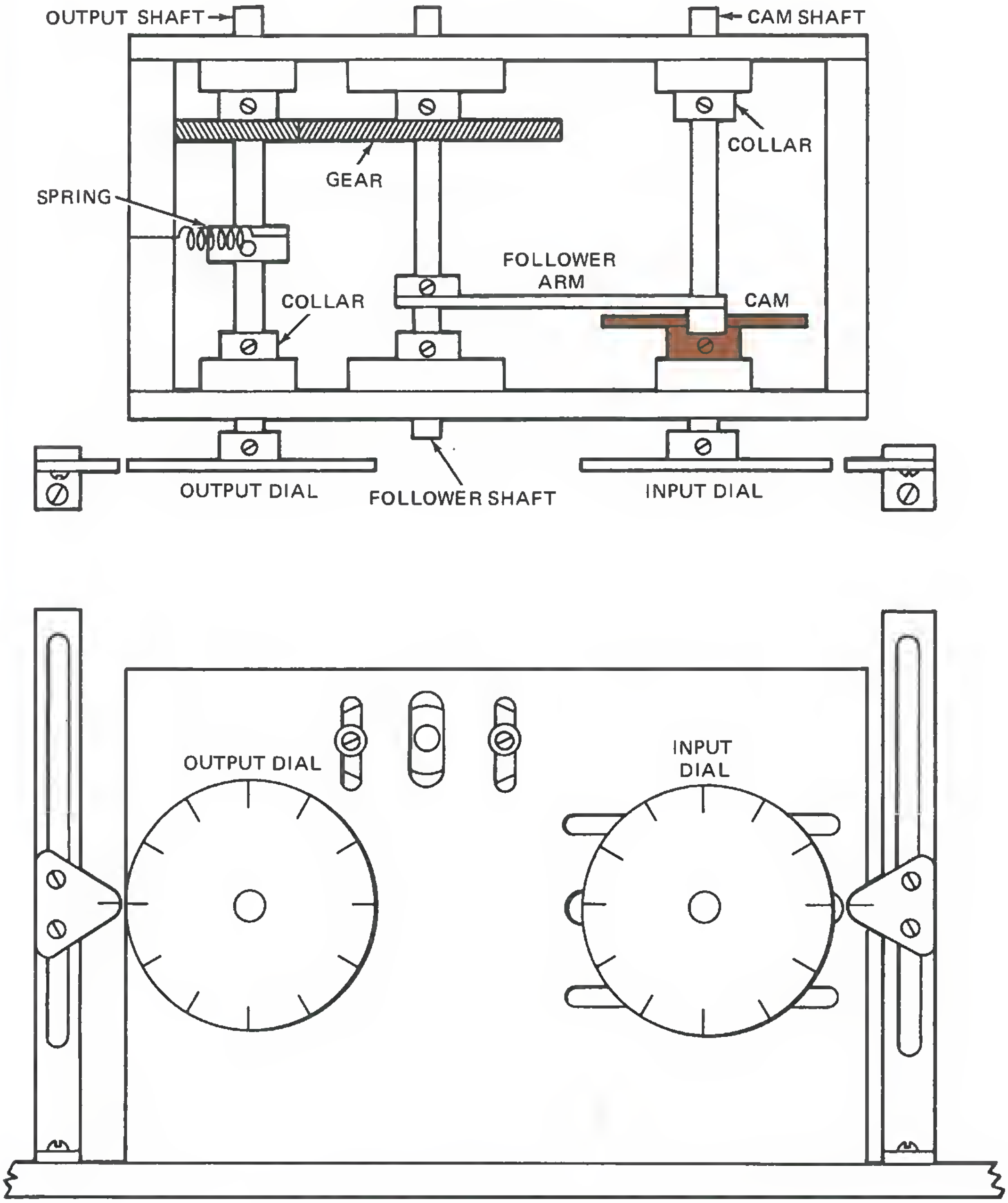


Fig. 17-8 The Bearing Plate Assembly

ANALYSIS GUIDE. Plot a graph of the data obtained during this experiment. Compare the measured displacement values with those computed during the layout of the cam. Explain any differences noted. Discuss the advantages and disadvantages of parabolic motion in comparison with linear and with simple harmonic motion when applied to cams. Discuss the reasons for using a pivoted cam follower. Add any comments of your own you deem appropriate.

Cam Profile

N_p	N_g	d

Gear & Follower Data

Θ_i	Θ_o	y

Fig. 17-9 The Data Tables

PROBLEMS

1. If the graph of displacement versus time is a second-degree equation, what degree equation represents velocity? What degree equation represents acceleration?
2. Is the displacement equation for parabolic motion during the first half of the follower rise a second-degree equation? Explain in detail, why or why not.
3. Draw a sketch showing the pressure angle for a roller follower having a short versus a long lever arm. What effect does the lever arm length have on the pressure angle?
4. Assume that the cam you used in this experiment rotates at a speed of 600 RPM. Draw the displacement, velocity, and acceleration graphs and list the maximum and minimum values for the ordinates of each graph.
5. Determine the angle through which the follower traveled in this experiment. If the cam rotated at 600 RPM, what was the maximum angular velocity and the average angular velocity of the cam follower assembly?

experiment 18 MULTIPLE CAM TIMING

INTRODUCTION. Cams are often used in groups to produce motions which have fixed time relationships. In this experiment we shall examine a simple example of such multiple cam timing.

DISCUSSION. Cams are often used in groups to establish definite time relationships between independent operations. Let's consider the two cams shown in figure 18-1. In this case the two cams are gear-coupled and, therefore, have related angular positions. As the lefthand gear rotates, the position of follower a is determined by the profile of cam A. Similarly, the position of follower b is determined by the profile of cam B. Since the two cams are gear-coupled, the followers' motions are definitely related to each other.

If we sketch follower position versus time, the result will be somewhat like figure 18-2. In this particular case the gear ratio between the two cams is one-to-one. Moreover the cams are set up so that when one follower is "on" its cam, the other is "off" its cam.

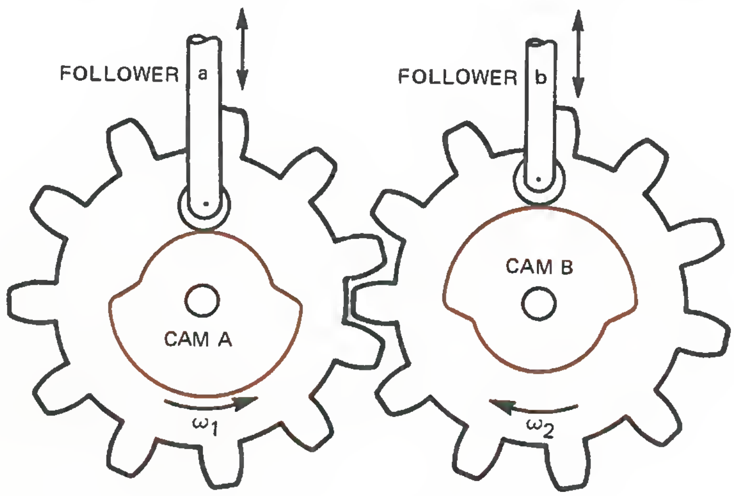


Fig. 18-1 Coupled Cams

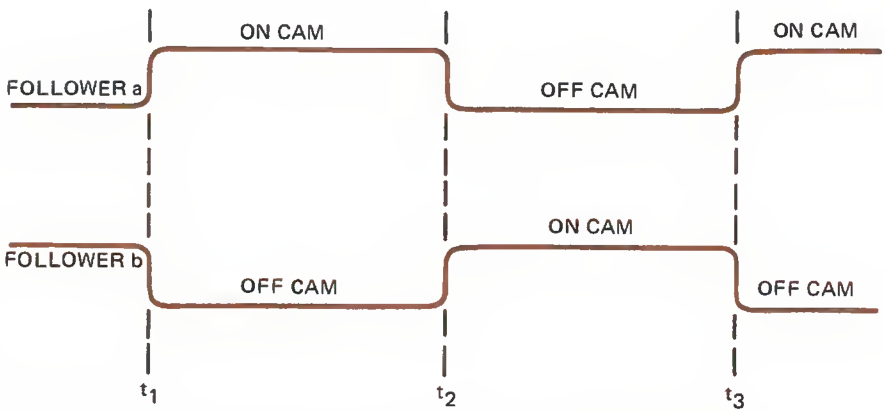


Fig. 18-2 Follower Positions Versus Time

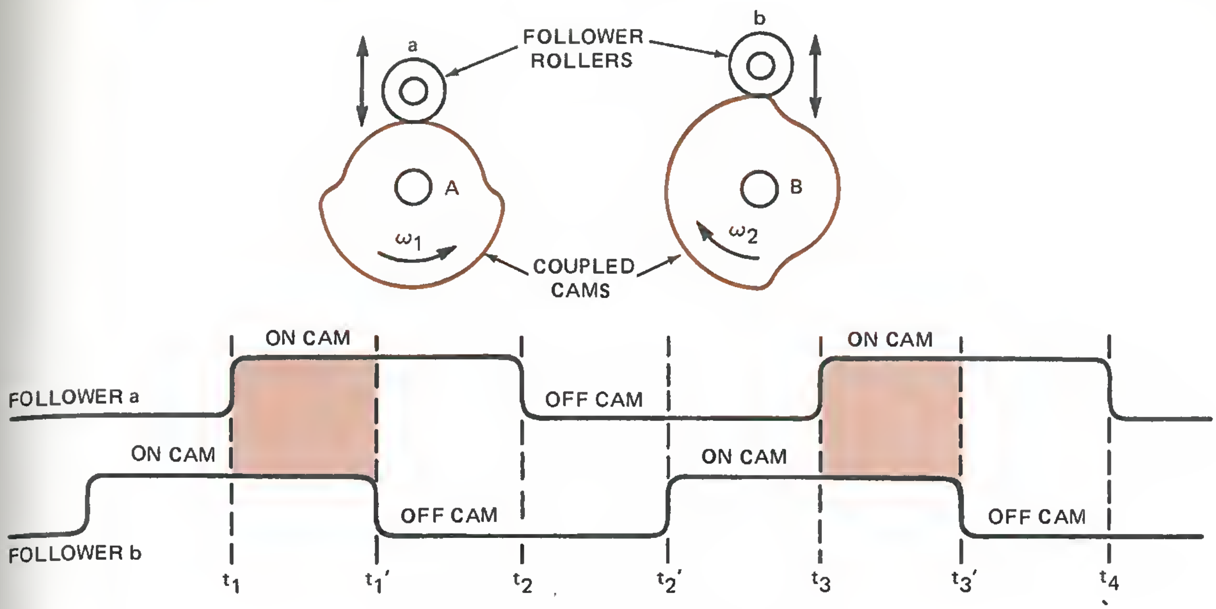


Fig. 18-3 Overlapping Cam Action

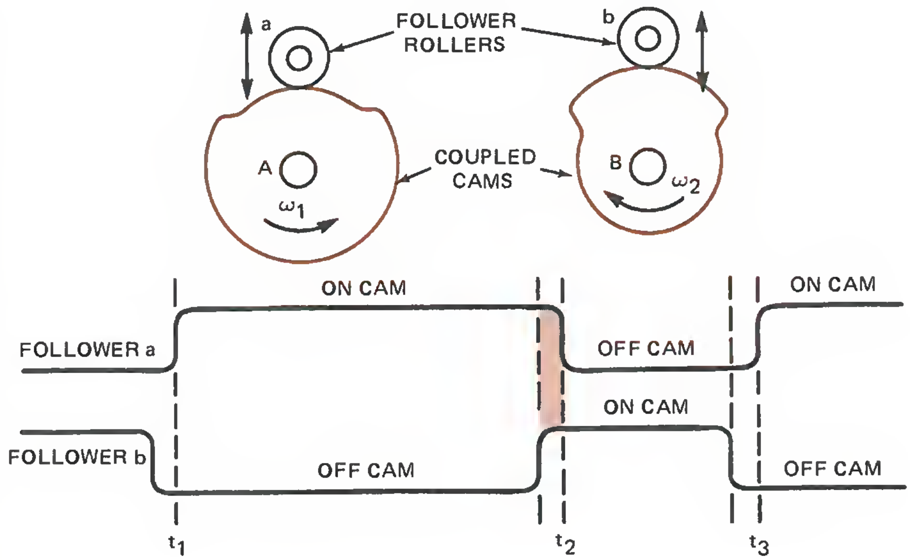


Fig. 18-4 Unequal Synchronized Dwell

By rotating one cam with respect to the other, we can get overlapping action as shown in figure 18-3. The colored areas represent the times during which both followers are on the cam simultaneously. In this illustration the follower rods and cam couplings have been omitted for simplicity.

It should be apparent that by rotating one cam with respect to the other we can produce any desired amount of overlap.

So far we have used two cams which have had approximately equal dwell angles. This is, of course, not at all necessary. Many applications require unequal but synchronized dwell times. Figure 18-4 shows such an arrangement. Also shown is a small amount of overlap.

By adjusting the angular velocity of a cam and its dwell angle, we can produce a wide range of dwell times.

The coupling between cams may be virtually any type of positive drive mechanism. Gears, tooth belts, chains, rigid couplings and solid shafts are all used occasionally to couple cams together. Up until now we have considered only cams coupled by a 1:1 velocity ratio. It is certainly possible to use other ratios. Figure 18-5 illustrates two cams coupled by a ratio of approximately 2:1.

In the case of a 2:1 velocity ratio, the cam on the pinion would rotate twice as fast as the one on the gear. This would cause the pinion cam follower to go through its cycle twice as often as the gear cam follower. The same possibilities for overlap and unsymmetrical dwell are, of course, still possible.

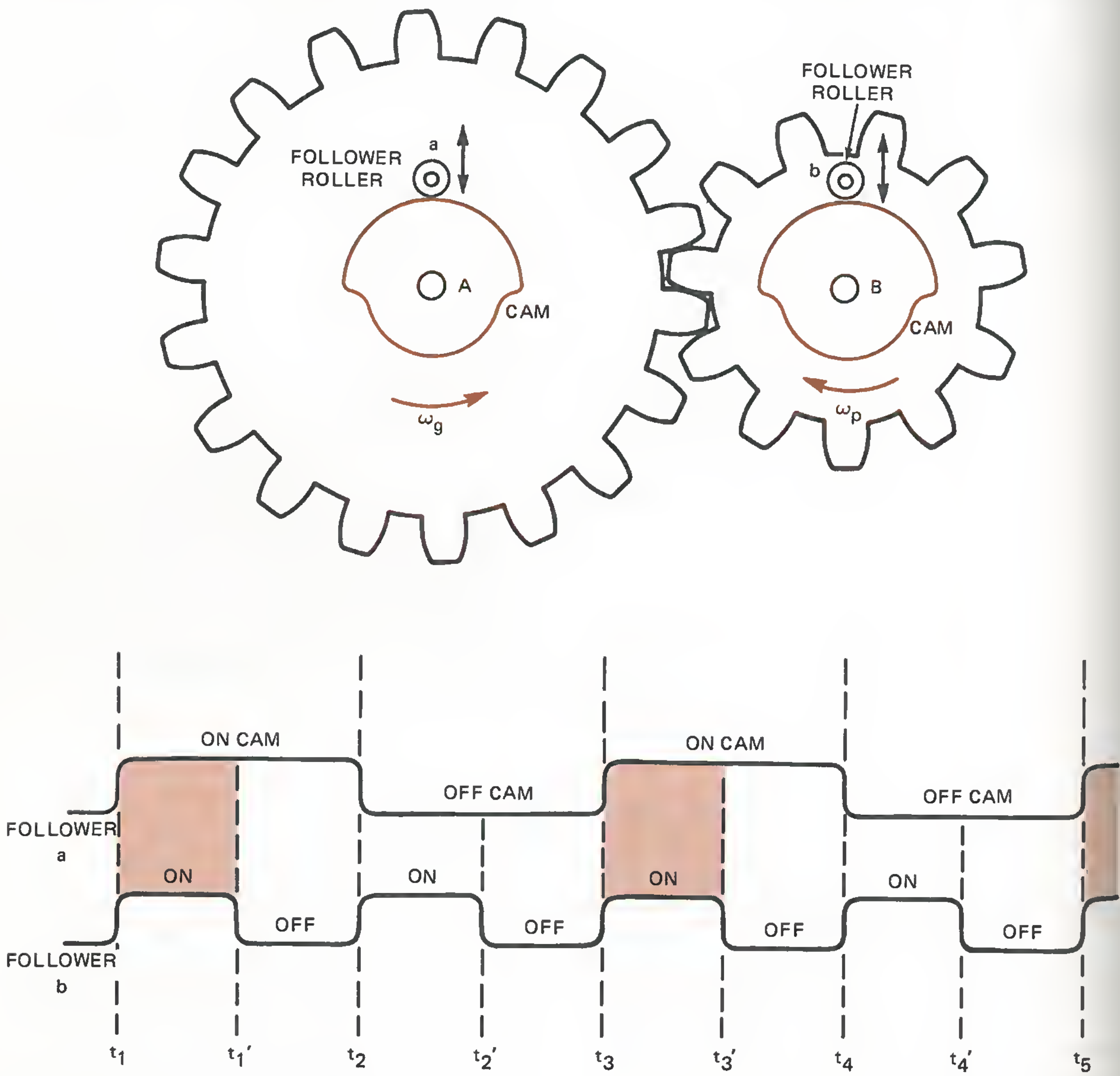


Fig. 18-5 Two-to-One Cam Coupling

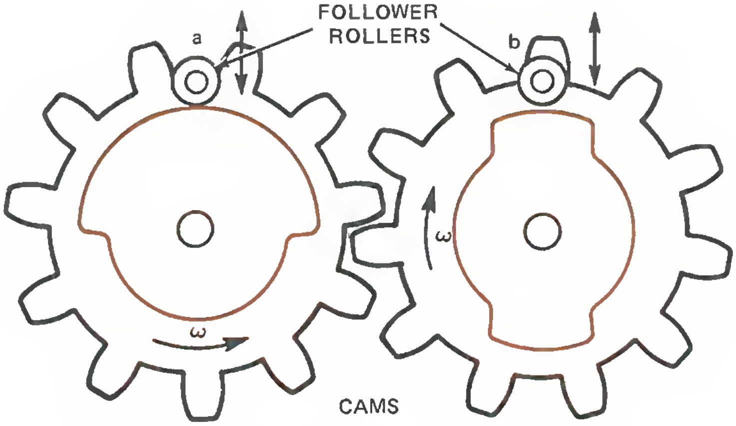


Fig. 18-6 A Multilobe Cam Timing Device

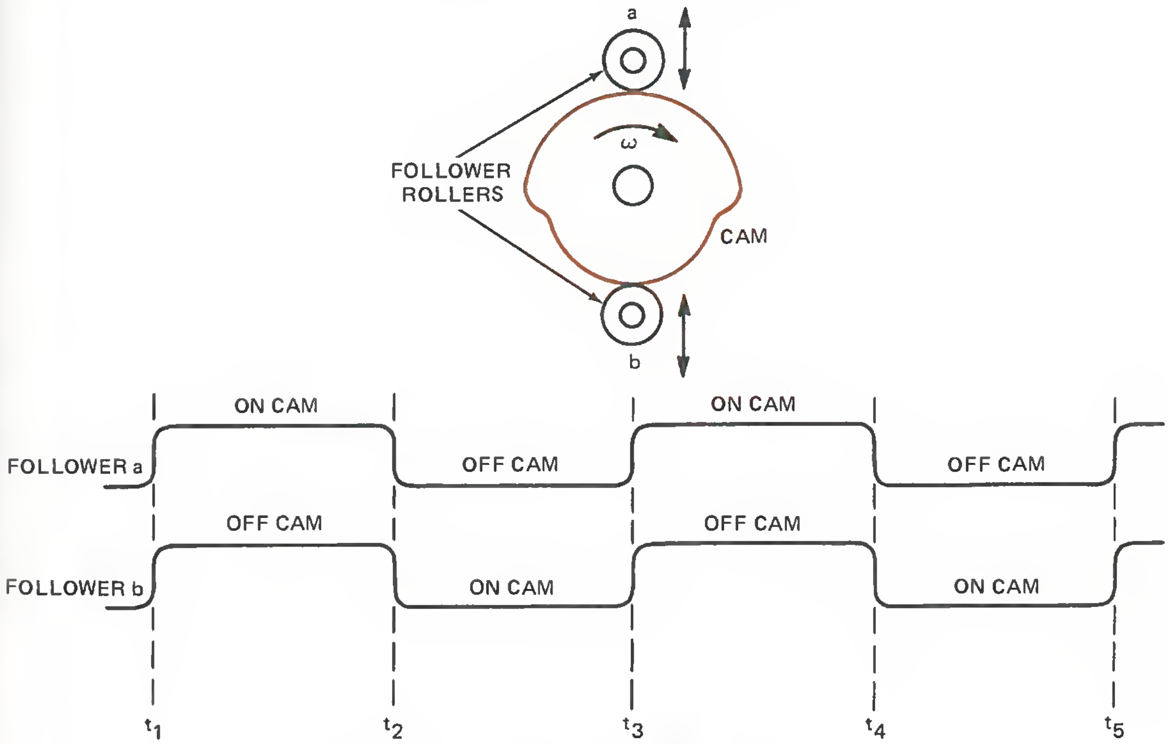


Fig. 18-7 Cam with Two Followers

Another way to produce substantially the same type of follower relationship is to use multilobe cams. Figure 18-6 shows such an arrangement.

In all these cam configurations the follower motions may be used to actuate mechanical, electrical, or other devices.

In many cases two or more cams have the same profile. In such instances it is often possible to use multiple followers. Figure 18-7 shows one such case. Notice that while both followers in this case have the same motion, they are alternately on the cam. If the desired output is produced when the follower is on the cam, then the two followers

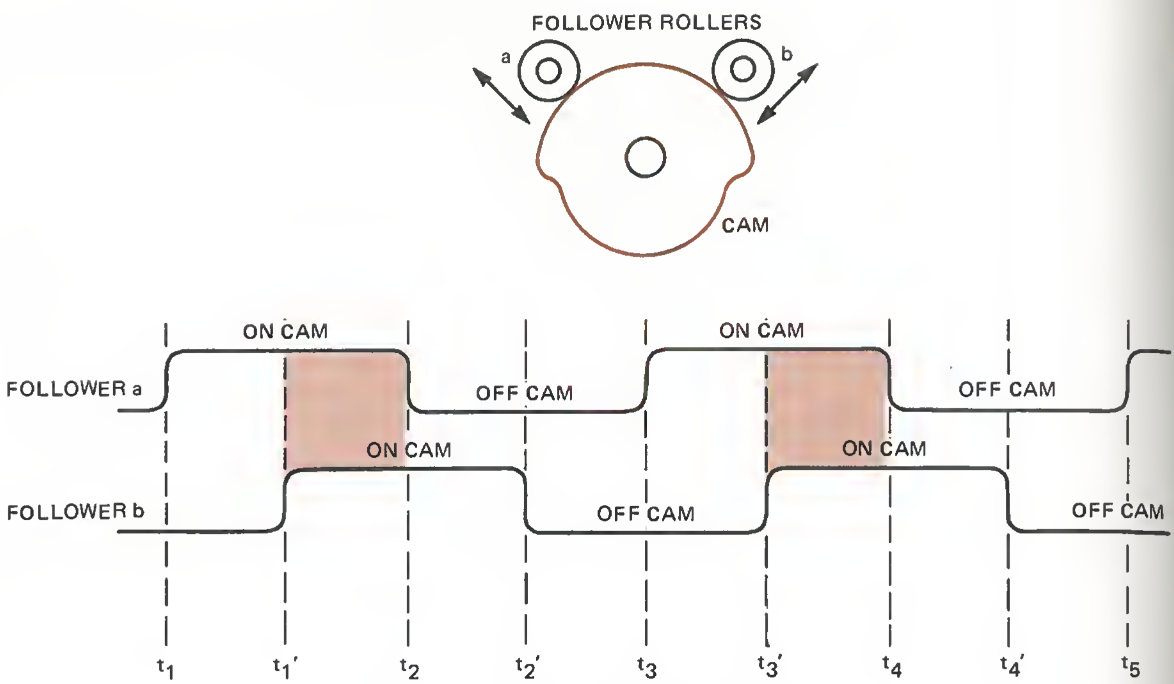


Fig. 18-8 Overlapping Follower Action

alternate output action. This arrangement then is equivalent to the one shown in figure 18-1.

By locating the followers appropriately we can produce overlap in a multiple follower configuration. One possibility is illustrated in figure 18-8. In this case the follower motions are substantially the same as those of figure 18-3.

When using multiple followers in this way, we can produce overlap from 0 to 100% depending on follower location, cam size, and follower size. In order to get 100% overlap it is necessary to use a cam that is thick enough to allow side-by-side follower mounting.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
6 Bearing mounts with bearings
2 Shaft hangers with bearings
4 Shafts 4" x 1/4"
2 Adjustable cams with 1/4-in. bore hubs
2 Index mounts
2 Microswitches with mounting hardware
6 Collars
1 Worm
1 Worm wheel
1 Bevel pinion with 1/4-in. bore hub
1 Bevel gear with 1/4-in. bore hub
2 Spur pinions approximately 3/4 in. OD with 1/4-in. bore hub
2 Spur gears approximately 2 in. OD with 1/4-in. bore hub
1 DC motor with mount
1 DC power supply 0 - 30V
1 Lamp assembly (3 lamps)with connecting leads

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Construct the worm drive assembly shown in figure 18-9. The dimensions indicated are only approximate.
3. Construct the bearing plate assembly shown in figure 18-10. The dimensions indicated are only approximate.
4. Mount the bearing plate assembly, the worm drive assembly, motor, switches, and lamp assembly on the breadboard as shown in figure 18-11.
5. Adjust the various shaft spacings for smooth operation of the entire mechanism.
6. Connect the lamp assembly, switches, motor, and power supply as shown in figure 18-12. Carefully arrange the wires so that they will not foul the mechanism.
7. Turn on the power supply and set the voltage to about 15V. The mechanism should run freely, and the lamps should blink as the cams rotate. If all the lamps do not blink, check the switch mountings to insure that the cams are operating the switches. If the lamps still don't blink, check your wiring and the lamps themselves.
8. Adjust the cams so that the lamps light in the following sequence:

   First lamp 3 on for 1/2 of a cam revolution,
   then lamp 2 on for 1/6 of a cam revolution,
   then lamp 1 on for 1/3 of a cam revolution,
   finally return to lamp 3 on.

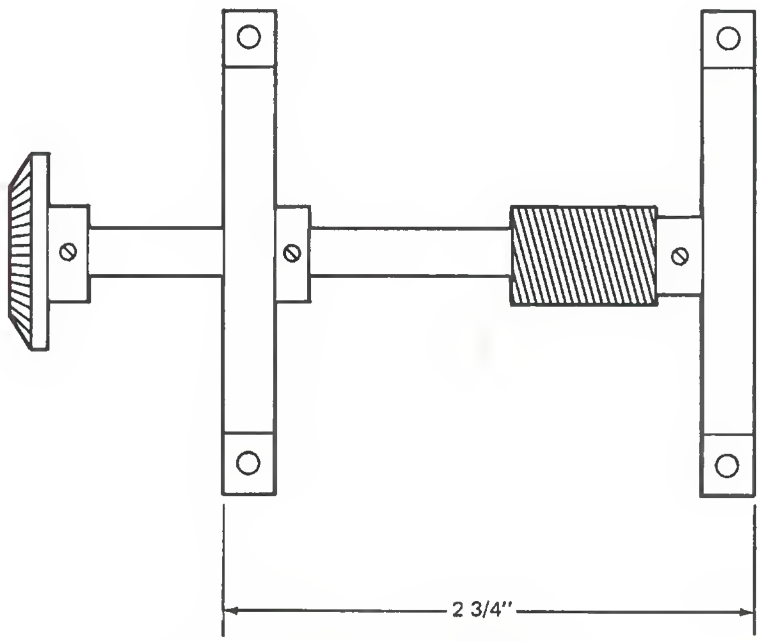


Fig. 18-9 The Worm Drive Assembly

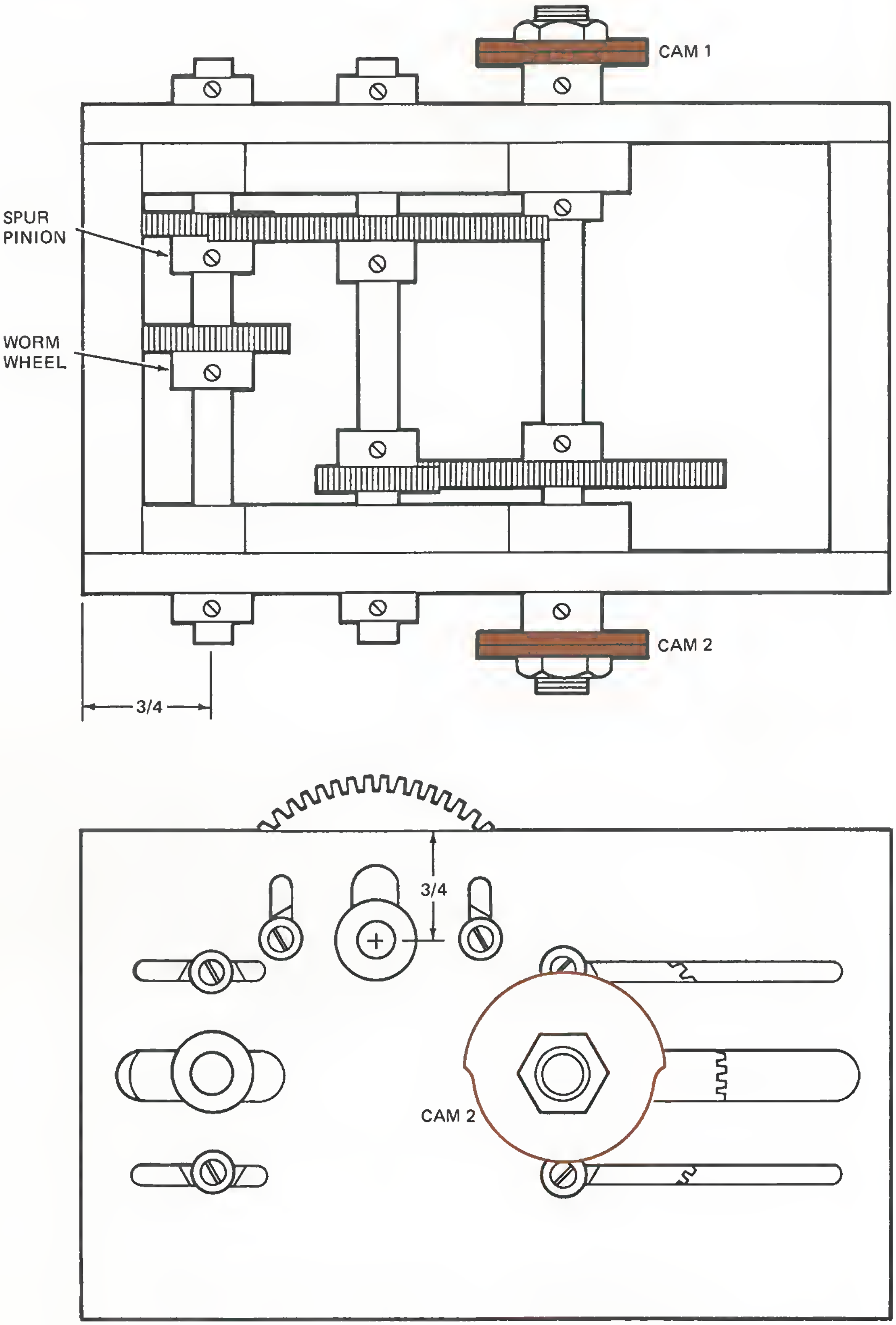


Fig. 18-10 Bearing Plate Assembly

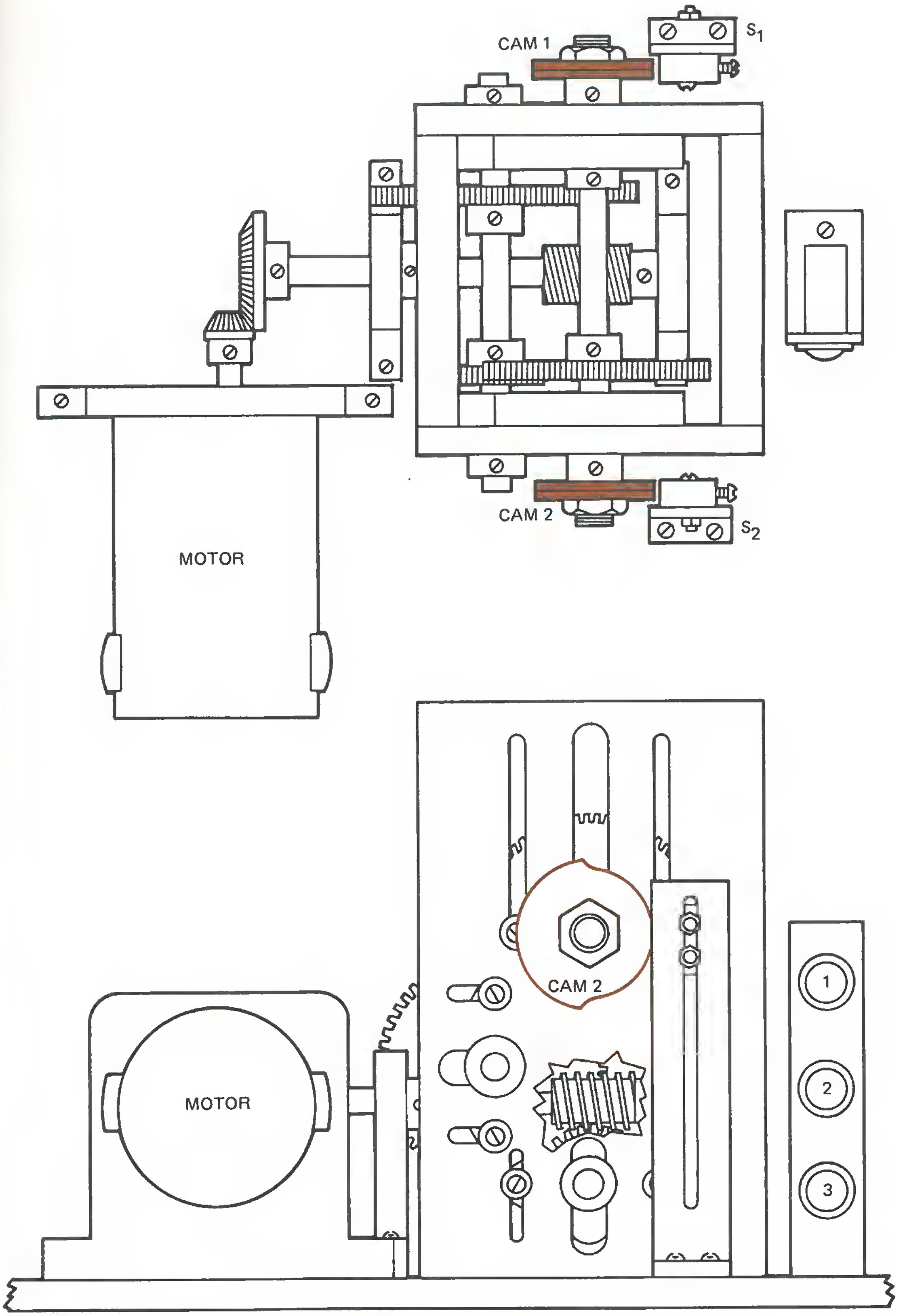


Fig. 18-11 The Experimental Mechanism

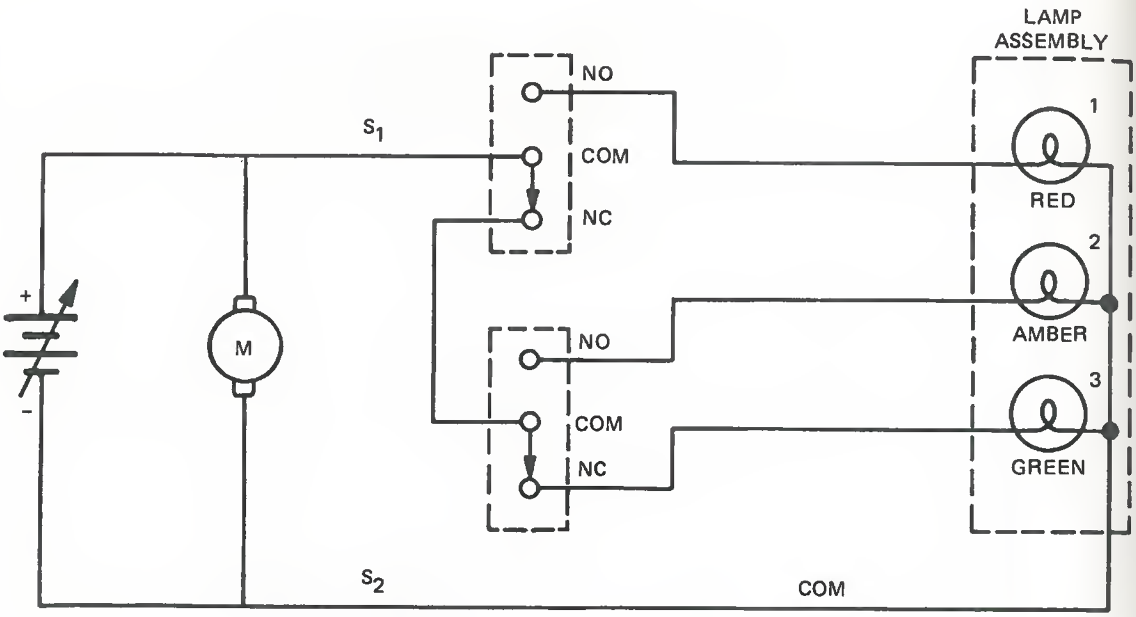
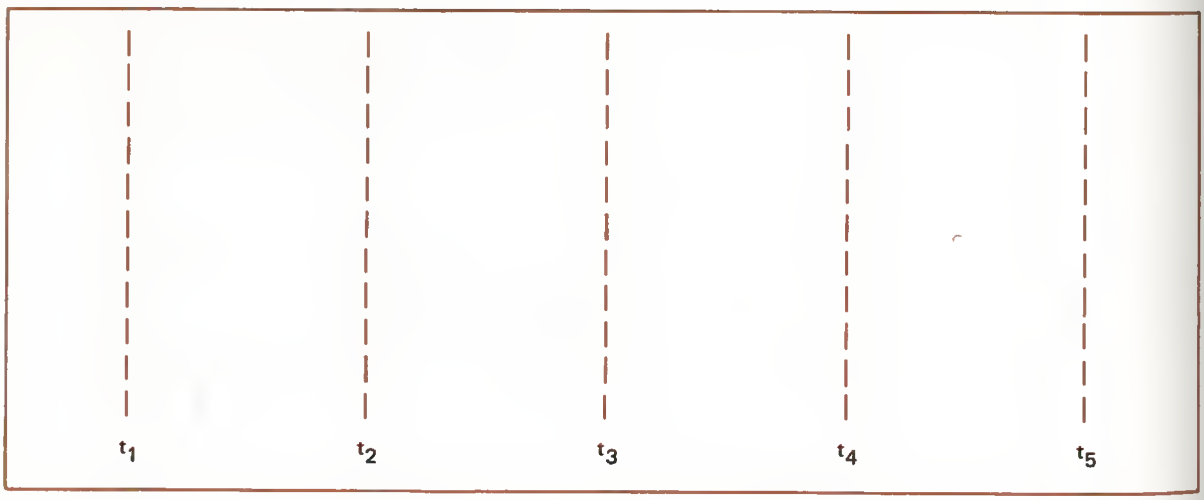


Fig. 18-12 The Electrical Circuit

9. Draw a scale sketch of the relative follower positions similar to those in the discussion. Label each segment showing which lamp is on.

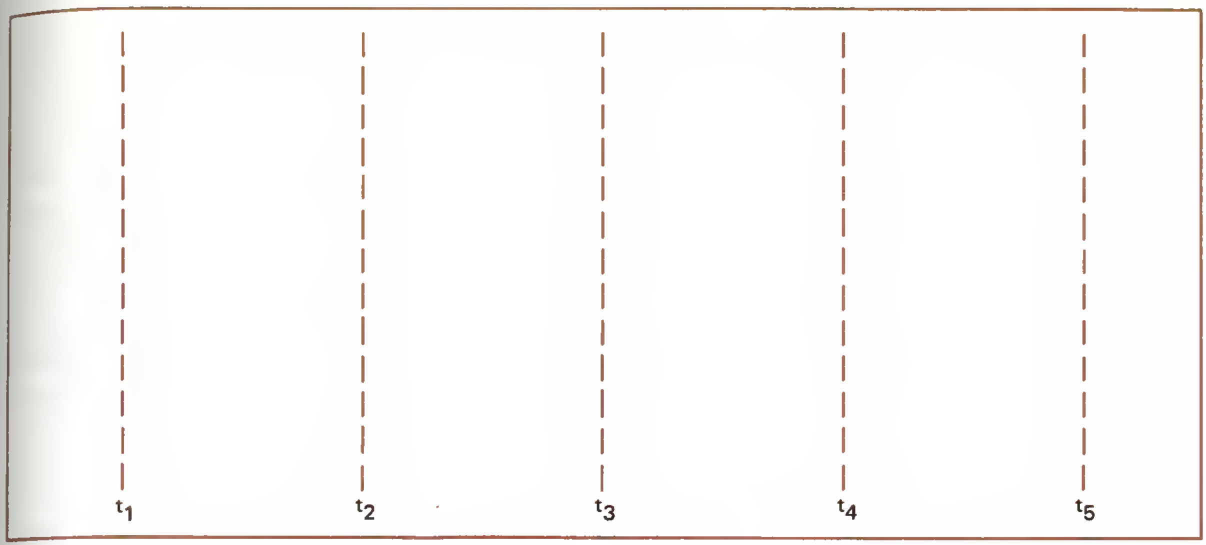
10. Readjust the cams so that the lamps light in a sequence that is the reverse of the one in step 8.

11. Repeat step 9 for the sequence in step 10.



RESULTS FROM STEP 9

Fig. 18-13 The Experimental Results



RESULTS FROM STEP 11

Fig. 18-13 The Experimental Results (Cont'd)

ANALYSIS GUIDE. In the analysis of these results you should compare the follower patterns for each setup. Discuss the similarities and differences between the follower patterns. Finally, list and discuss at least three applications of multiple cam timing.

PROBLEMS

1. Make a sketch showing how three cams could be used to produce the same results as those observed in the experiment.
2. Repeat problem 1 using only one cam and two followers.
3. If a cam has a dwell angle of 180° and is rotating at 35 RPM, how long (in seconds) is the follower on the cam?
4. What would be the result if the dwell angle in problem 3 were 110°?
5. How long is the follower in problem 4 off the cam?
6. The cam in problem 4 is directly coupled to a second cam with a dwell angle of 80°. If the lobes are displaced from each other by 45°, what would be the overlap time? (Assume that the 110° cam has its follower come on the lobe 45° before the other one comes up on its lobe.)

experiment 19 HARMONIC DRIVES

INTRODUCTION. In modern mechanical power transmission systems, high-ratio gear speed reductions are often necessary. This task can sometimes be accomplished effectively with a harmonic drive transmission. In this experiment we will examine one type of harmonic drive.

DISCUSSION. The harmonic drive transmission is an efficient, small, light-weight method for getting gear speed reductions. Ratios of more than 300:1 can be efficiently achieved in a single reduction. Harmonic drives are relatively inexpensive, simple, and can have low to zero backlash.

The basic harmonic drive transmission consists of three parts: a wave generator, flexspline, and circular spline. Figure 19-1 illustrates the parts. For speed reductions the input goes to the wave generator, and output is taken from the flexspline with the circular spline held stationary.

Figure 19-2 illustrates the components of the transmission meshed together for operation as a gear speed reducing device. The wave generator distorts the shape of the smaller diameter flexspline, and when rotated, the reduction ratio is

$$\boxed{\frac{\omega_o}{\omega_i} = \frac{n}{n - N}} \qquad (19.1)$$

where n is the number of teeth on the flexspline and N, the number on the circular spline. Because of the greater circumference of the circular spline, it contains more teeth and the ratio will be a negative quantity, indicating that the output rotational direction is opposite to the input.

Various metals and plastics may be used in the manufacturing of harmonic drives.

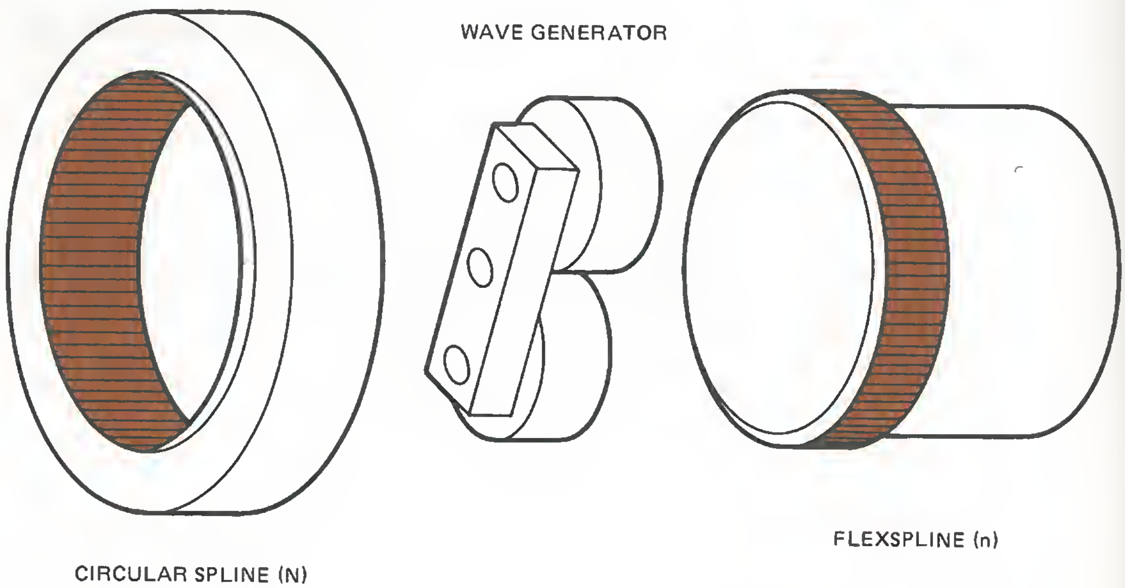


Fig. 19-1 Components of a Harmonic Drive

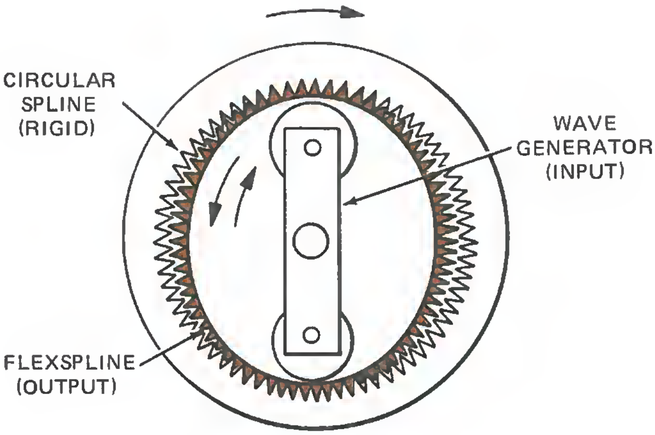


Fig. 19-2 Harmonic Drive

Stainless steel is a common material for making flexsplines for units to be used in heavier load applications. Harmonic drives also come in a variety of configurations, all utilizing the same mechanical principles. Figure 19-3 illustrates some of the possibilities.

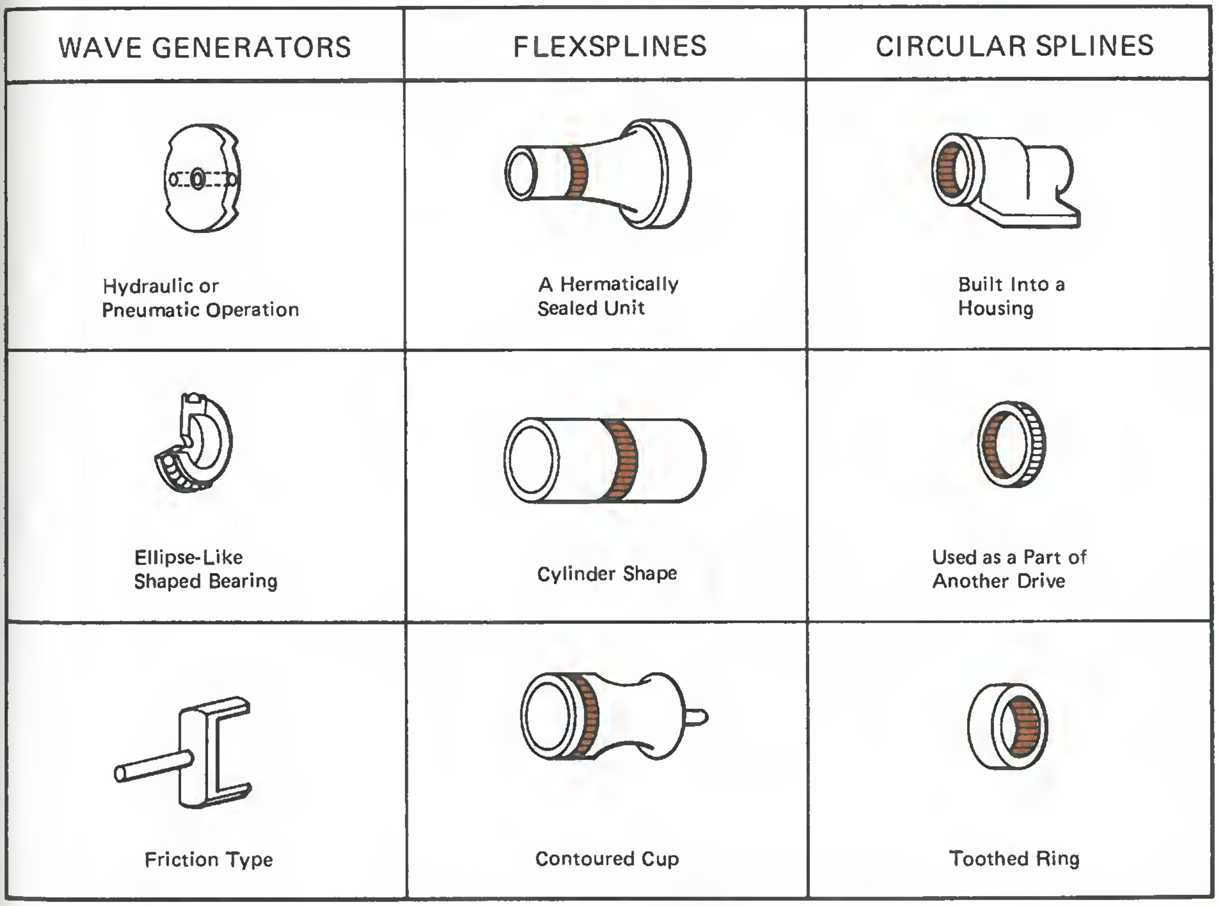


Fig. 19-3 Variations of Harmonic Drives

MATERIALS

- 1 Harmonic drive with mount
- 2 Bearing plates with spacers
- 1 Stroboscope
- 1 DC motor with mount
- 1 Universal joint
- 1 Microswitch mounted on an index mount
- 1 Lamp and holder
- 1 Power supply
- 4 Bearing holders with bearings
- 1 Breadboard with legs and clamps
- 1 Adjustable cam with 1/4-in. bore hub
- 2 Spur gears, approx. 3/4-in. OD with 1/4-in. bore hubs
- 2 Spur gears, approx. 2 in. OD with 1/4-in. bore hubs
- 2 Collars
- 2 Shafts 4" x 1/4"

PROCEDURE

1. Examine all of the components to insure they are not damaged.
2. Locate the spur gears. Count and record the number of teeth on each. Calculate the ratios of each reduction shown in figure 19-4.
3. Identify the flexspline and circular spline of the harmonic drive. Count and record the number of teeth on each.
4. Calculate the speed ratio from the motor to the output shaft $\frac{\omega_m}{\omega_o}$.

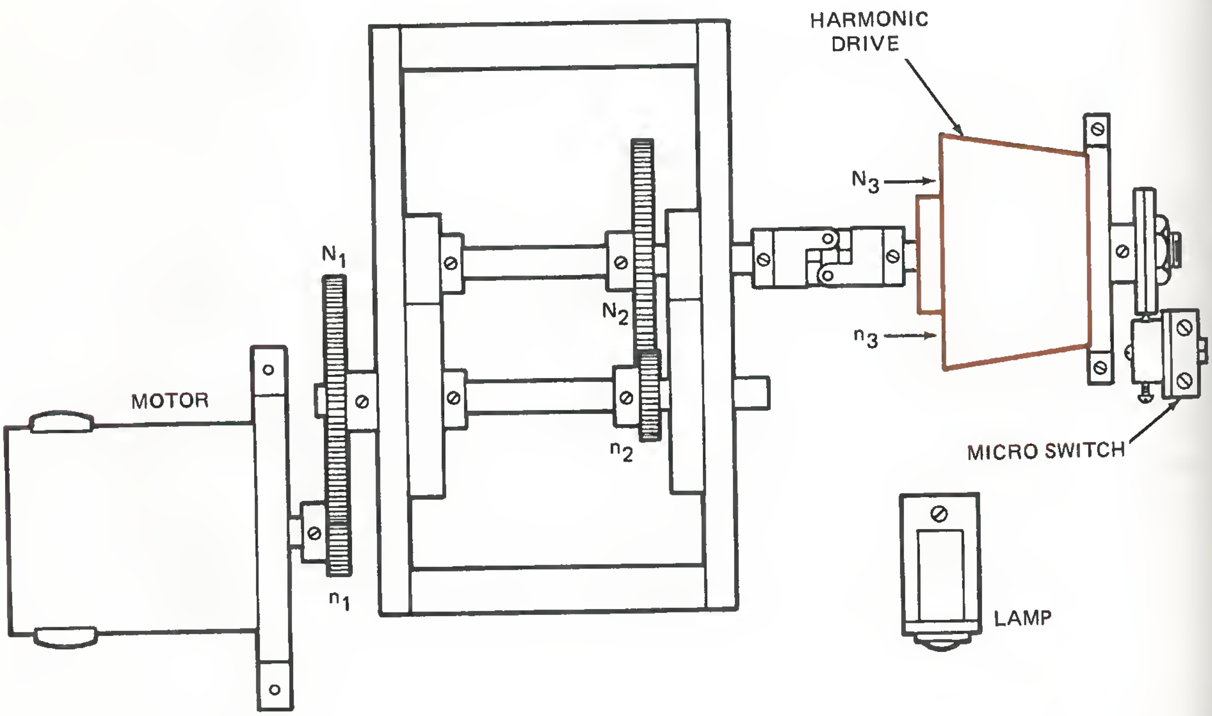


Fig. 19-4 Experimental Setup

5. Assemble the mechanism shown in figure 19-4. Connect the lamp to the microswitch and power supply so that it will turn on when the plunger is depressed by the lobe on the cam.
6. Hand rotate mechanism to insure proper operation before applying power to the motor.
7. Connect the motor to the DC power supply and set the voltage to about 20 volts.
8. Using the stroboscope measure the angular velocity of the motor shaft (ω_i) and the universal joint (ω_u).
9. Count the output angular velocity (ω_o) by using the second hand on your watch and counting the light flashes. Better accuracy can be achieved by counting for three minutes and dividing result by 3 to get revolutions per minute.

n_1 (3/4 in. OD)	N_1 (2 in. OD	n_2 (3/4 in. OD)	N_2 (2 in. OD)	$\frac{N_1}{n_1}$	$\frac{N_2}{n_2}$

Spur Gears

n_3	N_3	$n_3 - N_3$	Ratio $\frac{n_3}{n_3 - N_3}$

Harmonic Drive

	ω_i	ω_u	ω_o	$\frac{\omega_i}{\omega_o}$ ratio
Calculated				
Measured				

Fig. 19-5 The Data Table

ANALYSIS GUIDE. In the analysis of your results you should explain how a velocity reduction is accomplished. Explain the relation of the harmonic drive to the cams you have studied. (That is, explain why a harmonic drive could be considered a special cam application.) Give some applications for the harmonic drive transmission.

PROBLEMS

1. Is the harmonic drive positive or does some slippage exist?
2. Explain how the harmonic drive should be connected to get a gear speed increase.
3. The input is applied to the wave generator, the flexspline is held stationary and the output is taken from the circular spline. What is the *direction* of output rotation compared to the input?
4. The circular spline is held stationary, flexspline used as the input and the output taken from the wave generator. Is this application of the harmonic drive a speed reducer or increaser?
5. Determine a set of gears for a harmonic drive that will give a reduction of 200:1.
6. If for some application the motor is delivering a torque of 50 in.-oz, what would be the output torque for 80% overall efficiency?

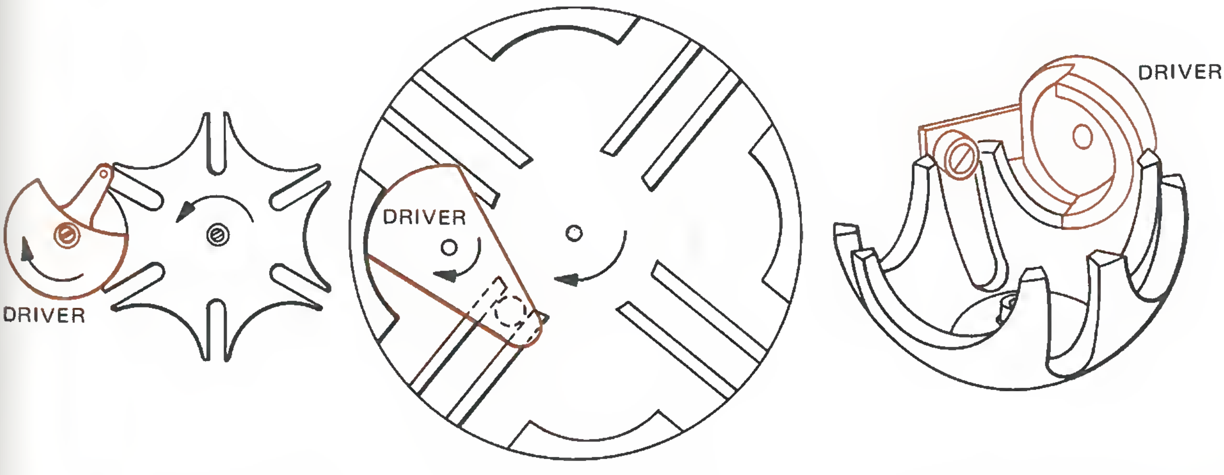
experiment 20 INTRODUCTION TO THE GENEVA MECHANISM

INTRODUCTION. A mechanism producing intermittent motion with a constant velocity input is sometimes needed in mechanical drive systems. In this experiment we will explore the basic operation of such a mechanism, the geneva wheel. Such wheels are often used to get an intermittent mode of operation.

DISCUSSION. Geneva wheels come in three basic configurations; they may be external, internal, or spherical. Each of these types of geneva mechanism is illustrated in figure 20-1. The wheels may have from 3 to a great many slots but usually have from 4 to 18. Fewer slots give high accelerations and a large number of slots make the diameter of the star wheel relatively large.

As the driving member of the geneva wheel is rotated, the roller engages the slots of the driven member (star wheel) and turns it. The distance turned depends on the number of slots in the star wheel. The dwell time (time the roller is not engaged in the slots) varies with each style of geneva wheel and the number of slots.

Because the roller always enters and leaves the slot of a spherical wheel at the equator of the sphere, it is engaged for 180 degrees and therefore has a dwell angle of 180 degrees. The external type has a dwell of more than 180 degrees and the internal one's dwell is less than 180 degrees. The long drive time of the internal type gives the advantage of lower accelerations because of the greater time available to reach necessary velocity. However, the internal geneva wheel mechanism is more difficult to mount because the input shaft cannot be a through shaft. The



(A) SIX SLOT EXTERNAL (B) FOUR SLOT INTERNAL (C) FOUR SLOT SPHERICAL

Fig. 20-1 Geneva Drive Mechanisms

crank must be fastened to the overhanging end of the input shaft. Because of the relative mechanical simplicity, the external geneva drive is most often used in cases where dwells of more than 180 degrees are acceptable.

An external geneva wheel with 8 slots would turn a total 1/8 of 360 degrees for each rotation of the driving wheel. A 4-slot wheel would turn 1/4 of 360 degrees and, in general, the rotation angle of the star wheel, Θ, is

$$\boxed{\Theta = \frac{360^\circ}{n}} \qquad (20.1)$$

where n is the number of slots.

Observing the diagram in figure 20-2

$$a = \ell_1 \cos \alpha$$

and

$$b = \ell_o - \ell_1 \cos \alpha$$

Since the maximum value of α measured from the center line is $180^\circ/n$, then

$$\boxed{\ell_o = \frac{\ell_1}{\sin \frac{180^\circ}{n}}} \qquad (20.2)$$

which is the distance required between the input and output shafts. ℓ_1 is the length of the crank arm and n is the number of slots.

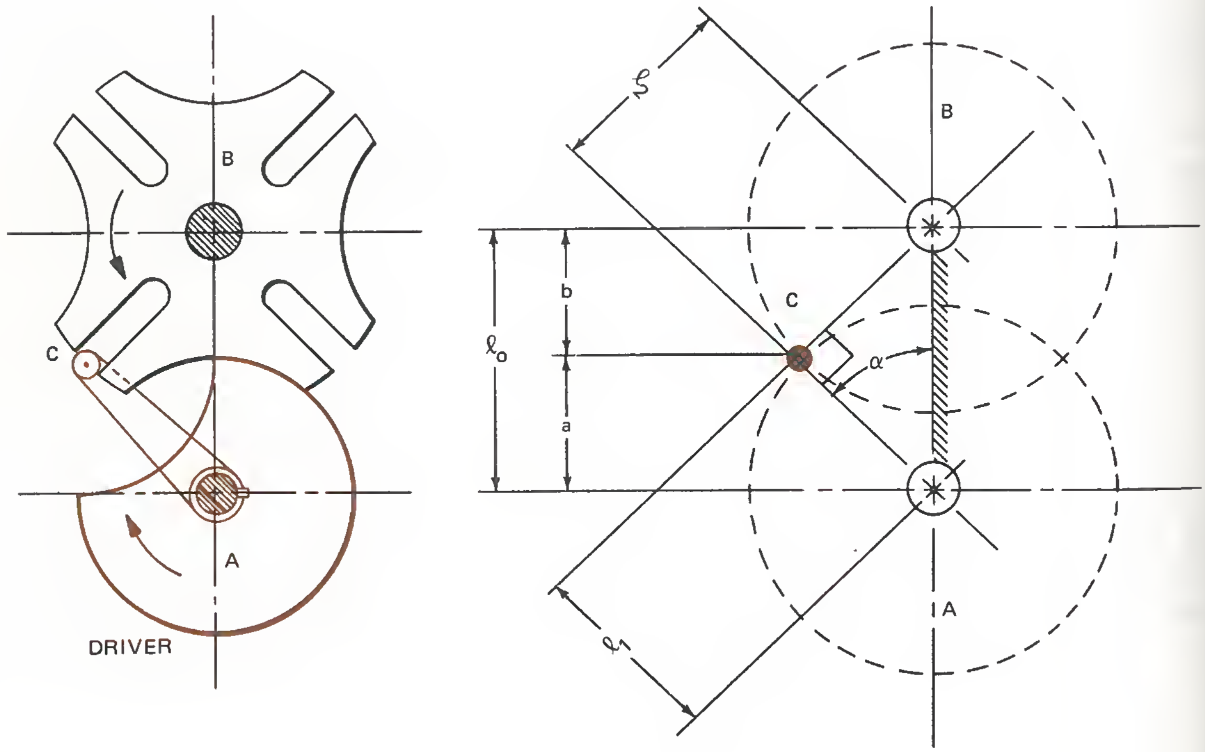


Fig. 20-2 Geneva Wheel with Simplified Sketch

MATERIALS

1 Breadboard with legs and clamps
2 360° disk dials
2 Dial indices with mounts
2 Bearing plates with spacers
2 Shafts 4" x 1/4"
4 Bearing mounts with bearings
1 Geneva wheel mechanism
4 Collars
1 Dial caliper (0 - 4 in.)

PROCEDURE

1. Inspect each component you plan to use to insure that it is undamaged.
2. Identify the geneva wheel and place the parts together with the roller *not* engaged in a slot of the star wheel.
3. Measure the length of the crank arm of the driving mechanism and the distance between the shafts. Record them in the data table.
4. Calculate the distance between the shafts using equation 20.2.

	Crank arm length, ℓ_1	Distance between shafts, ℓ_o
Measured		
Calculated		

Input Angle (degrees)	Output Angle (degrees)
0	
5	
10	
15	
25	
29	
30	
35	
40	
45	
50	
55	
60	

Input Angle (degrees)	Output Angle (degrees)
65	
70	
75	
80	
85	
90	
100	
150	
200	
250	
300	
350	
360	

Fig. 20-3 The Data Tables

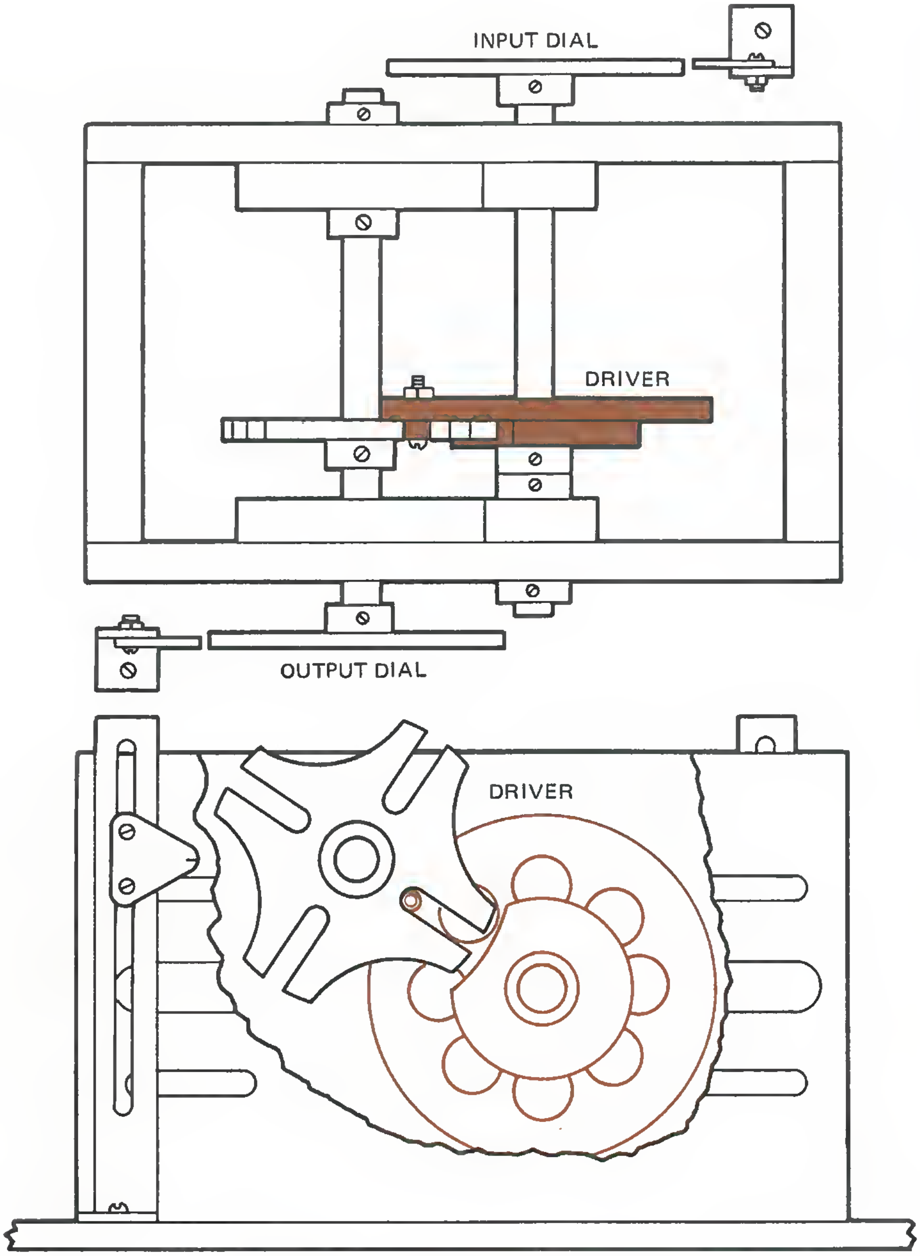


Fig. 20-4 The Experimental Setup

5. Construct the mechanism as shown in figure 20-4.

6. Set the geneva wheel so that the roller is just beginning to enter the slot of the star wheel.

7. Hold the geneva wheel fixed and set both the input and output dials to zero.

8. Read and record output angular displacement for each value of input given in the data table.

9. Plot a graph of the angular displacement of the driving crank (input) versus the angular displacement (output) of the driven member.

ANALYSIS GUIDE. In your discussion of the results achieved in this experiment include a description of the output motion discussion. Explain how the velocity of the output changes as the input rotates through 360 degrees. Use your graph to illustrate your explanation.

PROBLEMS

1. Calculate the distance between the shafts of an 8-slot geneva wheel that has a driving crank length of 1 inch.
2. How does the ratio of input and output velocity vary with the number of slots of a geneva wheel (averaged over long time periods)?
3. What are some of the possible problems that one might encounter in driving geneva wheel mechanisms at high RPMs?
4. The external geneva wheel has a greater dwell than drive time and the opposite relationship is true for the internal device. For equal input velocities, which would you expect to experience lower acceleration and why?
5. How many degrees does a 10-slot star wheel turn during each revolution of the driven member?
6. What is the dwell time for a 10-slot external geneva wheel mechanism?
7. What is the major advantage of the internal type compared to the external type geneva mechanism?

experiment 21 LOADING GENEVA MECHANISMS

INTRODUCTION. Varying load conditions are an important consideration in the operation of mechanical devices. In this experiment we will examine an example of how the load on a driven shaft can vary even though there is a constant frictional load on the output of the mechanism.

DISCUSSION. The driven member of the geneva mechanism is part B in figure 21-1 and the driving member is part A. As the driver rotates and engages the driven member, the effective lever arm length ℓ_2 changes as the roller moves through the arc between points C to E. The relationship of ℓ_2 to the angle α is

$$\ell_2 = \ell_1\sqrt{1 + m^2 - 2m\cos\alpha}$$

where

$$m = \frac{1}{\sin\frac{180°}{n}} \quad \text{and} \quad n = \text{No. of Slots}$$

During the rotation from C to E, lengths a_1 and b_2 vary according to

$$a_1 = \ell_1 \cos\alpha$$

and

$$b_2 = \ell_0 - \ell_1 \cos\alpha$$

The distance between the shafts, ℓ_0, is

$$\boxed{\ell_0 = \frac{\ell_1}{\sin\frac{180°}{n}}}$$

The input force is applied to the shaft of

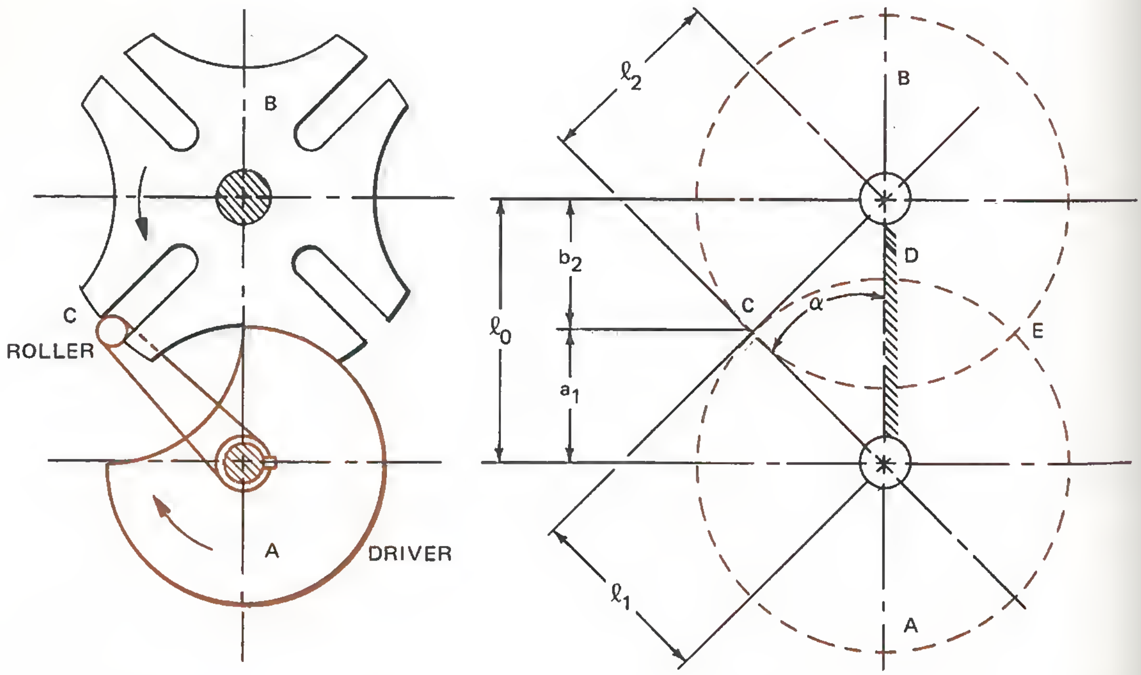


Fig. 21-1 The Geneva Wheel

the driving member and is delivered as a force tangent to the arc C - E. Neglecting the roller diameter, the lever arm length ℓ_1 remains fixed. The force delivered to the star wheel may vary because the effective lever arm length ℓ_2 varies from its shortest length at point D to its longest length at point C and E. For example, ℓ_2 at 0° (point D) is

$$\ell_2 = \ell_1\sqrt{1 + m^2 - 2m\cos\alpha}$$

$$m = \frac{1}{\sin\frac{180°}{n}} = \frac{1}{\sin 45°} = 1.414$$

$$\ell_2 = \ell_1\sqrt{1 + (1.414)^2 - 2(1.414)(1)} = 0.414\ell_1$$

whereas ℓ_2 at 45° (points C to E) is

$$\ell_2 = \ell_1\sqrt{3 - 2.818(0.707)} = 0.969\ell_1$$

As the lever arm length varies, the angle at which the force is applied also varies. At points C and E the torque arm, ℓ_2, is at its longest, providing the maximum mechanical advantage, but only the component acting to produce rotation (the part of the force that is tangent to the circle inscribed by the circumference of part B) constitutes the effective force. As the roller moves through the arc from C to D, the lever arm of the driven member effectively gets shorter but the angle becomes such as to deliver more torque to part B. As an example let's suppose α = 22.5 degrees. Referring to figure 21-2, the roller is half-way between points C and D. The force vector producing the torque is along the tangent line from F to G.

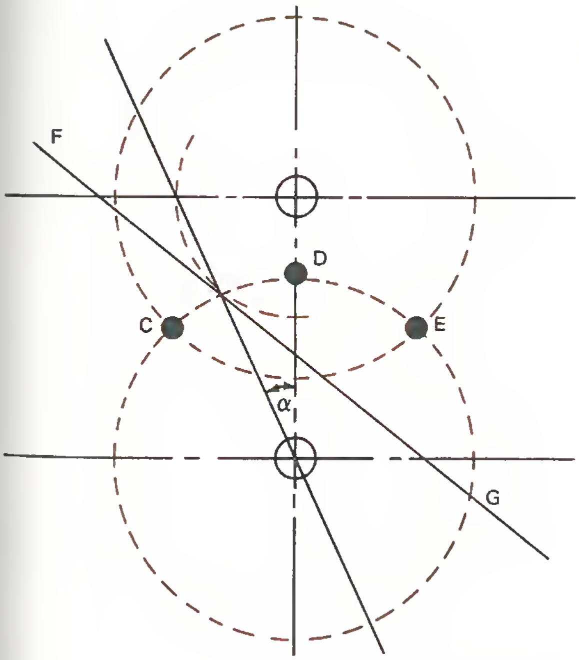


Fig. 21-2 Forces in a Geneva Mechanism

If the lever arm length of the driven member decreases in effective length but the angle becomes more favorable for producing the torque and these occur in equal proportions, the load would not change from C to E. However, if one of these values changes more than the other, the load would vary as the roller moved through the arc from C to E.

MATERIALS

1 Breadboard with legs and clamps

1 360° disk dial

1 Dial index with mount

2 Bearing plates with spacers

2 Shafts 4" x 1/4"

4 Bearing mounts with bearings

1 Geneva wheel mechanism

4 Collars

2 Spring balances

2 Spring balance posts with clamps

2 Lever arms 2 in. long with 1/4-in. bore hubs

PROCEDURE

1. Examine all your parts to be sure they are not damaged.
2. Construct the mechanism shown in figure 21-3. Leave the adjustments on the spring balances loose.
3. Set the geneva mechanism so that the roller is just entering a slot in the star wheel from the bottom.
4. Hold the mechanism fixed and set the dial to read zero degrees.
5. Be sure that both of the lever arms are vertical and that both of the spring balances are horizontal before making any data readings.
6. Set the dial to the first angle listed in the data table and readjust the levers so that they are both vertical.
7. Hold the mechanism in this position and adjust the input spring balance to about 4 oz.
8. Adjust the output spring balance until the lever arms will remain in the vertical positions when released.
9. Record the input to output forces and compute their ratio.
10. Set up the next angle listed on the data table and repeat steps 7 through 10. Proceed until all of the data has been recorded.

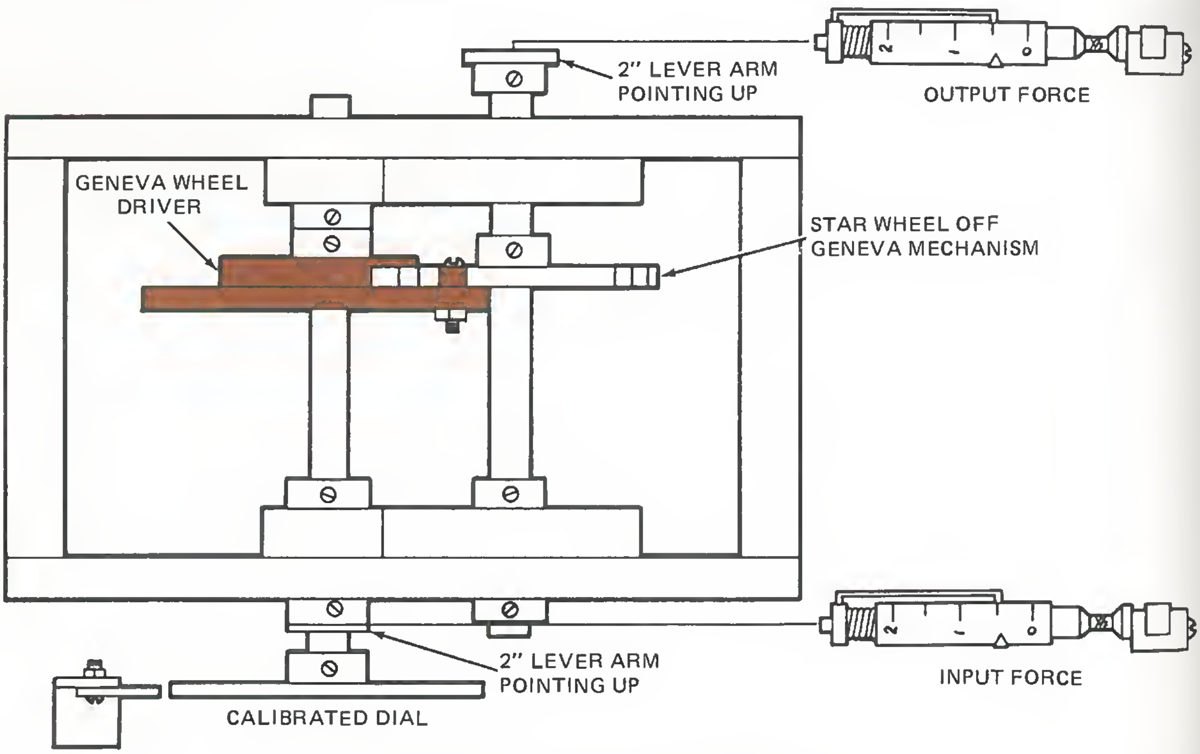


Fig. 21-3 Experimental Setup

Angle (degrees)	F_1 (oz)	F_2 (oz)	Ratio
20			
25			
30			
35			
40			
45			
50			
55			
60			
65			
70			
75			

Fig. 21-4 The Data Table

ANALYSIS GUIDE. Discuss your data emphasizing the pattern that the data makes with respect to the input angle. Explain why the results turned out as they did by extending the ideas presented in the discussion section.

PROBLEMS

1. Calculate the length of the driven crank, ℓ_2, at 0 and 90 degrees for a four-slot geneva wheel.
2. Calculate the length of the driven crank, ℓ_2, at 22.5° and 45° for a six-slot geneva wheel.
3. What is the approximate angle at which the roller will first engage in the slot of an eight-slot geneva wheel mechanism?
4. If a row of holes is to be drilled in a piece of steel plate by a semi-automatic machine tool, explain briefly how a geneva mechanism might be used to position the table of the machine tool.
5. If a four-slot geneva mechanism is used to drive a six-slot geneva mechanism, describe the output rotation pattern in relation to a constant input. How are the input and output RPM related?

experiment 22 SLIDING-LINK MECHANISM

INTRODUCTION. A specialized cam application which is used in many rotary machines is the sliding link. In this experiment we shall assemble and examine a simple example of this type of mechanism.

DISCUSSION. A sliding link can be used to couple parallel shafts and produce predictable output motion. Figure 22-1 shows a typical example of one type of sliding-link mechanism.

As the input link ℓ_1 rotates counterclockwise, the slider moves along the output link ℓ_2 causing it to turn also. In such an arrangement there are two limiting positions. These positions are shown in figure 22-2. Notice from the figure that the limiting positions occur when the angle between ℓ_1 and ℓ_2 is 90 degrees. At this point we see that

$$\sin\frac{\Theta}{2} = \frac{\ell_1}{\ell_o}$$

or

$$\Theta = 2\sin^{-1}\frac{\ell_1}{\ell_0} \qquad (22.1)$$

where Θ is the angle through which the output link moves as the input link rotates. Notice that the operation of this type of sliding link is very similar to that of a crank rocker four-bar mechanism. However, in this case, the coupling link has zero length and the output link length is variable.

The slider must be able to move freely from the minimum output link length to the maximum output link length. These positions occur when the output link, input link, and fixed link are colinear. Figure 22-3 shows both these positions.

From these sketches we can observe several conditions that must be satisfied to produce such a mechanism. For example, figure 22-3a illustrates that ℓ_1 must be the shortest link. Also we see that the minimum length ℓ'_2 of the output link must be

$$\ell'_2 = \ell_0 - \ell_1$$

while the maximum length ℓ'_2 must be

$$\ell''_2 = \ell_0 + \ell_1$$

Since the slider moves from ℓ''_2 to ℓ'_2, the working length (S) of ℓ_2 must be

$$S = \ell''_2 - \ell'_2$$

or

$$S = (\ell_0 + \ell_1) - (\ell_0 - \ell_1)$$

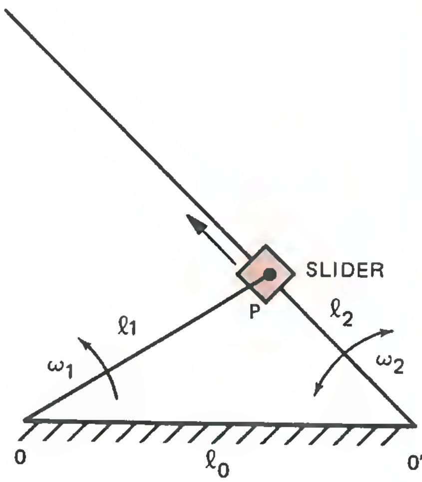


Fig. 22-1 A Basic Sliding-Link Mechanism

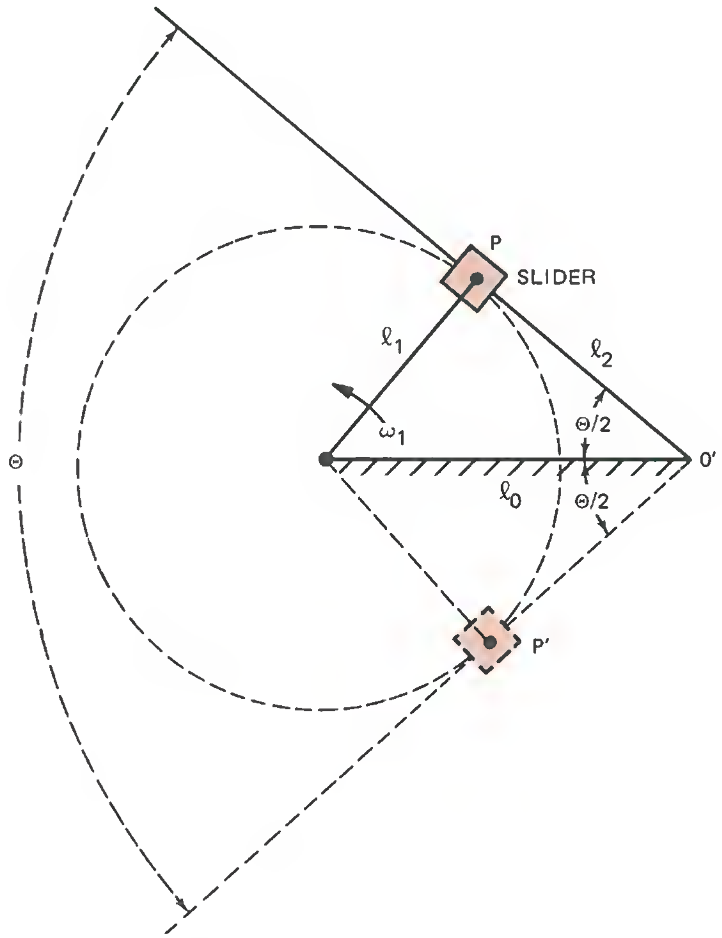
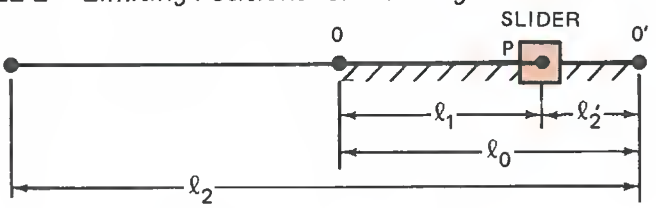
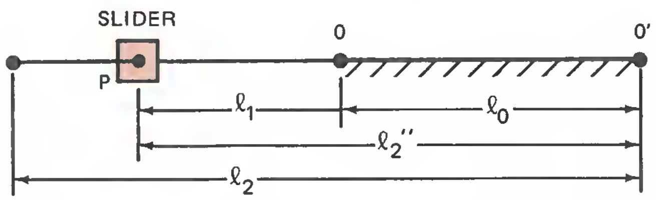


Fig. 22-2 Limiting Positions for a Sliding-Link Mechanism



(A) MINIMUM OUTPUT LINK LENGTH ℓ_2'



(B) MAXIMUM OUTPUT LINK LENGTH ℓ_2''

Fig. 22-3 Maximum and Minimum Output Link Lengths

so we have

$$S = 2\ell_1 \qquad (22.2)$$

as the working length of the output link.

If we change the relative lengths of ℓ_0 and ℓ_1 so that ℓ_0 is the shortest link, we will have a mechanism like the one shown in figure 22-4. This mechanism works in a manner that is similar to a four-bar drag-link. That is, when the input link turns a complete revolution, the output follows it around.

Either type of sliding-link mechanism can be used as a double rocker provided that the link lengths are appropriate for the desired output. Except for a double-rocker application the working length of ℓ_2 must not be physically limited. If it is, the slider will "bottom" against the limit and prevent normal operation. In some double-rocker applications physically limited sliders can be used if the limits lie outside the normal output stroke range.

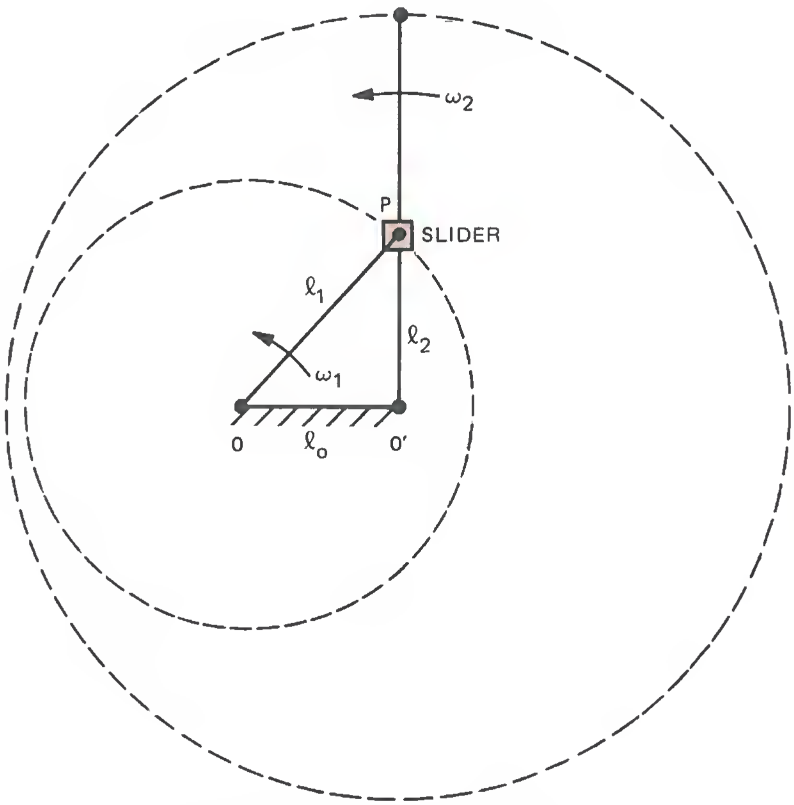


Fig. 22-4 Sliding Link with $\ell_0 < \ell_1$

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Bearing holders with bearings
2 Shaft hangers with bearings
2 Disk dials
2 Dial indices with mounts
1 Lever arm 1 in. long with 1/4-in. bore hub
1 Slotted lever 2 in. long with 1/4-in. bore hub
1 Flat head machine screw 2-56 x 1/2 in.
2 Flat washers No. 2 x 1/2 in. OD
2 Hex nuts 2-56 x 1/4 in.
1 Steel rule 6 in. long
1 Shaft 2" x 1/4"
1 Shaft 4" x 1/4"

PROCEDURE

1. Inspect your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 22-5. Be sure the slider operates freely and that the shafts are in line vertically.
3. Measure and record ℓ_0, ℓ_1, ℓ_2', ℓ_2'', and S for this mechanism. (*Each of the symbols is defined in the discussion.*)
4. With both lever arms pointing straight upward, set the dials to read zero.
5. Rotate the input dial in 10-degree steps and record both input angle and output angle. Continue for at least 360 degrees of input rotation.
6. Move the sliding link to the 1/2-in. position on the input lever and relocate the output shaft as shown in figure 22-6. The output dial must be located through one of the breadboard slots.
7. Repeat steps 3, 4, and 5.
8. Compute the value of S′ for each setup using equation 22.2.
9. From your data determine the total angular swing of ℓ_2. Record this value as Θ'.
10. Using equation 22.1 compute the output lever swing Θ''.

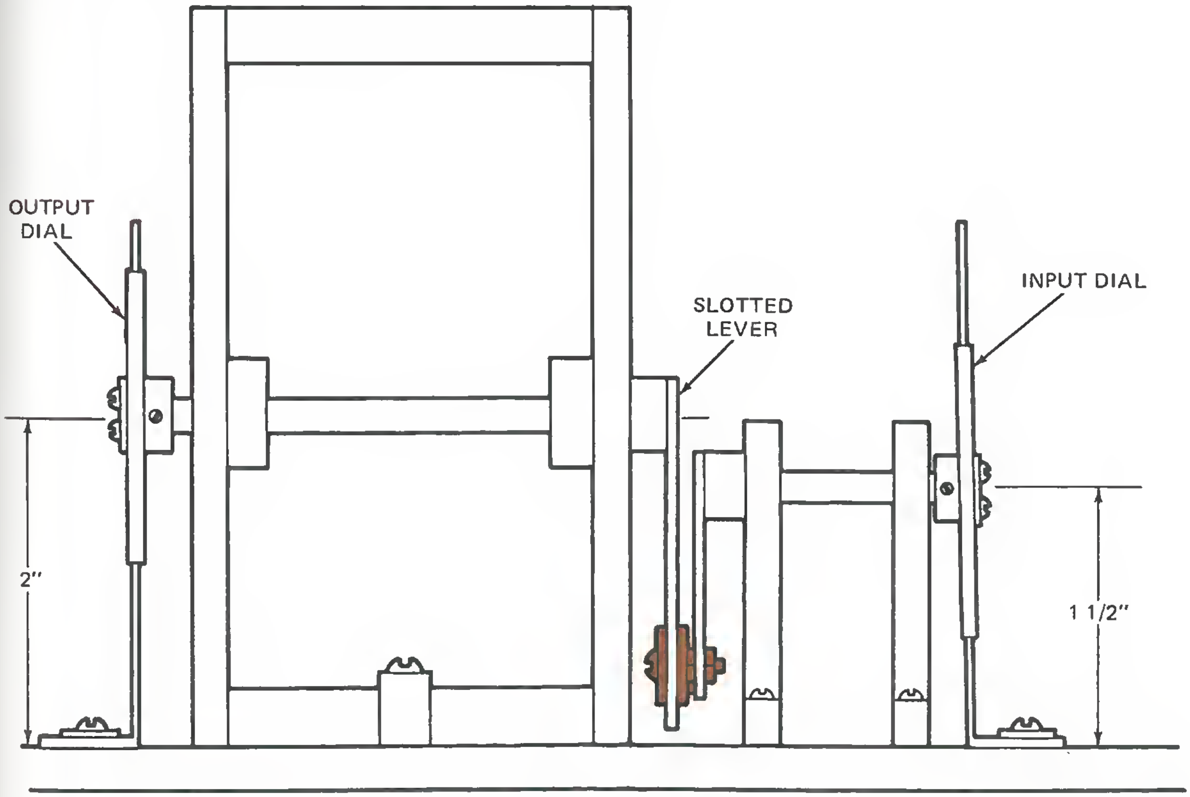


Fig. 22-5 The First Experimental Mechanism

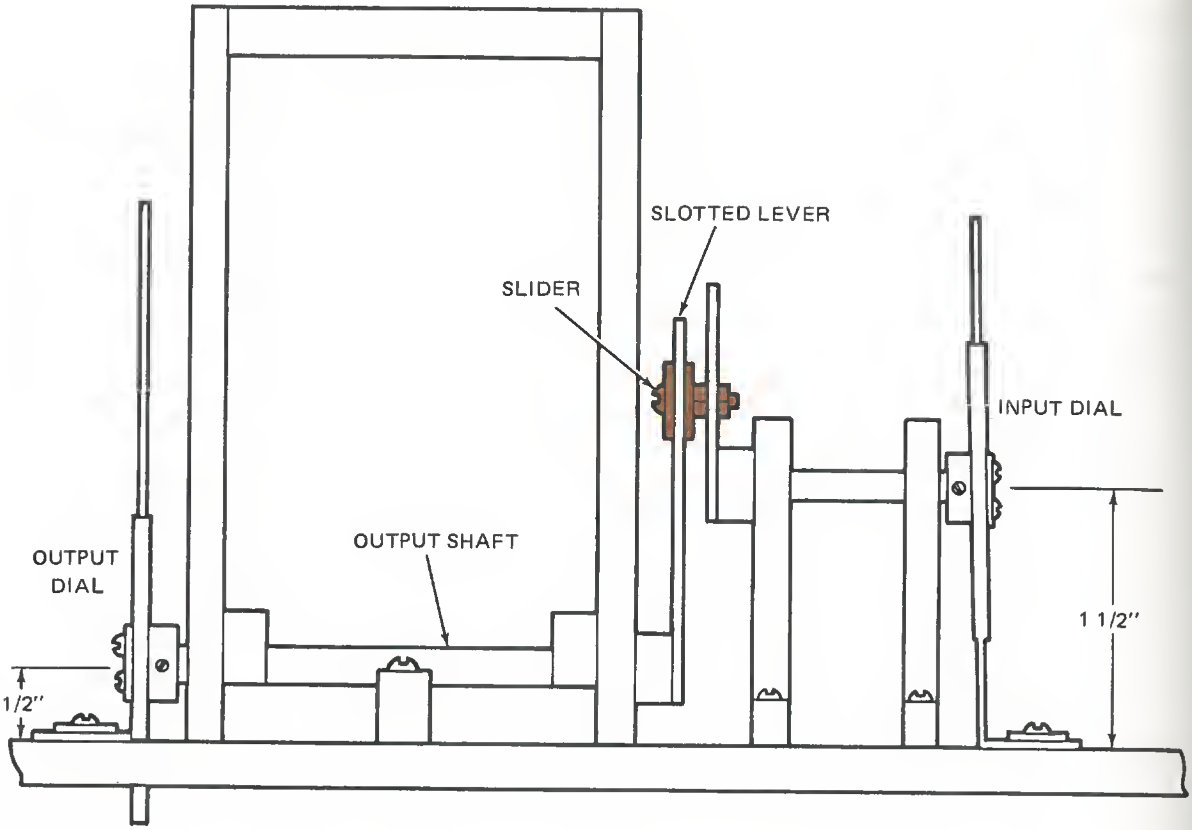


Fig. 22-6 The Second Experimental Mechanism

ANALYSIS GUIDE. In analyzing your results you should plot the input angle versus the output angle for each mechanism. Then discuss the nature of the plots. Consider also how well your values of S and S′ agreed in each case. How well did Θ' and Θ'' agree?

PROBLEMS

1. List several practical uses for a sliding-link mechanism.
2. A certain sliding-link mechanism has an input link that is 14 in. long. How long is the working length of ℓ_2?
3. If the fixed link in problem 2 were 18 in. long, what kind of output motion would result?
4. Through what angle would the output link in problem 3 swing?
5. If the fixed link in problem 2 were 12 in. long, what kind of output motion would result?
6. Through what angle would the output link in problem 5 swing?
7. If the input velocity in problem 5 were constant, would the output velocity also be constant? Explain your answer.

First Setup

ℓ_0 = ____________

ℓ_1 = ____________

ℓ_2' = ____________

ℓ_2'' = ____________

S = ____________

S′ = ____________

Second Setup

ℓ_0 = ____________

ℓ_1 = ____________

ℓ_2' = ____________

ℓ_2'' = ____________

S = ____________

S′ = ____________

Θ' = ____________

Θ'' = ____________

Input Θ_i (degrees)	Output Θ_o (degrees)

First Setup

Input Θ_i (degrees)	Output Θ_o (degrees)

Second Setup

Fig. 22-7 The Data Tables

experiment 23 QUICK RETURN MECHANISM II

INTRODUCTION. Mechanisms which produce linear motion with different velocities in different directions are widely used in practical applications. Quick-return mechanisms are one example of this type of motion. In this experiment we will examine a sliding-link type of quick return.

DISCUSSION. The sliding-link mechanism shown in figure 23-1 is operating in a manner similar to a four-bar crank-rocker. As the input link (ℓ_1) rotates through angle Θ in the clockwise direction, the output link (ℓ_2) swings through the arc (α) from A′ to A and the load moves from bottom dead center (BDC) to top dead center (TDC). We will call this motion the advance stroke. Then, as the input link rotates through angle θ, the output link swings back to A′ and the load returns to BDC. We will call this part of the cycle the return stroke.

Notice that the load moves the same distance during both the advance and return strokes. The input link (ℓ_1), however, rotates through a larger angle (Θ) during the advance stroke than it does (θ) during the return stroke. If the input link is rotating at some constant angular velocity (ω), then the load must travel faster during the return stroke than it does during the advance stroke. Because of this difference in speed we can call the whole mechanism a quick-return assembly.

The travel of the load is limited by the limiting positions of the output link. Figure 23-2 shows the limiting positions somewhat more clearly. Notice that the limiting positions occur when ℓ_1 and ℓ_2 are perpendicular. Also notice that the angle between ℓ_o and ℓ_2 at the limiting position is one half of the total swing of ℓ_2. Because of this and the fact that ℓ_1 and ℓ_2 are perpendicular, we can observe that the angle between ℓ_1 and ℓ_o at the limiting position is

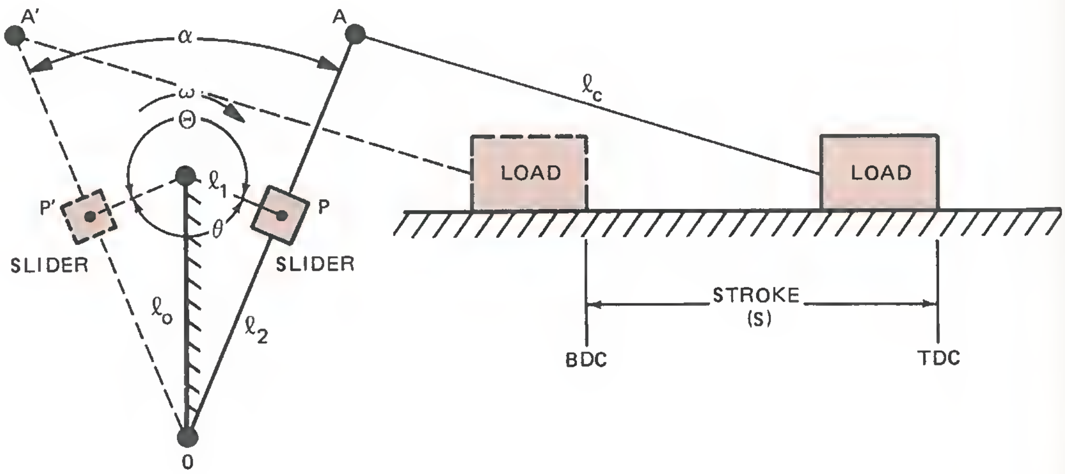


Fig. 23-1 A Sliding-Link Quick Return

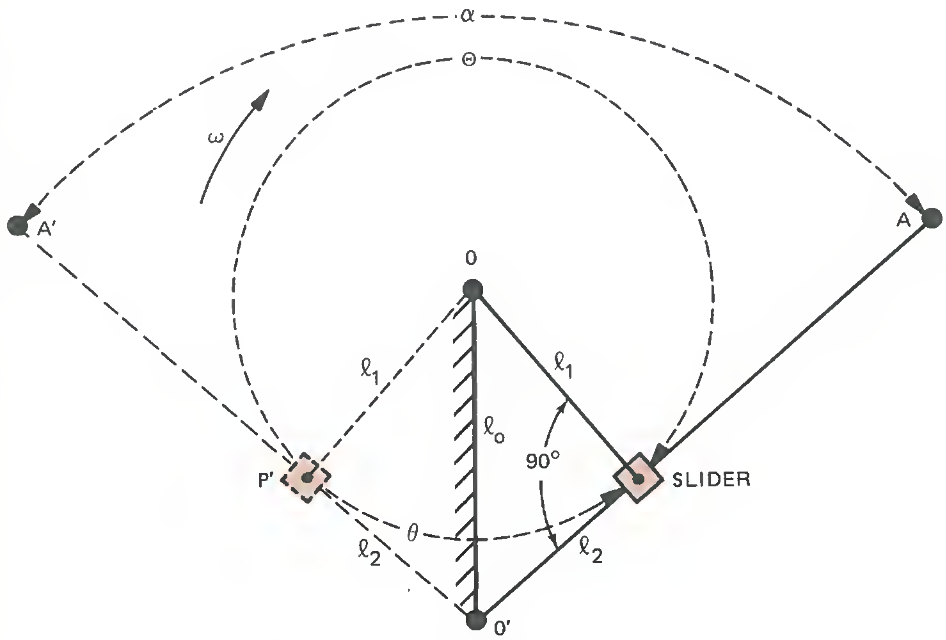


Fig. 23-2 The Sliding-Link Mechanism

$$\frac{\theta}{2} = 90^\circ - \frac{\alpha}{2}$$

or

$$\theta = 180^\circ - \alpha \qquad (23.1)$$

which is the angle through which ℓ_1 rotates during the return stroke of the quick-return mechanism.

From figure 23-2 we can observe that the angle through which ℓ_1 rotates during the advance stroke is related to that of the return stroke by

$$\Theta = 360^\circ - \theta$$

Substituting the relationship for θ we have

$$\Theta = 360^\circ - (180^\circ - \alpha)$$

or

$$\Theta = 180^\circ + \alpha \qquad (23.2)$$

for the advance stroke rotation of ℓ_1.

We can relate these angles to the link lengths by observing from figure 23-2 that

$$\sin\left(\frac{\alpha}{2}\right) = \frac{\ell_1}{\ell_o}$$

From this equation we can solve for α in the form

$$\alpha = \frac{1}{2}\sin^{-1}\left(\frac{\ell_1}{\ell_o}\right)$$

As a result we may write

$$\boxed{\Theta = 180^\circ + \frac{1}{2}\sin^{-1}\left(\frac{\ell_1}{\ell_o}\right)} \qquad (23.3)$$

and

$$\boxed{\theta = 180^\circ - \frac{1}{2}\sin^{-1}\left(\frac{\ell_1}{\ell_o}\right)} \qquad (23.4)$$

Then the ratio of these two angles Θ/θ is called the ratio of the speed-of-advance to the speed-of-return:

$$\boxed{\frac{\Theta}{\theta} = \frac{180° + \frac{1}{2}\sin^{-1}\left(\frac{\ell_1}{\ell_o}\right)}{180° - \frac{1}{2}\sin^{-1}\left(\frac{\ell_1}{\ell_o}\right)}} \qquad (23.5)$$

We can use this equation to determine this important ratio from the lengths of ℓ_1 and ℓ_o.

In making calculations based on this equation you should remember that Θ is normally more than 180 degrees while θ is normally less than 180 degrees.

Now let's go back and examine the output link and load relationship. Figure 23-3 shows a simplified sketch of this part of the mechanism.

For simplicity let's assume that the line of action of the load is perpendicular to the fixed link ℓ_o. This will not always be true but analysis is simpler when it is the case.

When the line of action of the load is perpendicular to ℓ_o, then the distance A to A′ is equal to stroke distance S. This distance is a chord subtending the arc AA′ at a radius equal to ℓ_2. In this case ℓ_2 is the total effective length of the output link.

From analytical geometry we know that such a chord can be found by

$$S = \ell_2 \sin\frac{\alpha}{2}$$

but we have already determined that

$$\sin\frac{\alpha}{2} = \frac{\ell_1}{\ell_o}$$

so we have

$$\boxed{S = \frac{\ell_1 \ell_2}{\ell_o}} \qquad (23.6)$$

for the stroke distance of the load.

Actually this is the maximum value that S can have. If we allow the fixed link to have an angle other than 90 degrees to the line of load action, the stroke will be reduced. When the fixed link and load action are colinear, the stroke will be at its minimum value.

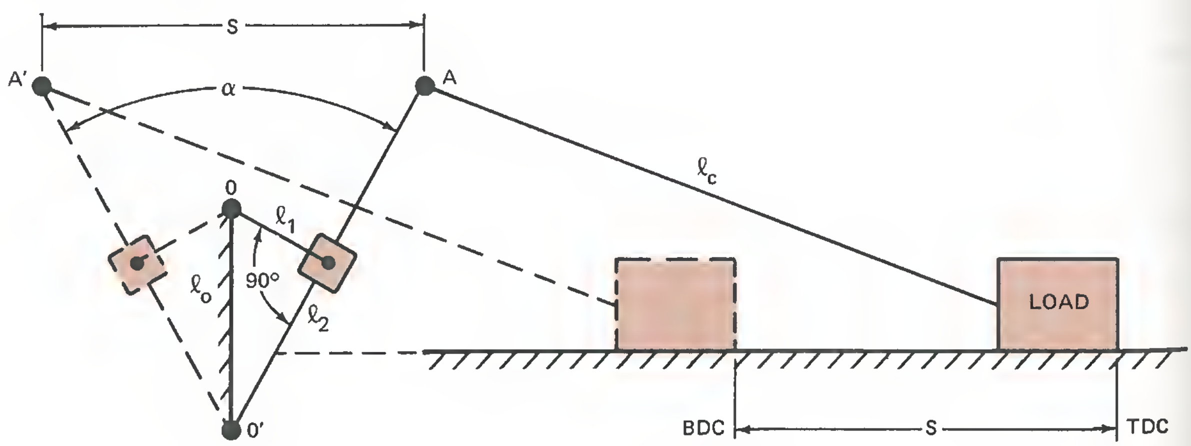


Fig. 23-3 The Stroke Distance

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
2 Bearing holders with bearings
4 Shaft hangers with bearings
1 Disk dial with 1/4-in. bore hub
1 Dial index with mount
1 Lever arm 1 in. long with 1/4-in. bore hub
1 Slotted lever 2 in. long with 1/4-in. bore hub
1 Lever arm 2 in. long with 1/4-in. bore hub
1 Flat head machine screw 2-56 x 1/2 in.
2 Flat washers No. 2 x 1/2 in. OD
2 Hex nuts 2-56 x 1/4 in.
1 Steel rule 6 in. long
1 Shaft 2" x 1/4"
2 Shafts 4" x 1/4"
2 Collars
1 Rigid coupling
1 Roundhead machine screw 6-32 x 1/4
1 Spacer No. 6 x 1/8 with 1/32 wall thickness
*1 Wire loop link 3 in. long

*For details of wire link construction refer to appendix A.

PROCEDURE

1. Inspect your components to insure that they are undamaged.
2. Construct the experimental setup shown in figure 23-4(a) and 23-4(b).

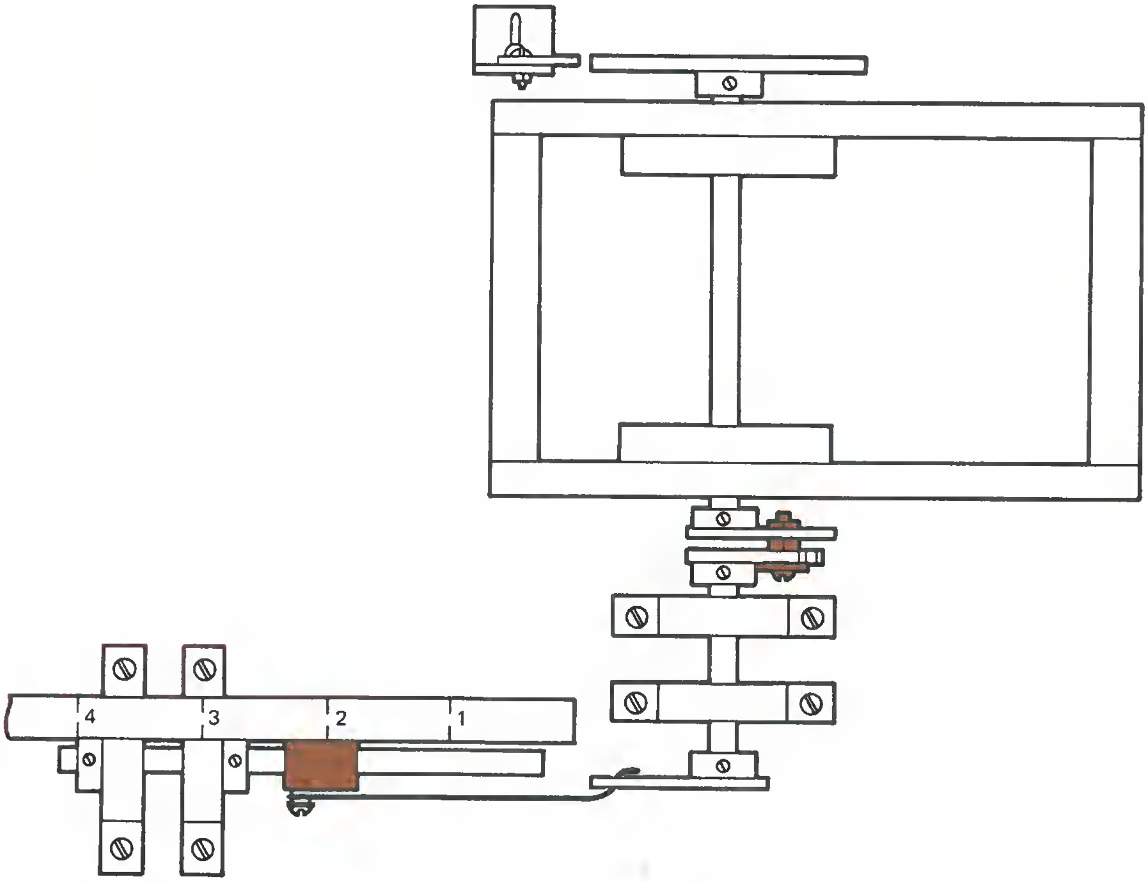


Fig. 23-4(a) Experimental Setup Top View

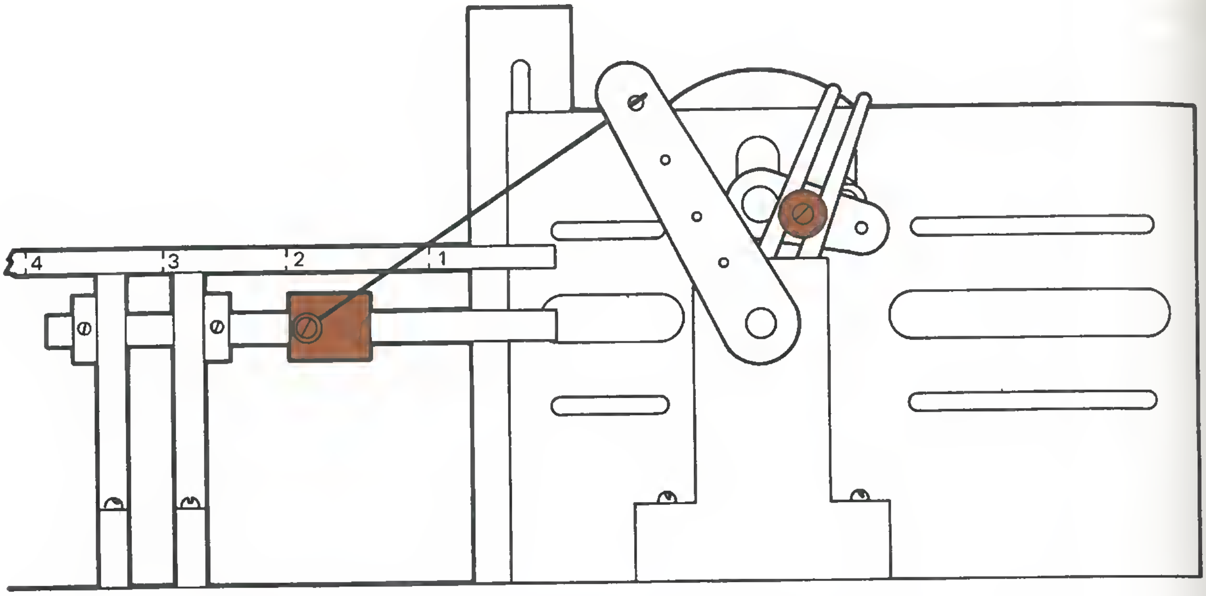


Fig. 23-4(b) Experimental Setup Front View

3. Rotate the dial several times to insure that the mechanism is properly aligned and rotates freely. Lubricate the load shaft if necessary.

4. Rotate the mechanism until the crank lever (ℓ_1) and the slotted lever both point straight upward. Now adjust the 2-inch lever that drives the load so that it points in the same direction as the slotted lever.

5. With the load at TDC, set the input dial to zero.

6. Lay the 6-inch steel rule across the load shaft hangers so that its zero end lines up with the end of the shaft. Tape it in position if necessary.

7. Starting with zero degrees on the dial, measure and record the dial angle (β) and the load displacement (X) every 20 degrees for one full dial revolution in the clockwise direction.

8. Repeat step 7 for one full dial revolution in the counterclockwise direction.

9. Measure and record the length of each link in the mechanism (ℓ_1, ℓ_o, ℓ_c and ℓ_2). (Notice that ℓ_2 is not the slotted lever.)

10. Measure and record the stroke (S) of the load.

11. Adjust the 2-inch lever that drives the load so that it is pointing in the direction at 90° to that of step 5. Relocate the load shaft as necessary.

12. Repeat steps 7 and 8. Record the data as β' and X′.

13. From your data determine Θ and θ for the first setup arrangement only. (Θ and θ are defined in the discussion.)

14. Using the results from step 13 compute Θ/θ.

ℓ_1	ℓ_o	ℓ_c	ℓ_2	S	Θ	θ	Θ/θ
Clockwise		Counterclockwise		Clockwise		Counterclockwise	
β	X	β	X	β'	X'	β'	X'

Fig. 23-5 The Data Table

ANALYSIS GUIDE. In your analysis of these data you should plot a curve for each set of β and X. On the curve identify the regions of the load travel from TDC to BDC and from BDC to TDC. For the first experimental arrangement determine the ratio of time-of-advance to time-of-return using equation 23.5. Also determine the stroke using equation 23.6. How do these results compare to your experimental values? Why were the results different when you moved the output lever?

PROBLEMS

1. A quick-return mechanism of the type shown in figure 23-1 has the following dimensions:

   ℓ_1 = 8 inches ℓ_o = 6 inches

   ℓ_2 = 18 inches ℓ_c = 28 inches

   Will such a mechanism work satisfactorily? Explain your answer.

2. If the fixed link in problem 1 were increased in length by 6 inches, would the mechanism work like the one in figure 23-1? Explain your answer.

3. What would be the ratio of speed-of-advance to speed-of-return in problem 2?

4. What would be the stroke in problem 2?

5. List three applications of a quick-return mechanism.

experiment 24 COMPUTING MECHANISMS (ALGEBRA)

INTRODUCTION. Mechanical devices are often used to perform computing functions. In this experiment we will investigate some of the more popular mechanisms used for computing algebraic functions.

DISCUSSION. Mechanisms used in computing operations could be classed as analog computers. The term *analog* comes from the word *analogy* which means a similarity between two different things. With mechanisms, a distance or angle may be used to represent time, miles per hour or gallons per minute. Another displacement could represent pounds of force, work or power. In each case the displacement of the mechanical device is used to represent (or be analogous to) something else.

A gear and pinion is a simple type of computing device. Recall that the velocity ratio of the gear and pinion is

$$\frac{\omega_g}{\omega_p} = -\frac{n}{N}$$

where ω_g and ω_p are the angular velocities of the gear and pinion, and n and N are the number of teeth of the pinion and gear, respectively. Solving for the angular velocity of the pinion gives us

$$\omega_p = -\frac{N}{n}\omega_g$$

The ratio of $-N/n$ is a constant for any given gear and pinion set; thus, the pinion angular velocity is a constant times the gear angular velocity. Then in this case, we have a type of analog computer that multiplies by a constant. It should be noted that the constant can be less than one in which case the device becomes a "divide by a constant" analog computer.

Figure 24-1 shows a mechanism that can be used for adding quantities. The sum, Σ, is the displacement of the center pointer and is related to x and y by

$$\boxed{\Sigma = x + y}$$

The device could be used to find any one of the quantities if the other two are known. If x is 3 and Σ is 12 and the values are set on the device, then y would register 9.

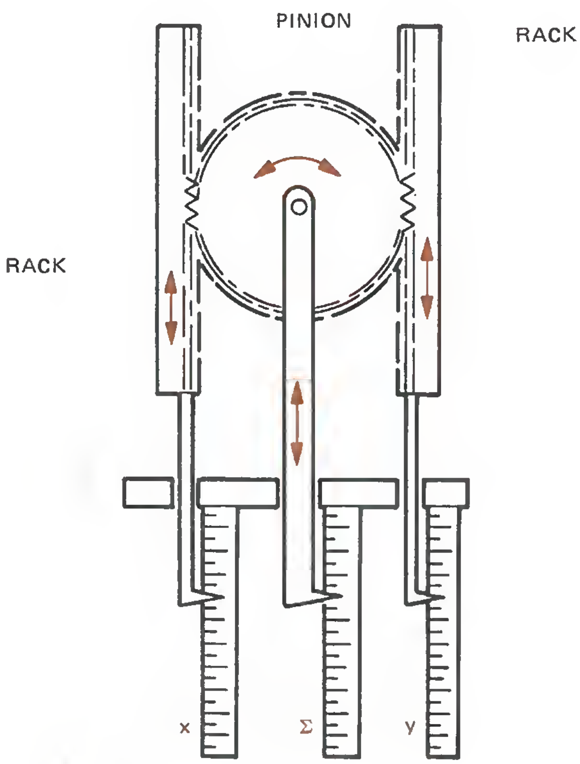


Fig. 24-1 Mechanical Summing Mechanism

MATERIALS

1 Breadboard with legs and clamps
2 Dial indices
3 Dial index mounts
1 Shaft 2" x 1/4"
1 Roller chain approximately 10 inches long
1 Sprocket approximately 1-1/4 in. OD with 1/4-in. bore hub
1 Spring balance
1 Spring balance post with clamp
1 Pulley approx. 1 in. OD with 1/4-in. bore hub

PROCEDURE

1. Inspect all of the parts to insure that they are in proper working condition.
2. Reproduce the two scales shown in figure 24-2 and prepare them to be taped onto the breadboard.

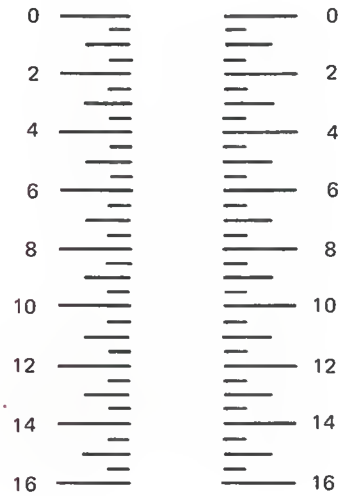


Fig. 24-2 Calibrated x and y Scales

3. Assemble the sprocket and pulley on the shaft as shown in figure 24-3. The pulley is used only as a standard for the sprocket and shaft.
4. Remove the master link from the chain and put the link in a safe place so that it doesn't get lost.
5. Construct the mechanism shown in figure 24-4. Be sure to use a flat washer between the chain and nut to avoid burring the chain links.
6. Position the spring balance and the index mounts so that zero is indicated on each when the chain is taut. The ounce scale of the spring balance will be used as a displacement measure in this experiment.
7. Set each of the values of x and y indicated in the data table and record the respective output readings (ounce scale) from the spring balance. Make all readings to the nearest whole number.
8. Remove the chain and replace the master link. Then disassemble the rest of the apparatus.

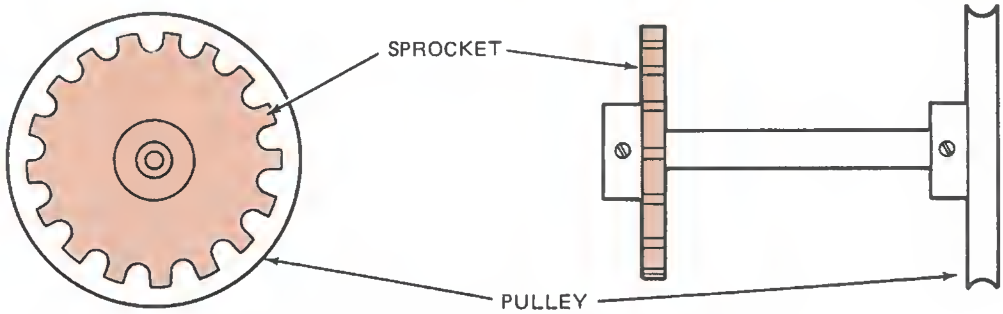


Fig. 24-3 Sprocket and Pulley Assembly

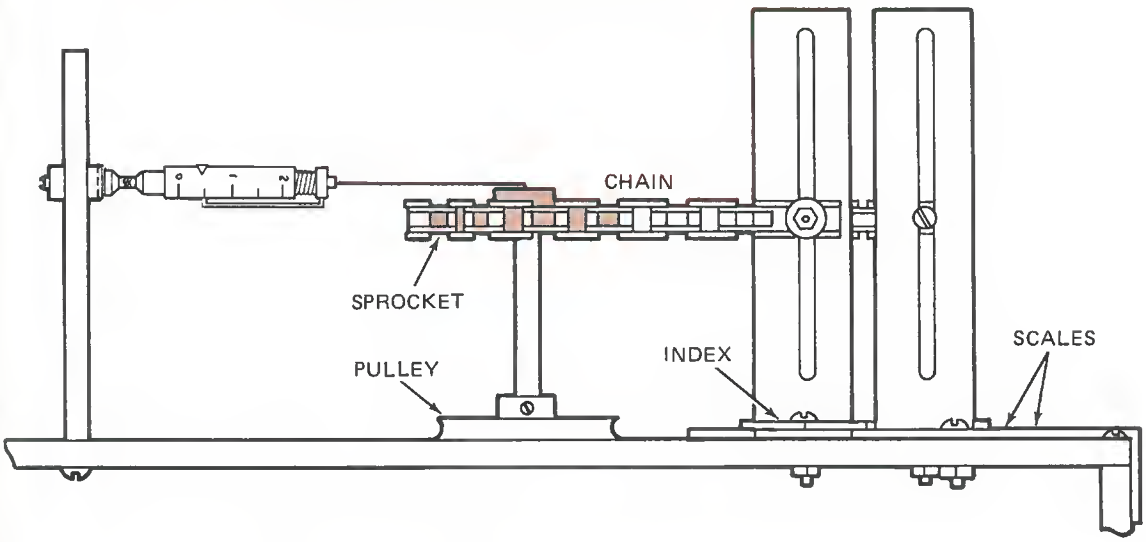


Fig. 24-4(a) Experimental Setup Side View

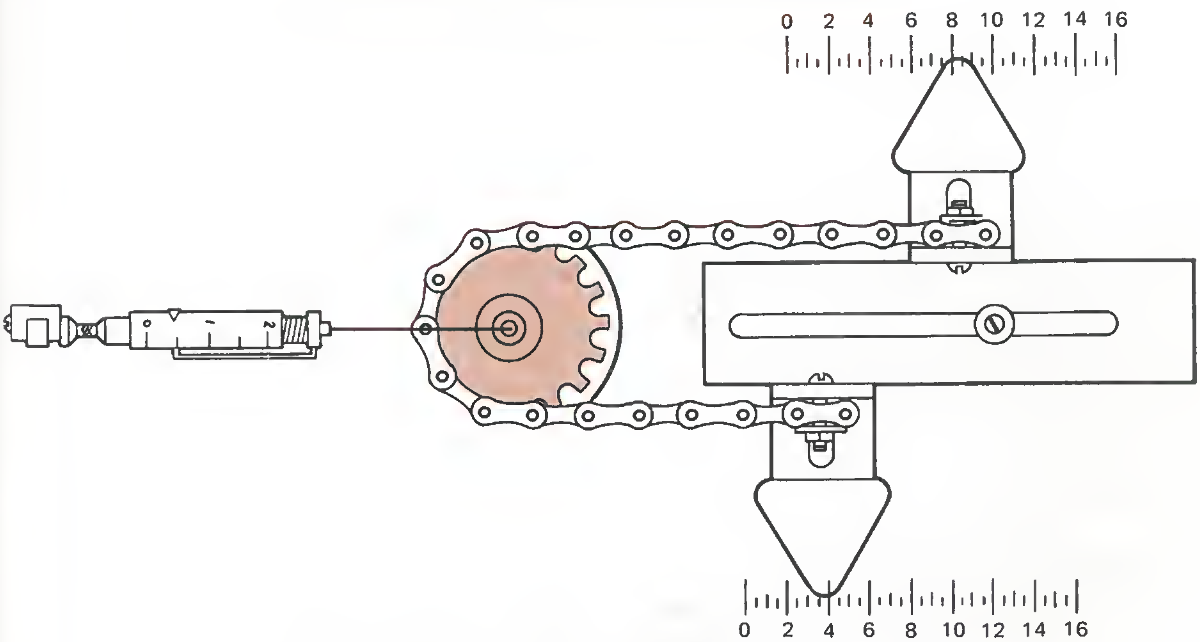


Fig. 24-4(b) Experimental Setup Top View

x	y	output (z)
4	8	
7	12	
8	4	
10	12	
14	16	
11	17	
17	10	
5	13	
12	12	
8	11	

Fig. 24-5 The Data Table

ANALYSIS GUIDE. Examine the results of the data table and determine the relationship between x, y and z. Discuss some of the possible sources of error in this experiment. What are some of the practical applications that this kind of mechanism could satisfy?

PROBLEMS

1. Suppose that we misalign the x and y scales in the experimental mechanism an inch or so by sliding one to the left and the other to the right in figure 24-4. If the three pointers were zeroed on the scales, would the misalignment affect the accuracy of subsequent data?
2. Compare the calibration of the x and y scale to the scale on the spring balance. How do they compare?
3. Explain why the scale is different for the x and y compared to the z scale.
4. Use the spring equation to explain why the force calibration of a spring balance can be used to express linear displacement.
5. Explain how to program and solve the equation

$$A + 16 = 25$$

   on the experimental setup of this experiment.
6. Sketch a mechanism using a lever and spring balances in such a way as to allow you to substitute forces for the displacements used in the experiment.
7. Explain in detail an example problem solution using your device from problem six.

experiment 25 COMPUTING MECHANISMS (TRIG)

INTRODUCTION. Computations of physical quantities that involve angles frequently require the use of trigonometry. In this experiment we will examine some basic trigonometric computing mechanisms.

DISCUSSION. A mechanism used to solve trigonometric functions would be classed as an analog device. The relationship between the input and the output may involve sine, cosine or tangent functions. Sometimes they also involve secant, cosecant or cotangent functions but no information that is not contained in the first three is added.

A device can be considered a sine, cosine or tangent function generator when its output displacement, velocity, or other quantitative measure is proportional to that function of a chosen angle. In other words, when the output is plotted versus the input angle, the graph would be proportional to the curve you would get by plotting the values for that function from a trig table. The shapes of the sine, cosine and tangent functions are illustrated in figure 25-1.

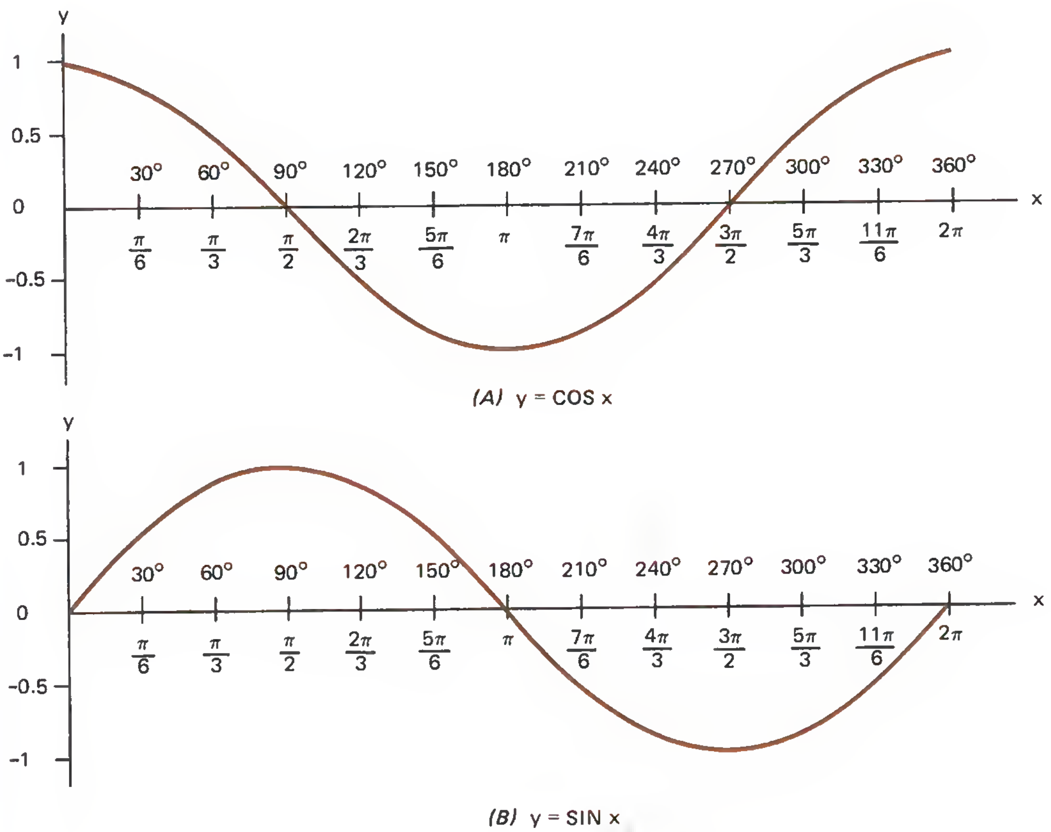
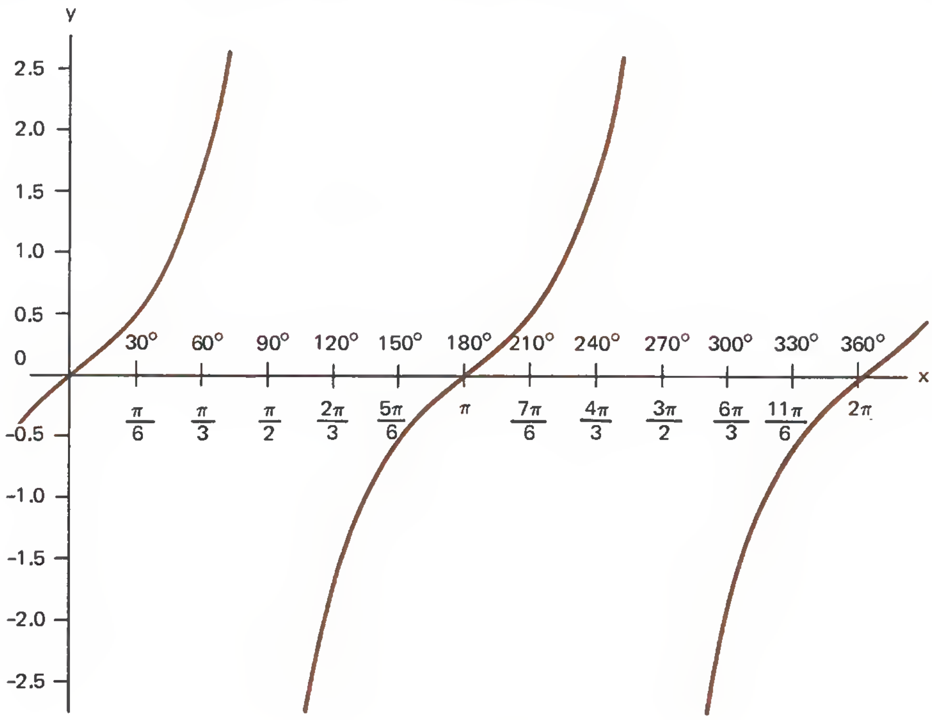


Fig. 25-1 Graph of Trigonometric Functions



(C) y = TAN x

Fig. 25-1 Graph of Trigonometric Functions (Cont'd)

You should already be familiar with a disk cam that drives a follower in a simple harmonic motion. The cam shown in figure 25-2 would be a device for generating a displacement equivalent to the sine function (or cosine, which is simply displaced 90° from the sine). It is important to understand that the follower displacement graph is called a

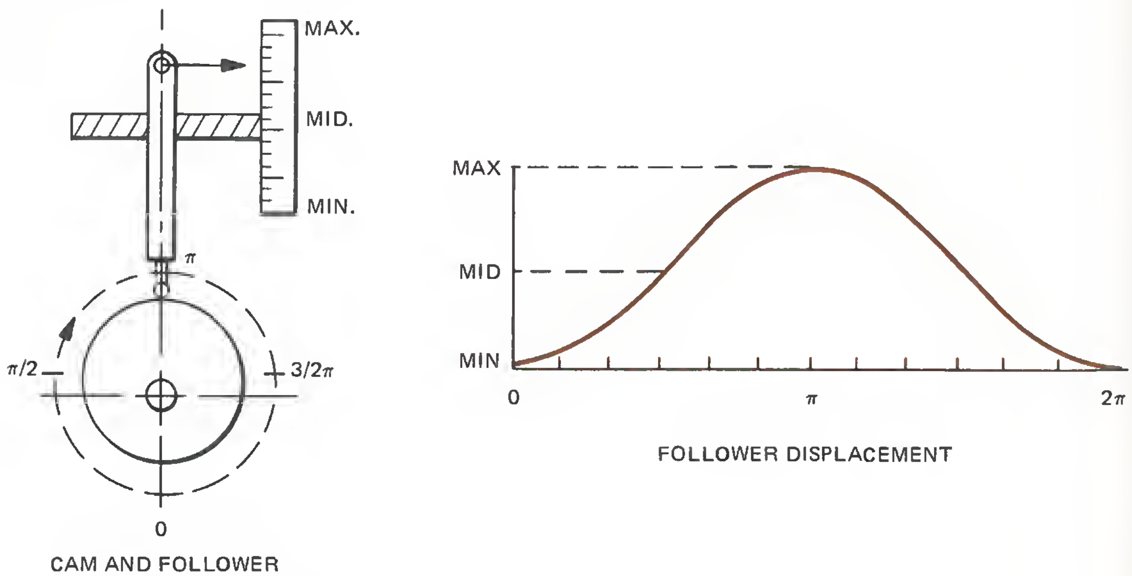


Fig. 25-2 Sine Cam Used as a Function Generator

sine or cosine function because of its *shape.* We can start at any point recording data and then pick the reference which gives the desired curve.

The scotch yoke in figure 25-3 is another popular mechanism for solving problems involving the trigonometric functions. Link ℓ_1 is a crank with a length ℓ_1 and is fixed at point P. The input angle is the crank angle shown as α in the diagram. The magnitudes of the x and y components correspond to the distances traveled by the slotted parts and are marked accordingly on the diagram. Length ℓ_1 is the analog distance used to represent the hypotenuse and while it is shown as a fixed distance in the figure, a slot in part A could make it a variable. The relationships between ℓ_1, x, and y are

$$y = \ell_1 \sin \alpha$$

$$x = \ell_1 \cos \alpha$$

Solving each equation for ℓ_1 gives us

$$\ell_1 = \frac{x}{\cos \alpha} = \frac{y}{\sin \alpha} \qquad (25.1)$$

As an illustration of this, let's say that a radar receiver shows that a flying target is 1500 yards away, and the antenna is positioned at an angle of 20 degrees above the horizontal as shown in figure 25-4. Choosing

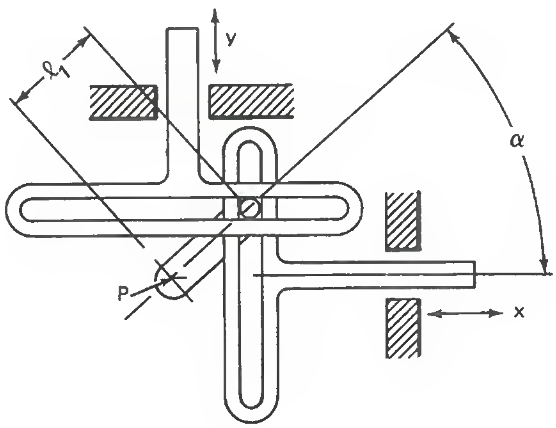


Fig. 25-3 The Scotch Yoke

a scale factor of 1 inch = 500 yards, the distance, ℓ_1, would be adjusted to three inches. The crank would be rotated to give an angle α of 20 degrees. If the distance x is measured and it is 2.8 inches, we have

Horizontal Range = 2.8 X scale factor
2.8(500 yds) $\simeq$ 1400 yards

Then, if the distance y is measured to be one inch, we have

Height = 1.0 X scale factor
1.0(500) = 500 yards

Precision mechanisms can be used to give high levels of accuracy but the principle of operation is the same as that described above.

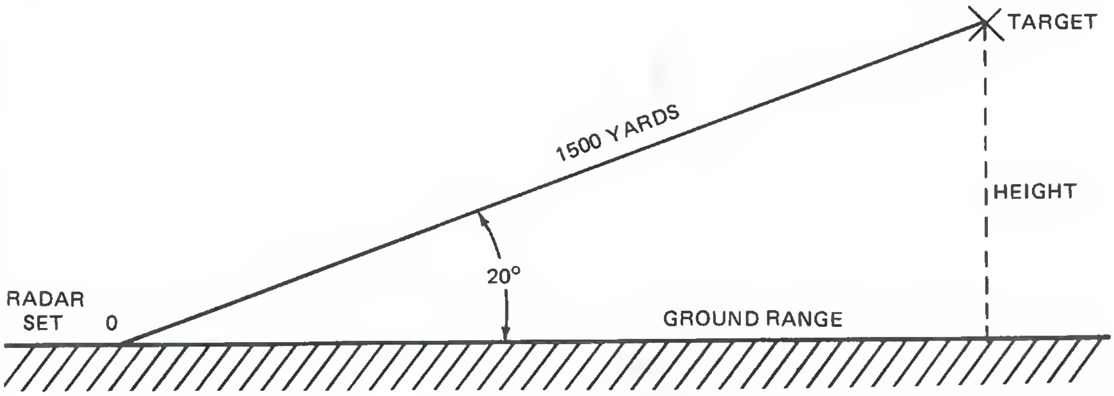


Fig. 25-4 Example Problem

MATERIALS

- 1 Breadboard with legs and clamps
- *1 Wire loop link approx. 3 in. long
- 1 Steel rule 6 in. long
- 1 Disk dial with 1/4-in. bore hub
- 1 Dial index with mount
- 1 Shaft 2" x 1/4"
- 2 Shaft 4" x 1/4"
- 4 Shaft hangers with bearings

*See appendix A for wire link construction details.

- 3 Collars
- 1 Rigid coupling
- 1 Lever arm, 1 in. long with 1/4-in. bore hub
- 1 Machine screw 6-32 x 1/4 roundhead
- 1 Spacer No. 6 x 1/8-in. long x 1/32-in. wall thickness
- 2 Bearing plates with spacers
- 2 Bearing mounts with bearings
- 1 Slotted lever, 2 in. long, with 1/4 in. bore hub

PROCEDURE

1. Inspect your parts to be sure they are in satisfactory working condition.
2. Construct the mechanism shown in figure 25-5.
3. Begin with the crank arm pointing up and the dial at 0°. Adjust the ruler to measure the travel of the follower. **Be sure the screw holding the link to the rigid coupling is in the same place for each reading you make.**
4. Record the displacement of the follower for each 20 degrees of rotation of the crank arm from 0 to 360° (call displacements left of the starting point a negative quantity).

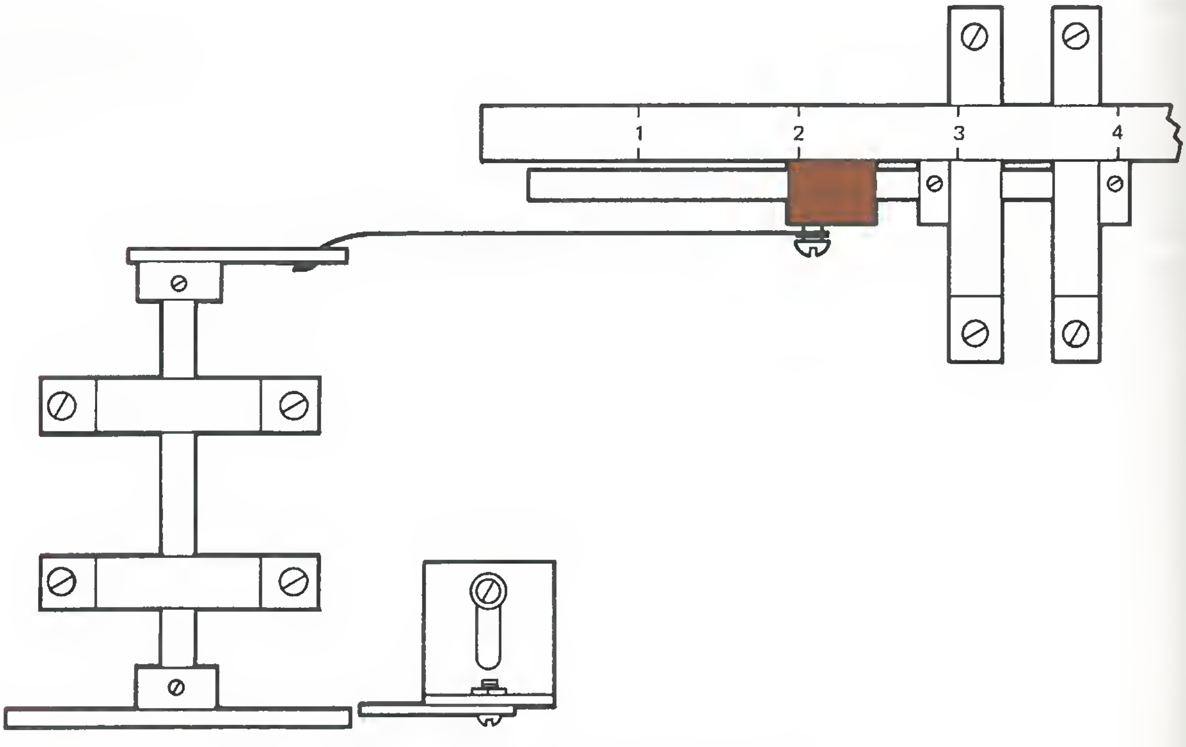


Fig. 25-5 Experimental Setup I

5. Construct the mechanism shown in figure 25-6.
6. Start with the slotted lever pointing vertically and adjust dial to read zero degrees.

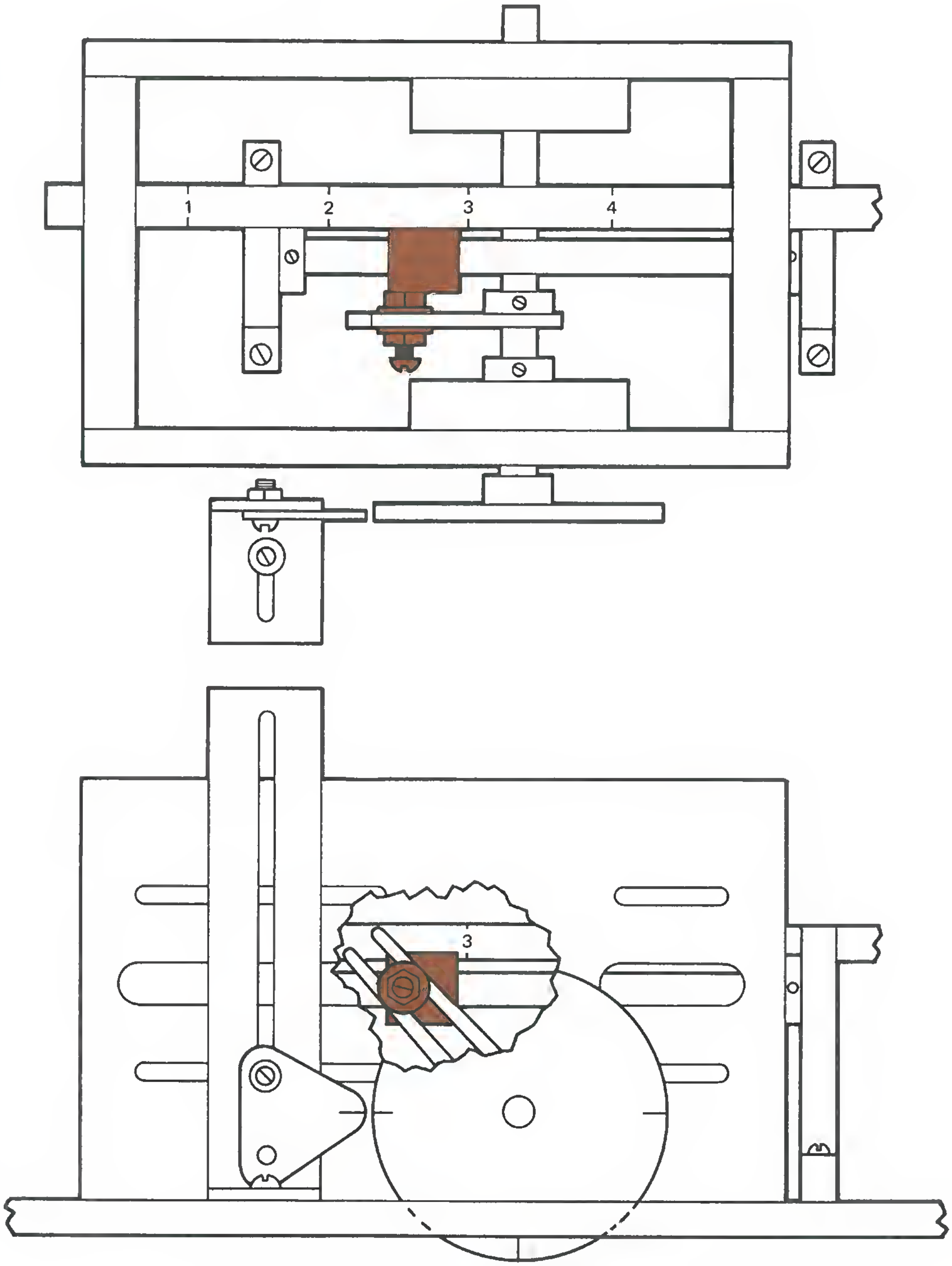


Fig. 25-6 Experimental Setup II

Fig. 25-5		Fig. 25-6	
Crank Angle α in degrees	Displacement in inches	Lever Angle α in degrees	Displacement in inches
0			0
20			1/8
40			1/4
60			3/8
80			1/2
100			5/8
120			3/4
140			7/8
160			1
180			1-1/8
200			- 1/8
220			- 1/4
240			- 3/8
260			- 1/2
280			- 5/8
300			- 3/4
320			- 7/8
340			- 1
360			- 1-1/4

Fig. 25-7 The Data Table

7. Place the ruler so that one edge of the rigid coupling indicates three inches and call this zero displacement. Position the ruler and sliding mechanism so that you can measure the displacement equally in both directions.
8. Record the angles for the displacements listed in the data table.
9. Plot the curves of your data on separate sets of axes but on the same sheet of graph paper.

ANALYSIS GUIDE. In analyzing your data explain the type of curve you got in each case. Explain how you know the function generated to be as you stated. Discuss any difficulties you encountered in the experiment.

PROBLEMS

1. The mechanism of figure 25-4 has a maximum error (e) of

$$e = \ell_c - \sqrt{\ell_c^2 - \ell_1^2}$$

   where ℓ_c is the link length and ℓ_1 is the crank length. Calculate the maximum error for your setup.

2. Explain how you could use the experimental setup of figure 25-4 to get the cosine function.
3. Calling the displacement, y, write the equation relating the angle α and y for figure 25-4.
4. Write the equation relating the displacement, y, and the angle, α, for figure 25-5.
5. Look at the experimental setup in figure 25-5. Try to imagine rotating the lever to 90°. What would the distance of displacement approach?
6. Did your results of problem five agree with equation 25.1? Explain your reply.
7. Knowing the vertical and horizontal components of two quantities that are at right angles to each other, which of the mechanisms of this experiment would you use to determine the angle? Explain.
8. If you knew the vertical component and hypotenuse of a right triangle, which of the mechanisms you constructed would produce data proportional to the angle? Explain your answer.

experiment 26 COMPUTING MECHANISMS (CALCULUS)

INTRODUCTION. Certain physical quantities represent areas or integrals and others represent rates or derivatives. In this experiment we will examine some simple ways of mechanically computing some of these quantities.

DISCUSSION. A rate of change tells us how rapidly one quantity is changing in relation to another quantity. For example, if the gasoline mileage of a car goes from 22 mpg at 40 mph to 10 mpg at 60 mph, the *average* rate of change is the change in mileage, Δ mpg, divided by the change in speed, ΔV or

$$\text{average rate} = \frac{\Delta \text{ mpg}}{\Delta V} = \frac{10 - 22}{60 - 40} = \frac{-12}{20}$$

$$= -0.6 \text{ hr./gal}$$

This means that your gasoline mileage would decrease an average of 0.6 mpg for each mile per hour increase in speed over the interval considered.

Many rates are changes in quantities compared to time. Suppose you drove from here to a point 150 miles awav in three hours. Your average rate of change of position would be

$$\text{average rate} = \frac{\Delta S}{\Delta t} = \frac{150 \text{ mi.}}{3 \text{ hr.}} = 50 \text{ mph}$$

This is your *average* rate of change so on the average, your position changed 50 miles each hour you drove. It isn't difficult to perceive the car you drove as being stopped part of the time or doing 70 mph part of the trip and still averaging 50 mph.

Suppose exactly one hour after you left on this trip you looked at the speedometer and it read 62 mph. The instantaneous speed then would be 62 mph at time equal to one hour. An instantaneous rate is called a derivative and each delta, Δ, (rate of change) used in the equation would be replaced by d.

$$\text{average rate} = \frac{\Delta S}{\Delta t} = 50 \text{ mph for}$$

the 3-hour interval

$$\text{instantaneous rate} = \frac{dS}{dt} = 62 \text{ mph at } t = 1 \text{ hour}$$

Notice for average rates we talk about time intervals while for a derivative we specify instants of time.

Mechanisms that have a measurable output quantity proportional to a rate of change are often called differentiators. A good example is the speedometer of a car which gives the speed of the car at any given time. One method of doing this is shown in figure 26-1.

The cable from the wheel turns a permanent magnet inside a coil so that an electrical current is generated and measured by the current meter. The faster the car goes, the faster the magnet turns, inducing more current, causing the speedometer pointer (meter needle) to read higher. The meter face is then calibrated in mph. The output current or meter deflection is a quantity that is proportional to the rate of change of the position of the car so this instrument is a differentiator. It uses electrical currents to be analogous to a speed so it is also an analog computer.

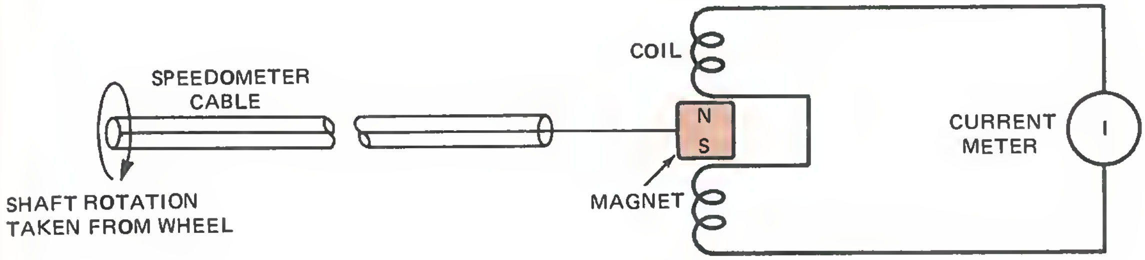


Fig. 26-1 Speedometer

Figure 26-2 shows a purely mechanical method of producing the same results. The faster the shaft rotates, the farther the weights are moved outwardly causing the sleeve to move up. The linkage makes the pointer needle indicate up scale.

Now let's look at integrating mechanisms. As an introductory example let's recall that one horsepower is 550 ft-lbs/sec. In other words, power is work per unit of time.

$$P = \frac{\text{work}}{\text{time}} = \frac{W}{t}$$

Solving for work gives us

$$W = Pt$$

If we want to compute work in terms of power and time, we would be calculating the area under a curve. Considering the ordinate values on the graph in figure 26-3 to be multipliers for the actual units, let's investigate the graphical relationship of some physical quantities.

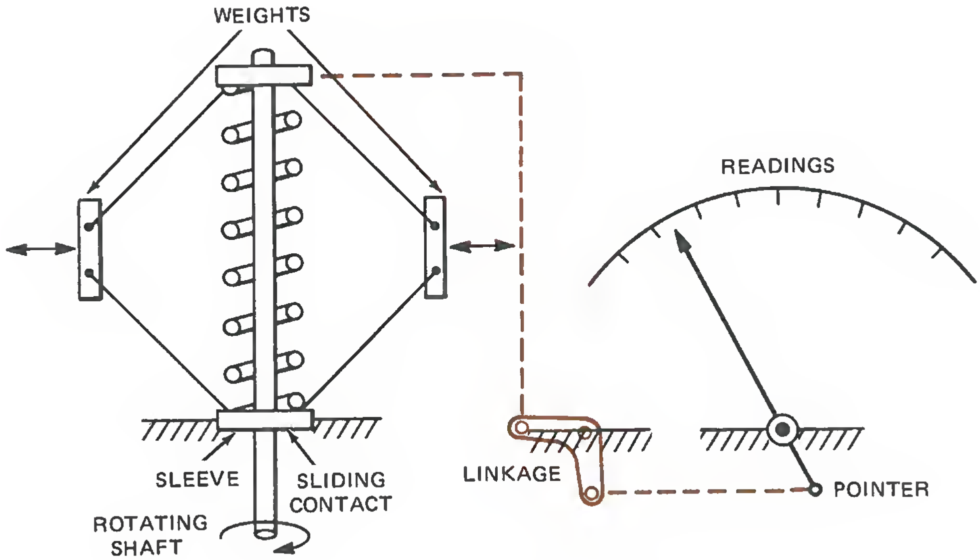


Fig. 26-2 Centrifugal Rate Indicator

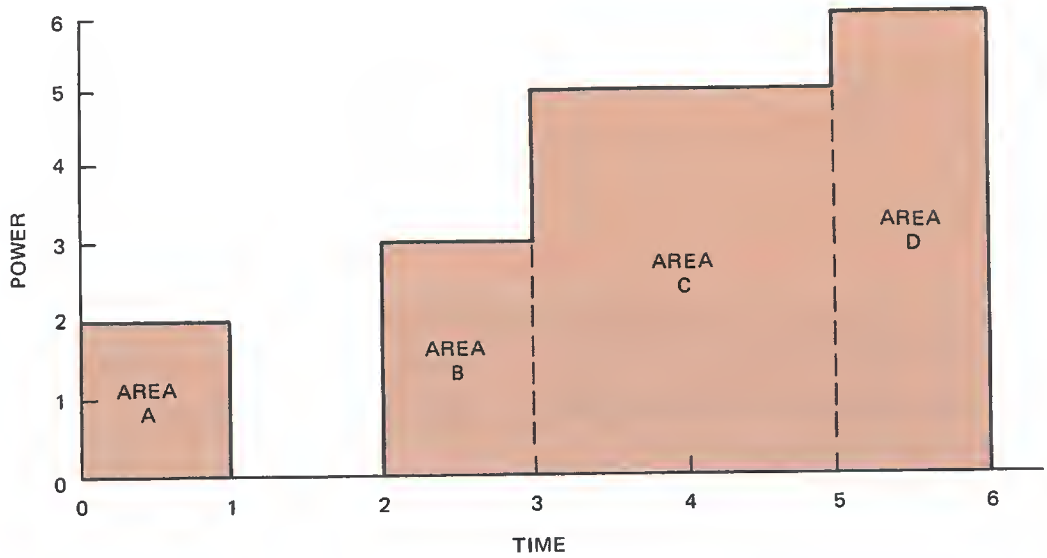


Fig. 26-3 Computing Work Graphically

Suppose a machine expends a continuous two units of power for each one unit of time. We would plot this on the graph of figure 26-3 as shown for time increment one, the work performed is

$$W = Pt = (2)\ (1) = 2 \text{ units of work}$$

Note that the product is the area under the power curve (area of a rectangle is length x width). Now, if no power is expended during the next time interval, no work is accomplished and the area from 1 to 2 is zero. For the time interval of 2 to 3, the power is 3 and the work is the area under the curve from 2 to 3 ($\Delta t = 1$)

$$W = Pt = (3)\ (1) = 3 \text{ units}$$

The area C under the curve (work) is

$$W = Pt = (2)\ (5) = 10 \text{ units}$$

Area D, by the same technique, is 6 units. The total work performed, then, is the total area:

$$W_t = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}4} + W_{4\text{-}5} = 2 + 0 + 3 + 10 + 6 = 21 \text{ units}$$

The computation of areas is relatively easy so long as the curve produces only rectangles. When the curve under which the area to be computed becomes other shapes, the problem becomes more complex and must be solved mathematically using integral calculus, or by some machine. Definite integrals are a method for calculating areas. For example, the integral from 0 to 1 (time axis) is the area under the curve of the function, f (t) and is written

$$W_{(0-1)} = \int_0^1 f(t)dt = 1 \text{ unit as before}$$

The interest here is not how you solve the integral but *what it means* in terms of the area, so

$$W_{(1\text{-}2)} = \int_1^2 f(t)dt = 0$$

$$W_{(2-3)} = \int_2^3 f(t)dt = 3$$

$$W_{(3-5)} = \int_3^5 f(t)dt = 10$$

$$W_{(5-6)} = \int_5^6 f(t)dt = 6$$

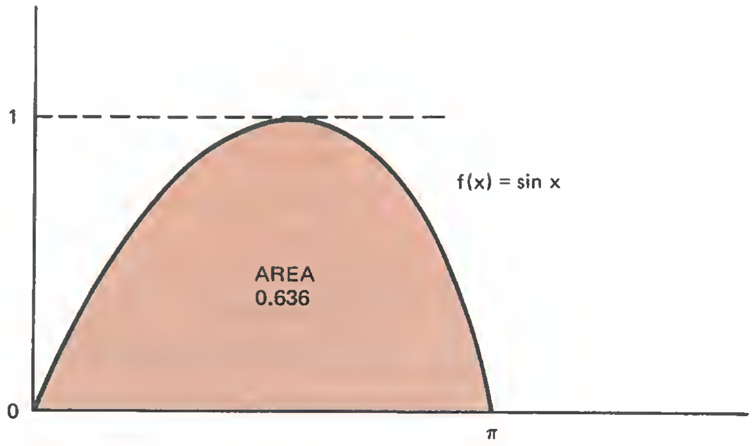


Fig. 26-4 Area of Half a Sine Wave

Thus, when we say a device is an integrator, we mean that it can compute the area under a curve for us.

Let's carry the example further by looking at figure 26-4. When we say that the definite integral from zero to π of a half sine curve is 0.636, we are saying that the area under the curve equals 0.636 and would be written

$$\text{Area} = \int_0^{\pi} f(x)dx = \int_0^{\pi} \sin x\, dx = 0.636$$

In some cases we are only interested in the way in which the area changes rather than its specific value. In these cases we are not interested in the end values shown at the top and bottom of the integral curve. For example, an intermediate step to calculating that the area of half a sine wave is 0.636 involves taking the indefinite (or antiderivative) of the function:

$$\int \sin x\, dx = -\cos x + k$$

Here k is called the constant of integration and shifts the waveshape up and down on the x, y axes. When k = 0, the cos x wave would be varying about (above and below) the x axis line.

Figure 26-5 shows the integrator that we will use in this experiment. The relationship between input, ω_i, output, ω_o and R is

$$\omega_o = \frac{1}{r}\int_0^t R d\omega_i$$

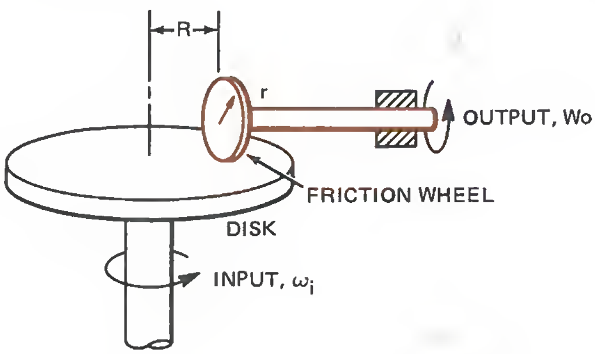


Fig. 26-5 Mechanical Integrator

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 1 DC motor 28 VDC
- 1 Pulley approximately 2 in. OD with 1/4 in. bore hub
- 2 Shafts 1/4 X 4″
- 2 Bearing mounts with bearings
- 2 Shaft hangers with bearings
- 1 O-ring approximately 1 X 1/8
- 1 Pulley approximately 1 in. OD with 1/4 in. bore hub
- 1 Disk dial
- 1 Steel rule 6 in. long
- 2 Collars
- 1 Flexible coupling
- 1 Stroboscope
- 1 DC power supply (0–40V)

PROCEDURE

1. Inspect your components to insure that they are undamaged.
2. Construct the bearing plate assembly shown in figure 26-6.
3. Snap the O-ring onto the smaller of the two pulleys. It should fit quite tightly.
4. Mount the bearing plate on the breadboard as shown in figure 26-7.
5. Loosen the clamps and slide the whole bearing plate assembly up snugly (not tightly) against the small pulley with the o-ring on it. **Note: The metal parts of the two pulleys should contact the o-ring only. Retighten the clamps.**

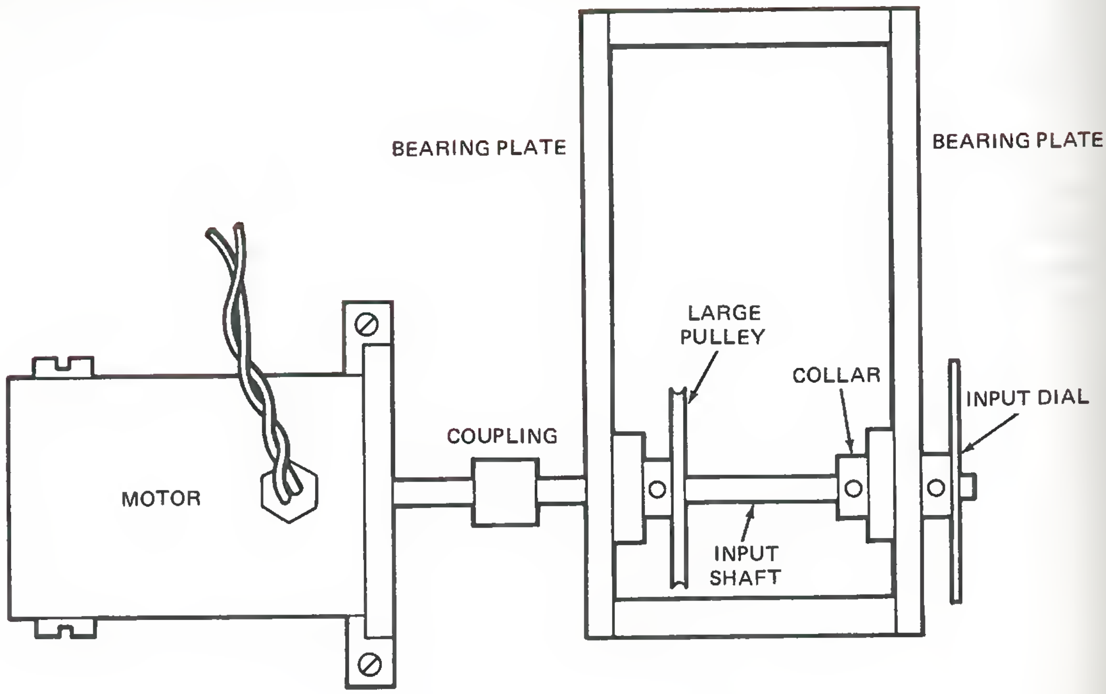


Fig. 26-6 The Bearing Plate Assembly

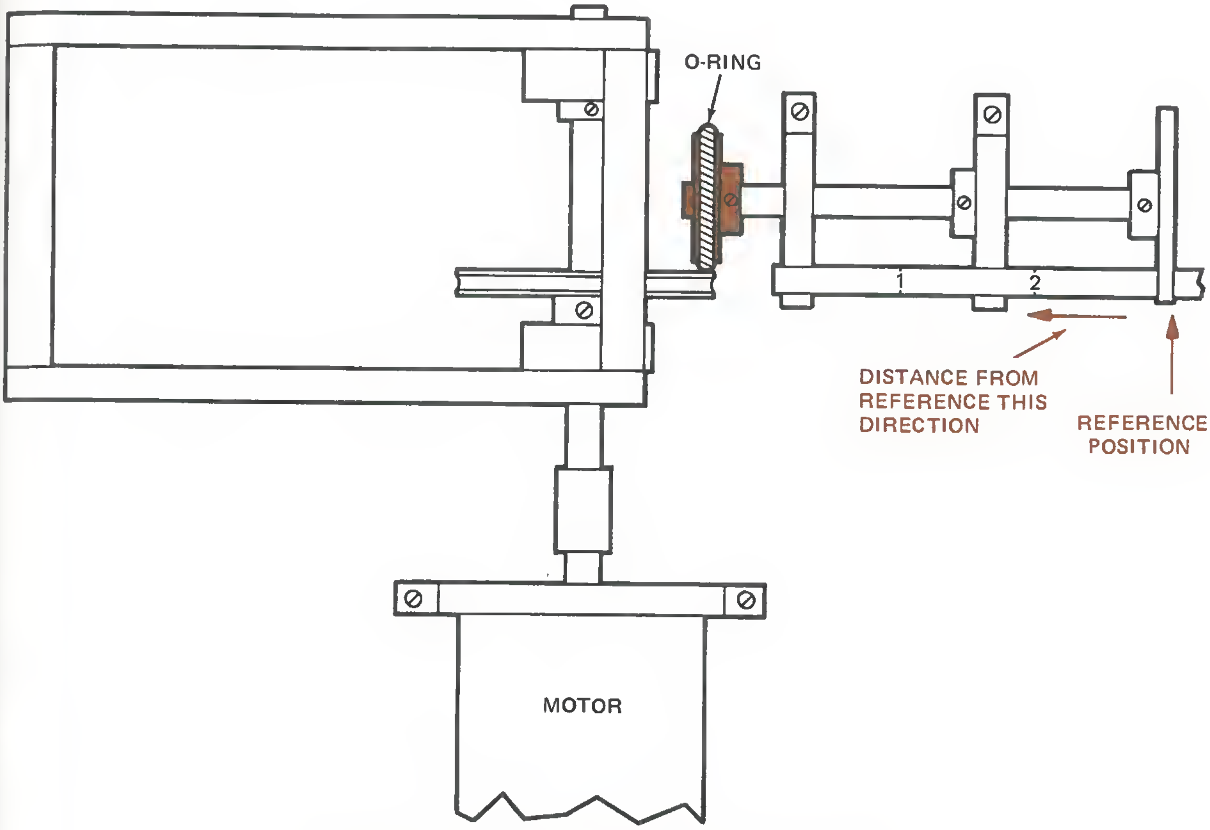


Fig. 26-7 Experimental Setup II Top View

6. Adjust the mechanism so that the rubber friction drive wheel is on the outer edge of the pulley wheel.

7. The forces will tend to move the rubber friction wheel away from the center of the drive wheel so the collar riding against the shaft hanger can serve to adjust the position of the friction wheel.

8. Place the ruler so that one edge of the dial indicates 3 inches. This will be the reference and will be called zero.

9. Set the power supply voltage to 10 volts and strobe the dial to determine the output RPM.

10. Move the dial the distance indicated (in 1/64 inches) in the data table (so that the friction wheel moves toward the center of the drive wheel) and reset the collar against the shaft hanger.

11. Strobe the output and repeat step 10 for each position indicated in the data table. The input is made to represent a sine wave by taking the values of displacement from the sine curve.

12. Your data represents slightly less than 90° of a cycle. Plot your data on the graph provided, choosing a scale to make the amplitude approximately the same as the sine wave provided.

Reading No.	Distance From Reference	Output RPM
1	0 (at Ref point)	
2	6/64 in.	
3	12/64 in.	
4	17/64 in.	
5	22/64 in.	
6	26/64 in.	
7	31/64 in.	
8	38/64 in.	

Fig. 26-8 The Data Table

13. Using your data for the first 75° and your knowledge of integration, plot how the remaining output curve should look if a complete cycle could be run.

ANALYSIS GUIDE. In analyzing your results from this experiment you should examine the data and discuss the relationship that exists between the input and output. Explain in your own words how the experimental mechanism could be used in a computing machine. Discuss any difficulties you encountered with the experiment.

PROBLEMS

1. After integrating a function you get a constant of integration which can be determined with some additional information. Suppose we solve the problem $y = \int dx = x + k$. In order to see the effect of k, plot $y = x + k$ below for $k = 0$, $k = 1$ and $k = 2$. Three points (say $x = 0$, 1 and 2) for each line should be sufficient.

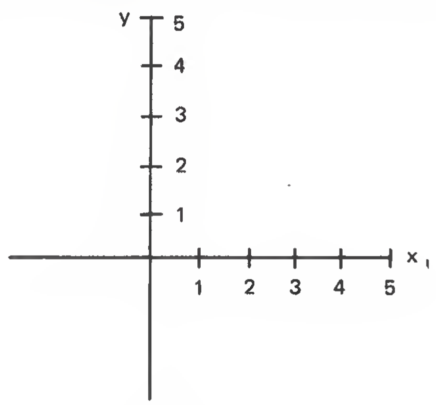


Fig. 26-9 Problem Axes

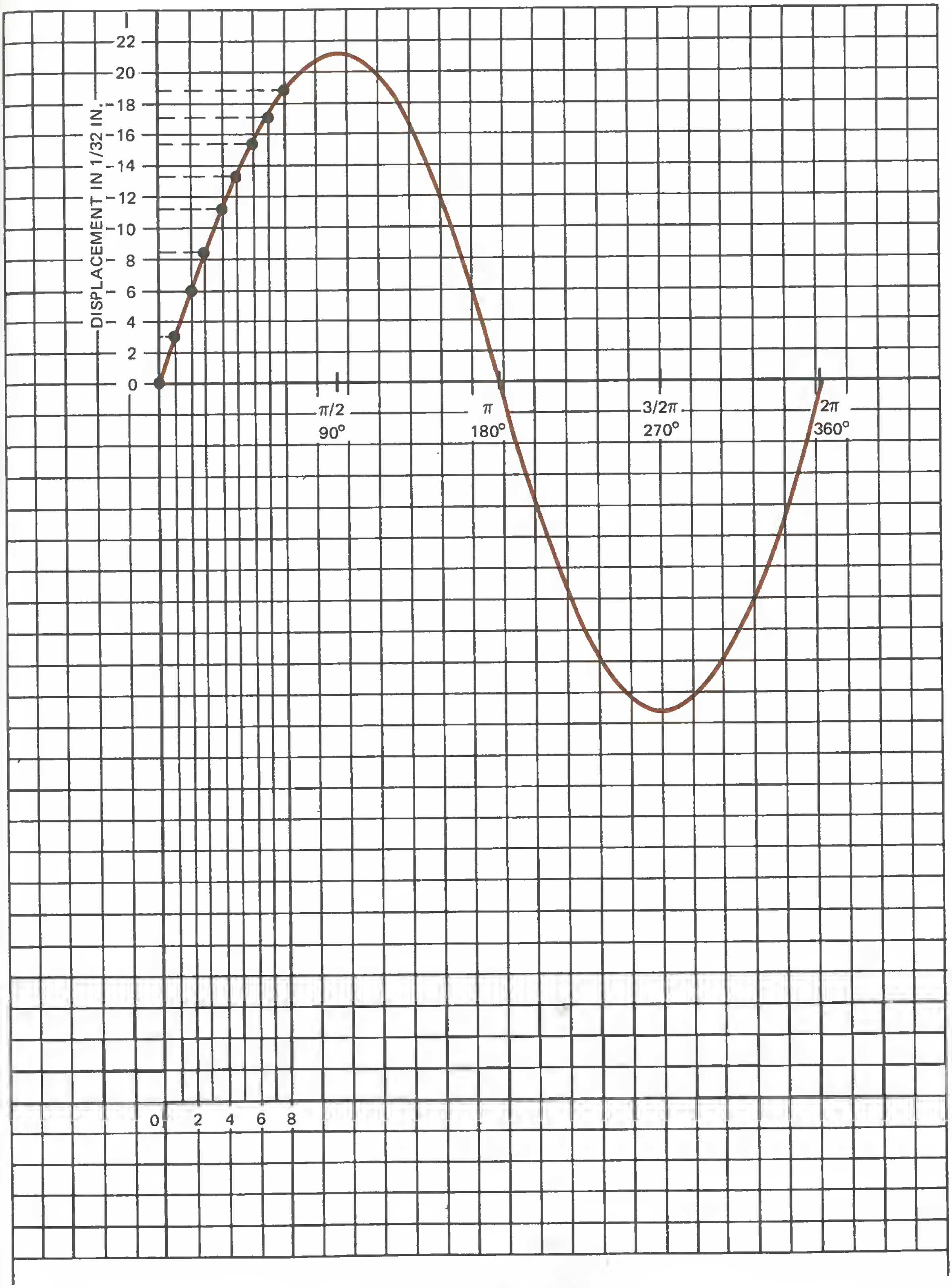


Fig. 26-10 Curves

2. The distance from the friction wheel to the center of the drive pulley, R, is varied by moving from the outside edge to the center at a constant velocity. This distance is changing at a linear rate and

$$\int x\,dx = \frac{x^2}{2} + k$$

   Sketch the output waveshape. Ignore the vertical position by letting k = 0.

3. If a flywheel changes its velocity from 10 RPM to 40 RPM in three seconds, what is the rate of change (average) in RPM per second?

4. A load is moved along a conveyor belt such that it takes 30 seconds to travel the full length of 100 feet. What is the average rate of change (velocity) in feet per second.

5. Sketch a way in which a mechanism could be connected to your experimental setup in order to drive the friction wheel back and forth in a sine function ($R = \sin \Theta$) (Check previous experiments for ideas about sine function generators).

6. Sketch the input and output waveforms for problem 5 above.

experiment 27 RATCHET MECHANISMS

INTRODUCTION. Many mechanisms require that motion be in only one direction. Others require that motion be intermittent although the input motion may be continuous. One method of achieving both of these types of motion is to use a ratchet. In this experiment we will examine some of the basic features of ratchets.

DISCUSSION. Ratchet mechanisms or ratchet gearing may be used to transmit motion of an intermittent nature, or to prevent a shaft from rotating backward. As an intermittent motion device, these mechanisms are useful for stepping or indexing. And, as a one-way motion device, they are useful as preventative or safety devices.

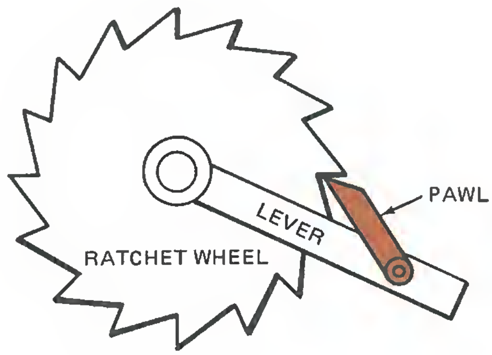


Fig. 27-1 Basic Ratchet Wheel

Figure 27-1 shows a ratchet in perhaps its simplest form. In this mechanism a movement of the wheel in a counterclockwise direction will occur when the lever is moved in that direction. When the lever is returned to its original position, the pawl or detent will slide over the wheel teeth but will not cause wheel rotation. In other words, when the lever is given an up and down oscillatory motion, the ratchet wheel will be given an intermittent rotary motion.

To prevent the ratchet wheel from rotating in the clockwise direction, the pawl could be secured to a stationary member. If the teeth of the ratchet wheel were square in shape, the pawl would then prevent motion in either direction. Square-toothed ratchet wheels are also used to provide reversing action. This approach is illustrated in figure 27-2. Here, the pawl is in the form of a plunger which has one tapered side and is free to

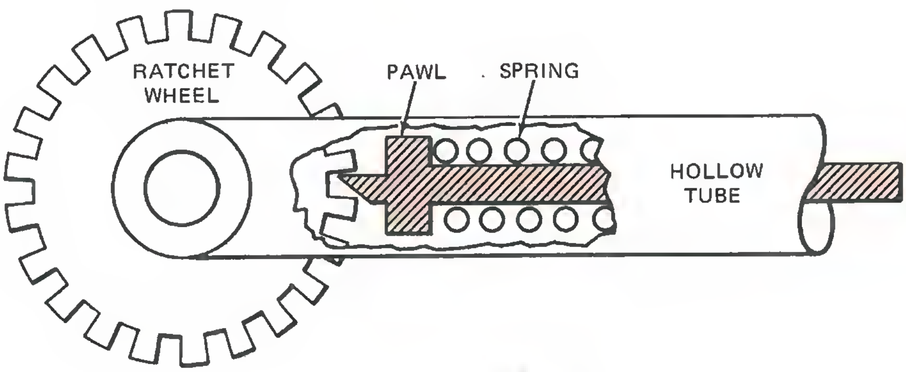


Fig. 27-2 Reversing Ratchet Mechanism

move but is held against the ratchet wheel by a small amount of spring force. When the pawl is lifted and turned 180 degrees, the flat driving face is reversed which will give the ratchet wheel motion in the opposite direction.

In all of the ratchets discussed so far, the number of indexing positions equals the number of teeth on the ratchet wheel. One way to increase the number of index positions is to increase the number of teeth; however, a large number of teeth mean smaller ones and thus less strength. Another method used to increase the number of stops made by the ratchet wheel is to use multiple pawls. For example, adding another pawl which is of a different length as shown in figure 27-3 will double the number of indexing positions. By placing a number of pawls side by side and proportioning their lengths according to the pitch of the teeth on the ratchet wheel, a quite fine feed can be obtained even though the ratchet wheel has a coarse pitch.

Another important type of ratchet is the frictional type. In this type there is no direct, positive engagement between the pawl and the ratchet wheel; rather, the intermittent motion is transmitted by frictional resistance. The ratchet wheel has a smooth surface. The pawl shape shown in figure 27-4 is such that motion in only one direction is encouraged. If the pawl is moved in the proper direction, the ratchet wheel will rotate. Conversely, if the ratchet wheel attempts to move in the opposite direction, the pawl friction will tend to prevent it. The action in this type of fric-

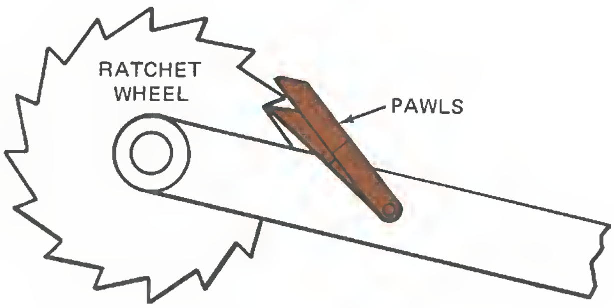


Fig. 27-3 Multiple Pawl Ratchet

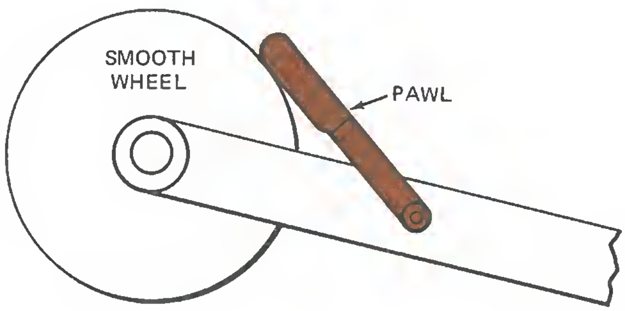


Fig. 27-4 Friction Ratchet

tion ratchet is substantially the same as the positive-action ratchet illustrated in figure 27-1.

A second type of friction ratchet is illustrated in figure 27-5. Rollers, or sometimes balls, are placed between the ratchet wheel and an outer ring which, when turned in one direction, causes the rollers or balls to be wedged between the wheel and ring as they move up the inclined edges. Rotation in the opposite direction will cause the rollers to move into the recessed areas of the teeth and thereby reduce the friction between the two surfaces.

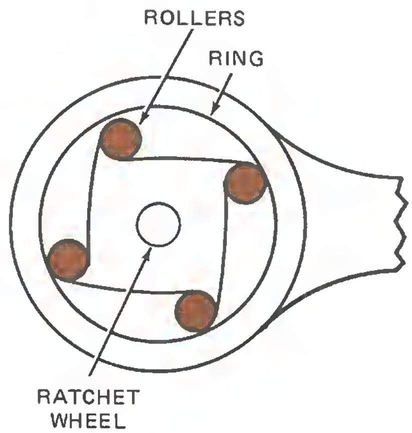


Fig. 27-5 Another Friction Ratchet

When designing a positive-action ratchet some consideration must be given to the way the pawl mates with the ratchet teeth. To insure that the pawl is automatically pulled in and engaged properly, an appropriately contoured tooth shape is important. One way to insure that relatively small forces are acting in the system is to make sure that the ratchet wheel center, the pawl pivot center, and the point of initial contact between the pawl and the ratchet all lie on the same circle. The normal to the line of contact at the point of initial contact between the pawl and the tooth face should pass through the center line of the ratchet and pawl pivot between their center points. This is illustrated in figure 27-6.

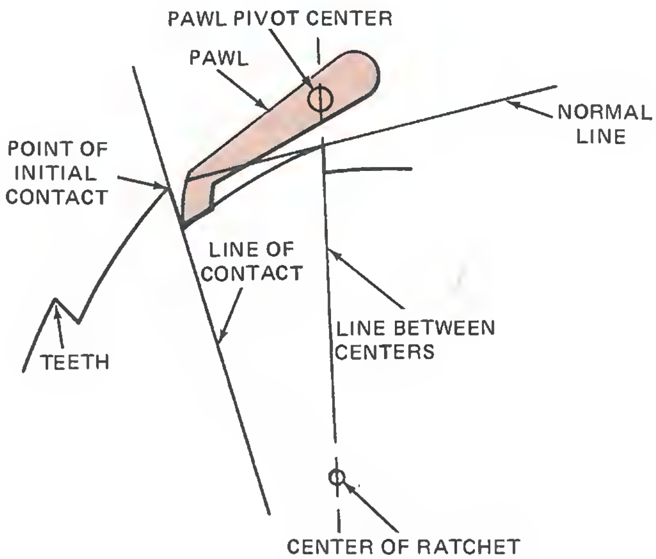


Fig. 27-6 Pawl-Tooth Mating Design

In some cases a four-bar mechanism is used to move a ratchet pawl. Figure 27-7 shows one of the many possible arrangements employing a four-bar mechanism. In this type of assembly the pawl steps over the ratchet teeth, engages one and pulls it to the right each time the crank rotates. The resulting ratchet wheel motion is, of course, intermittent.

The drive pawl used may have a single tooth or many teeth. In some instances a rack is used as a drive pawl. The intermittent motion may be either rotary as in figure 27-7 or linear. A linear motion mechanism would use a rack in place of the ratchet wheel and would probably have some means of returning the rack to its starting position from the end of its travel.

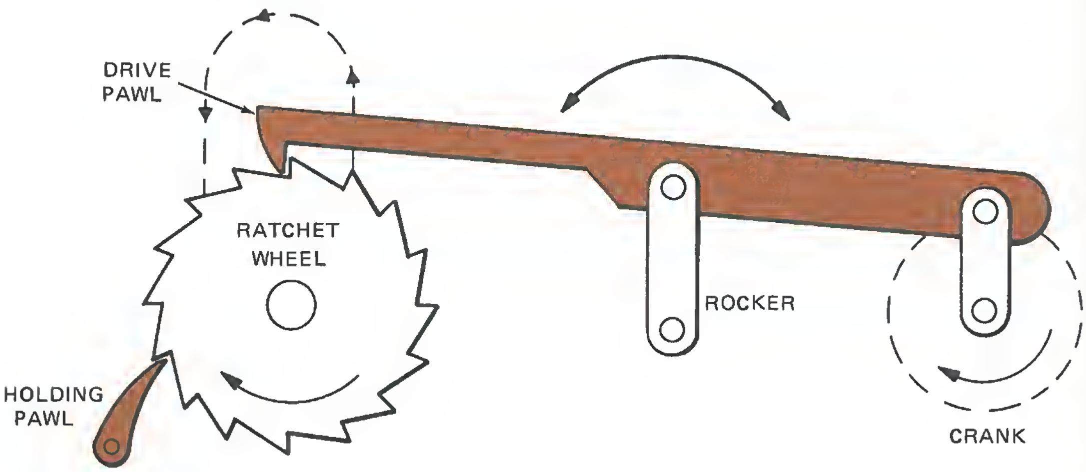


Fig. 27-7 A Four-Bar Ratchet Assembly

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 4 Bearing holders with bearings
- 2 Shafts 4" x 1/4"
- 1 Shaft 2" x 1/4"
- 2 Shaft hangers with bearings
- 2 Disk dials with 1/4-in. bore hubs
- 2 Dial indices with mounts
- 5 Collars
- 1 Rack
- 2 Lever arms 1 in. long with 1/4-in. bore hubs
- 1 Spur gear approx. 3/4 in. OD with 1/4-in. bore hub
- 2 Flathead machine screws 2-56 x 3/4 in.
- 4 Flat washers No. 2 x 1/2 in. OD
- 4 Hex nuts 2-56 x 1/4 in.
- 1 Steel rule 6 in. long
- 1 Rubber grommet approx. 1/2 in. OD x 1/4 in. thick with 1/4-in. hole

PROCEDURE

1. Inspect each of your components to insure that it is undamaged.
2. Mount the rack to the crank and rocker as shown in figure 27-8. The crank should be 1/2 in. long and the rocker should be 1 in. long.

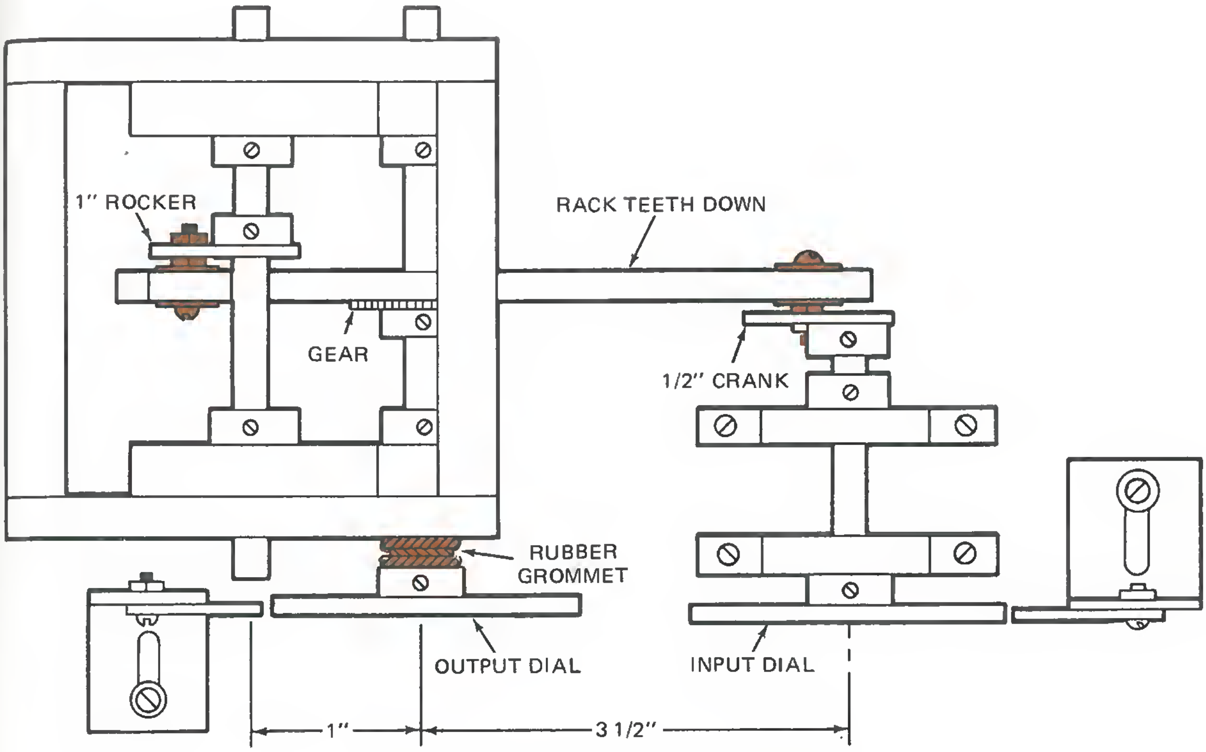


Fig. 27-8 The Experimental Setup

3. Construct the remainder of the mechanism shown in figure 27-8. The dimensions shown on the drawing are only approximate. Measure and record each link length (ℓ_1, ℓ_2, ℓ_c, ℓ_o).
4. Adjust the height of the rocker shaft and the spacing between the bearing plate assembly and crank shaft so that the rack properly meshes with the gear when both levers point vertically downward.
5. Adjust the dials to read zero when the rack is at its righthand limiting position.
6. Fit the output dial so that it squeezes the rubber grommet firmly against the bearing plate.
7. Starting at zero carefully rotate the input dial clockwise in 20-degree steps. Record both the input and output dial readings at each step. Continue until the output dial has rotated more than 360 degrees.
8. Reset the dials to the position given in step 5. Be sure the rubber grommet is firmly squeezed between the output dial and the bearing plate.
9. Repeat step 7 but rotate the input counterclockwise this time.
10. On graph paper plot the output positions versus the input positions for both sets of data.

Clockwise Rotation		Counterclockwise Rotation	
Input Θ	Output Θ	Input Θ	Output Θ
$\ell_1 =$	$\ell_c =$	$\ell_o =$	$\ell_2 =$

Fig. 27-9 The Data Table

ANALYSIS GUIDE. In analyzing this experiment you should consider several possible applications of a ratchet. How might a ratchet be used to obtain both quick return and intermittent motion? Discuss means of varying the timing of the intermittent motion. What was the purpose of the rubber grommet? Discuss any difficulties you encountered in assembling the mechanism and any other pertinent comments which you might have.

PROBLEMS

1. Discuss at least three similarities and three differences between a geneva mechanism and a ratchet mechanism.
2. What are the advantages of the friction type ratchet over the positive action type?
3. Draw a simple stick diagram of the experimental mechanism.
4. Graphically determine the amount of horizontal and vertical displacement experienced by the rack in the experiment.
5. Verify that the mechanism in the experiment satisfies the conditions required for a crank rocker.

experiment 28 FRICTION RATCHETS

INTRODUCTION. In many applications it is necessary to restrict rotation in one direction while allowing rotation in the opposite direction. Friction ratchets are sometimes used to provide single direction rotation. In this experiment we will examine a simple example of such a ratchet.

DISCUSSION. A friction ratchet is composed of a wheel and a pawl. Figure 28-1 shows one of the possible arrangements. In this type of mechanism the wheel may turn relatively freely counterclockwise. When it does, it tends to lift the end of the pawl against the force exerted by the pawl spring and that arising from the weight of the pawl itself. In some cases no spring is used and the pawl is held in contact with the wheel by gravity. In this event only small amounts of torque are required for counterclockwise rotation of the wheel.

In other cases the spring is used and the torque required to turn the wheel counterclockwise depends more or less directly on the spring force. Figure 28-2 shows a simplified diagram of a friction ratchet operation under this type of condition.

For purposes of analysis let's assume that we know the value (f_1) of the component of the spring (or gravity) force acting perpendicular to the pawl arm. When we don't know this value, we can determine it from the spring (or gravity) force and the pawl arm angle.

If the wheel is to turn, it must impart a force f_2 to the lever which is such that the moments acting on the pawl arm balance. That is, from figure 28-2,

$$Sf_1 = \ell_1 f_2 \qquad (28.1)$$

where both f_1 and f_2 are perpendicular to the pawl arm.

Now let's look at the angles between the forces and distances in the mechanism. Figure 28-3 shows the ones we will be interested in. Let's notice that the force f_2 acting to lift the pawl arm is one of the quadrature components of the tangential force F. The relationship

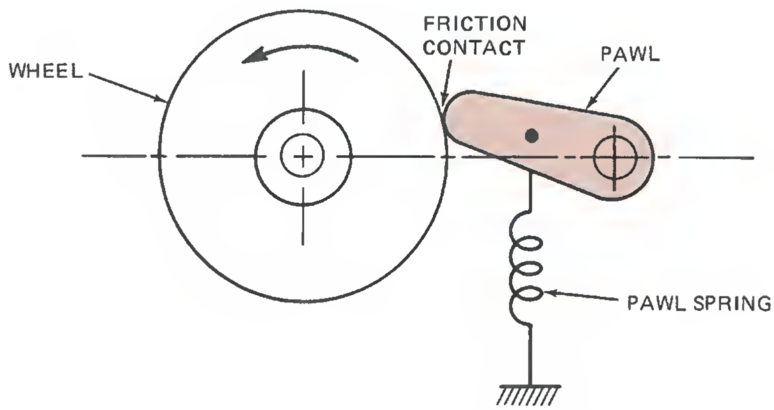


Fig. 28-1 A Friction Ratchet

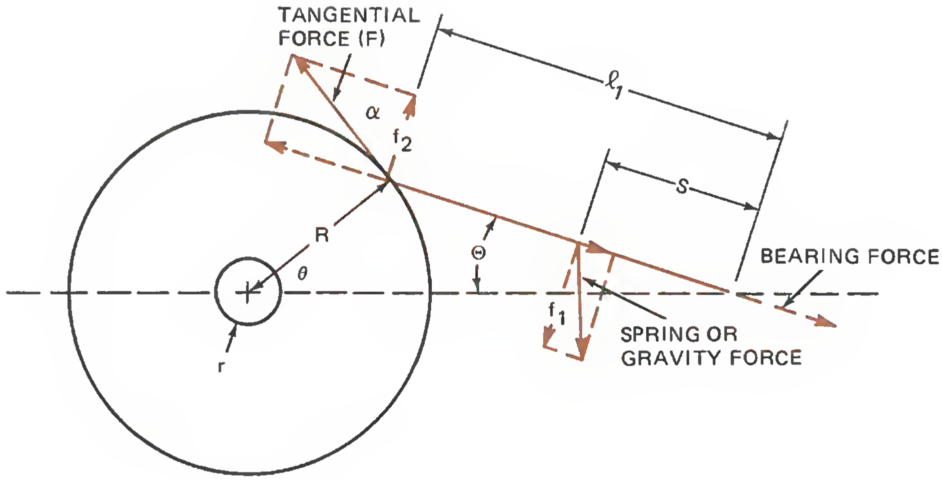


Fig. 28-2 Forces Acting on a Friction Ratchet

between these two forces is

$$F = \frac{f_2}{\cos \alpha} \qquad (28.2)$$

To evaluate this equation we must first determine the angle α between f_2 and F. To do this we observe that at the vertex of f_2 and F

$$\alpha + \beta + 180^\circ = 360^\circ$$

or

$$\alpha = 180^\circ - \beta \qquad (28.3)$$

So, if we know β, we can find α. Applying the law of sines to the triangle in figure 28-3 we have

$$\frac{C}{\sin \beta} = \frac{R}{\sin \Theta}$$

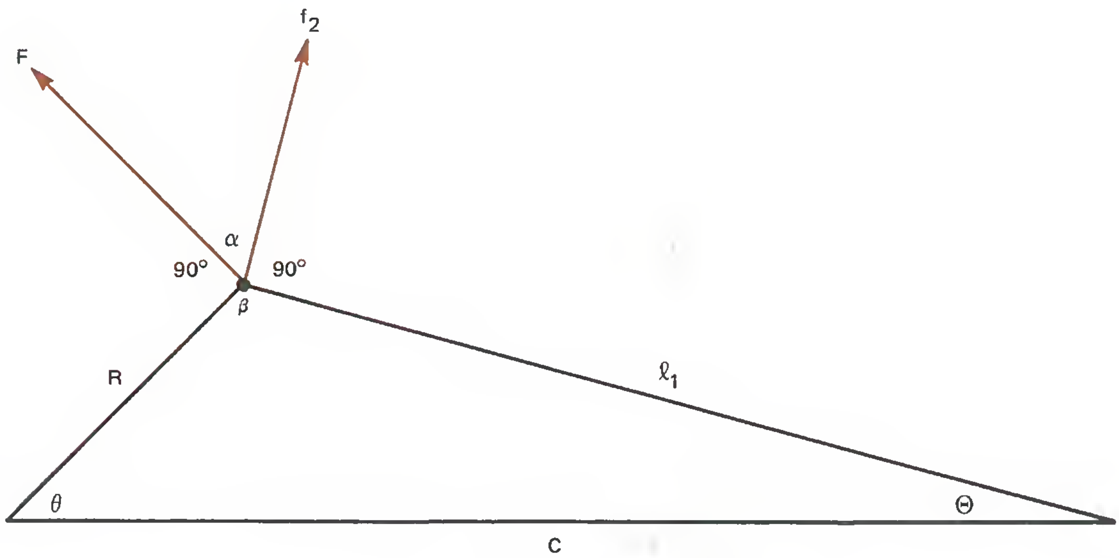


Fig. 28-3 Forces, Distances and Angles

or

$$\sin\beta = \frac{C}{R}\sin\Theta \qquad (28.4)$$

where R is the wheel radius, C is the center distance between shafts and Θ is the angle between the center line and the pawl arm. With this equation (28.4) we can determine β. Then using 28.3 we get α. Finally, if we know α, we can combine equations 28.1 and 28.2 rendering

$$F = \frac{Sf_1}{\ell_1 \cos\alpha} \qquad (28.5)$$

which relates the tangential wheel force to the spring (or gravity) force acting perpendicular to the pawl arm.

The wheel torque is, of course,

$$T = FR = \frac{SRf_1}{\ell_1 \cos\alpha} \qquad (28.6)$$

and the tangential force required at the wheel shaft is

$$F_i = \frac{T}{r} = \frac{SRf_1}{r\,\ell_1 \cos\alpha} \qquad (28.7)$$

where r is the shaft radius.

If the wheel in figure 28-1 attempts to rotate clockwise, the pawl arm is forced downward. This tends to jam the pawl against the wheel opposing its clockwise rotation.

MATERIALS

1 Breadboard with legs and clamps
2 Bearing plates with spacers
4 Bearing holders with bearings
4 Collars
1 Pulley approximately 2 in. OD with 1/4-in. bore hub
2 Lever arms 1 in. long with 1/4-in. bore hubs
1 Washer 1/4 in. ID, 1/2 in. OD, 1/16 in. thick
1 O-ring 1 in. average diameter
2 Spring balances
1 Spring balance post with clamp
1 Waxed string approximately 18 in. long
1 Protractor
1 Steel rule 6 in. long
1 Dial caliper 0 - 4 in.

PROCEDURE

1. Inspect all of your components to insure that they are undamaged.
2. Assemble the levers and O-ring as shown in figure 28-4.
3. Assemble the mechanism shown in figure 28-5.
4. Adjust the shaft center distance so that the lever arm is vertical when the O-ring is in firm contact with the pulley groove.
5. Tie a loop in one end of the waxed string.
6. Wind the string tightly in a single layer onto the pulley shaft so that the loop is accessible. The string must be wound on the shaft in the direction that will cause the pulley to turn forcing the pawl arm away when the loop end of the string is pulled.
7. Adjust the pawl spring balance so that it is horizontal and reads 4 ounces.

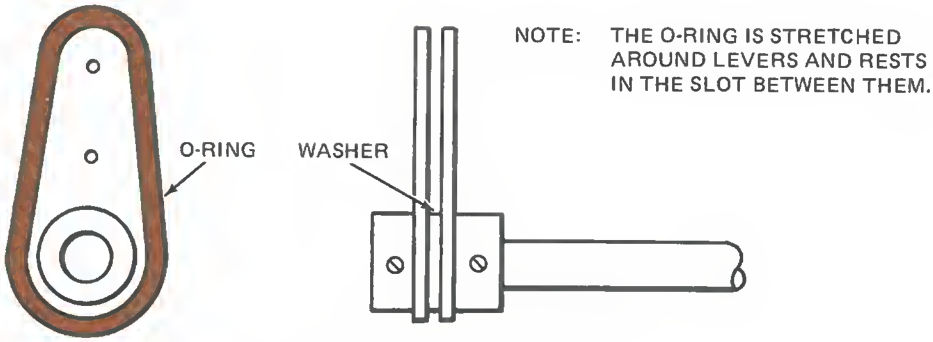


Fig. 28-4 Levers and O-Ring

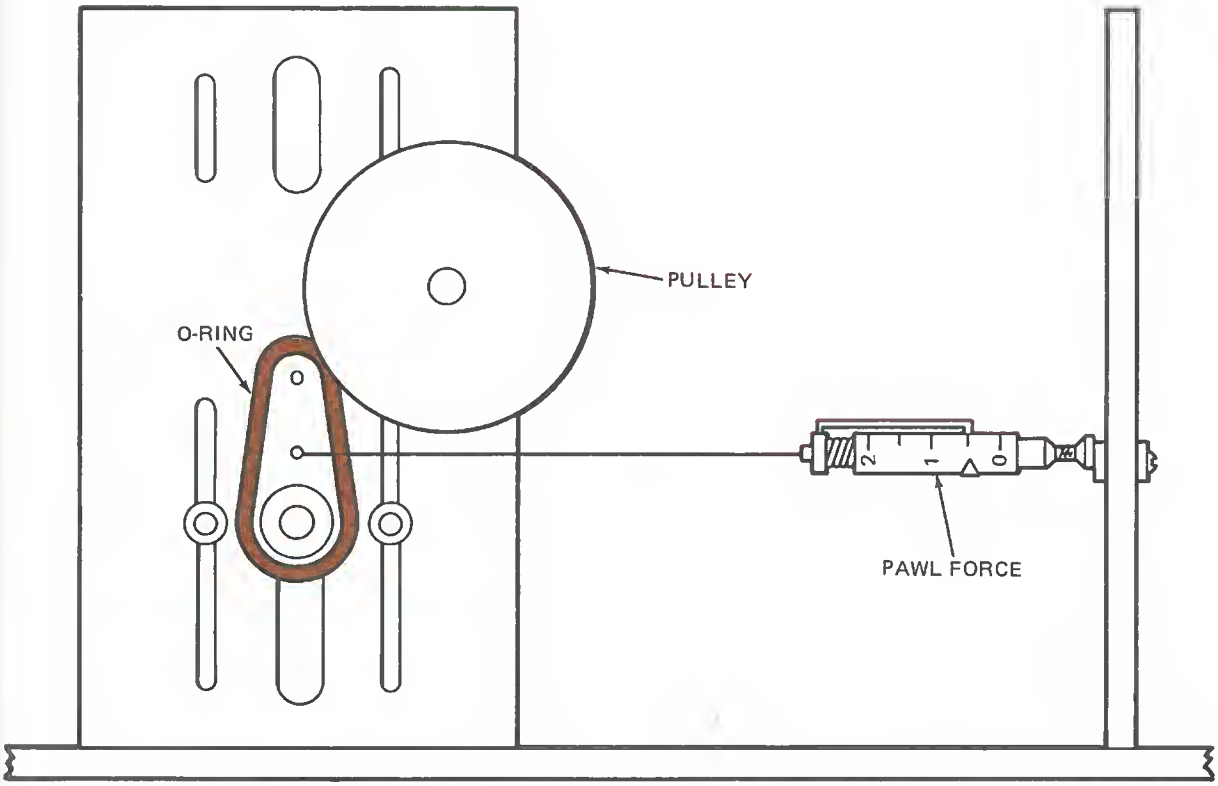


Fig. 28-5 The Experimental Mechanism

8. Hook the end of the remaining spring balance into the string loop.
9. Smoothly pull the spring balance causing the pulley to rotate at a constant velocity. Record the force required to maintain a constant pulley velocity. You may have to practice this operation several times to get it smooth enough.
10. Wrap the string back onto the shaft in the opposite direction.

11. Now repeat step 9.

12. Repeat steps 6 through 11 for pawl forces of 8, 12, and 16 ounces.

13. Measure and record the ratchet parameters listed near the bottom of the data table. (Θ is the angle between the pawl center line and the center line between the shaft centers.)

Pawl Force	Pulley Force Forward	Pulley Force Backward
4 oz		
8 oz		
12 oz		
16 oz		

ℓ_1	S	Θ	R	r

Fig. 28-6 The Data Table

ANALYSIS GUIDE. In analyzing your results from this experiment you should consider two main points:

1. Was the forward force required to rotate the pulley more-or-less proportional to the pawl force?
2. Did you observe a difference between the forward and backward pulley forces for a given pawl force?

Based on your answer to these points discuss the effectiveness of the experimental ratchet.

PROBLEMS

1. Using your ratchet parameters from step 13 and equation 28.4, compute the angle β.
2. Make a sketch of the mechanism similar to figure 28-3 and label Θ and β.
3. Using equation 28.3 compute the angle α and label it on your sketch.
4. With equation 28.7 compute the pulley force F_i and label s, R, f_1, r, ℓ_1 on your drawing. (Assume f_1 = 12 oz for this calculation.)
5. Compare F_i from problem 4 to the corresponding value in the data table. How well do they agree?

experiment 29 TOGGLE LINKAGES

INTRODUCTION. Basically, a toggle linkage is composed of two members joined together in such a way that a small force at the joint produces a large force at the ends. Toggles are used in a variety of presses, clamps and fasteners. In this experiment we will examine basic toggle action.

DISCUSSION. Let's consider the simple mechanism shown in figure 29-1. If we assume for a moment that the link lengths (ℓ_1 and ℓ_2) are equal, then we can draw the force diagrams shown in figure 29-2.

Notice that in figure 29-2a the compression forces, f_1 and f_2 in links ℓ_1 and ℓ_2, respectively, are equal to each other in magnitude. Moreover, these forces must add up vectorially to equal the force f applied to the toggle joint.

Looking at the load (point B), we see that the force here has two components. It has its share of the vertical load (f/2) and a horizontal component (F), From figure 29-2b we can determine the relationship between these two components as being

$$\frac{f/2}{F} = \tan \Theta$$

or

$$\boxed{\frac{f}{F} = 2 \tan \Theta} \qquad (29.1)$$

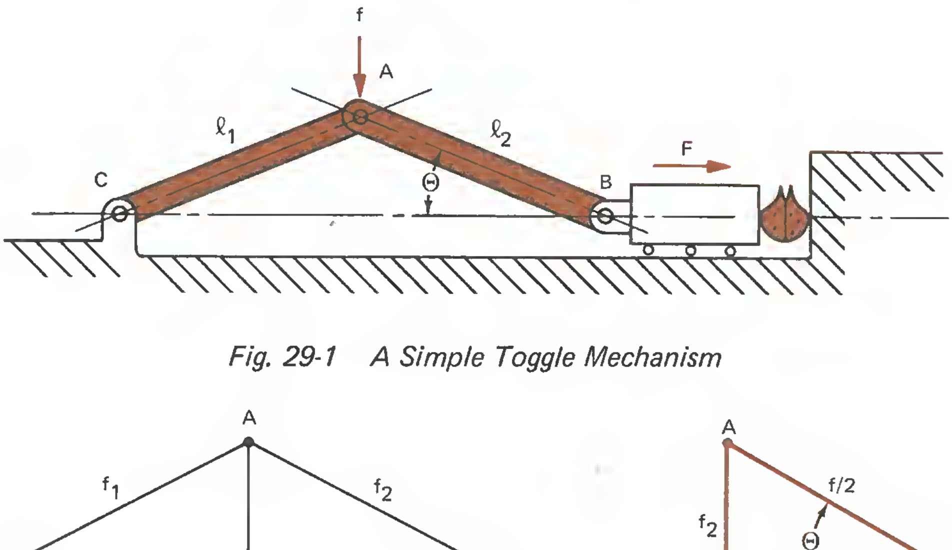
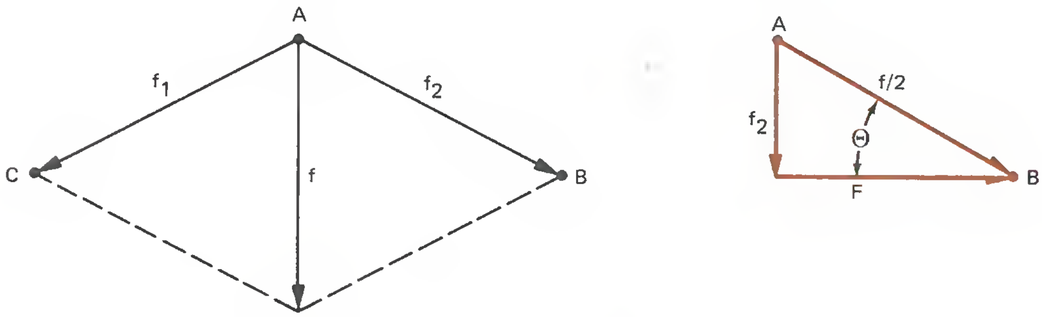


Fig. 29-1 A Simple Toggle Mechanism



(A) FORCES AT POINT A

(B) FORCES AT POINT B

Fig. 29-2 Forces in a Simple Toggle Mechanism

where θ is the angle between ℓ_2 and the center line BC.

Another way that this equation is sometimes written requires that we recall that

$$\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$$

Substituting into equation 29.1 gives us

$$\frac{f}{F} = 2\frac{\sin\Theta}{\cos\Theta}$$

or

$$\boxed{f\cos\Theta = 2F\sin\Theta} \qquad (29.2)$$

as an equivalent expression.

If we plot equation 29.1, the result is approximately as shown in figure 29-3. Notice that as Θ approaches zero degrees, the ratio f/F also approaches zero. Now, if f is a finite (non-zero) force, then F becomes extremely large. In theory at least, F would be infinite at $\Theta = 0$. However, in a practical case, such an idealistic condition is impossible due to play in the joints, compression in the links, etc.

In many applications the link lengths will not be equal. Figure 29-4 shows a typical case using unequal link lengths. In this case we can draw the force diagrams shown in figure 29-5. Notice that f_1 and f_2 are now the vertical force components at points C and B, respectively.

If the mechanism is in equilibrium, then the sum of f_1 and f_2 must equal the force f applied at the toggle joint:

$$f_1 + f_2 = f$$

Also, the moments acting on points C and B must be equal so

$$bf_1 = af_2$$

Focusing on the forces at point B (figure 29-5b), we see that f_2 and F are related by

$$\frac{f_2}{F} = \tan\Theta \qquad (29.3)$$

Using the moment equation we can solve for f_1 in terms of f_2 with the result

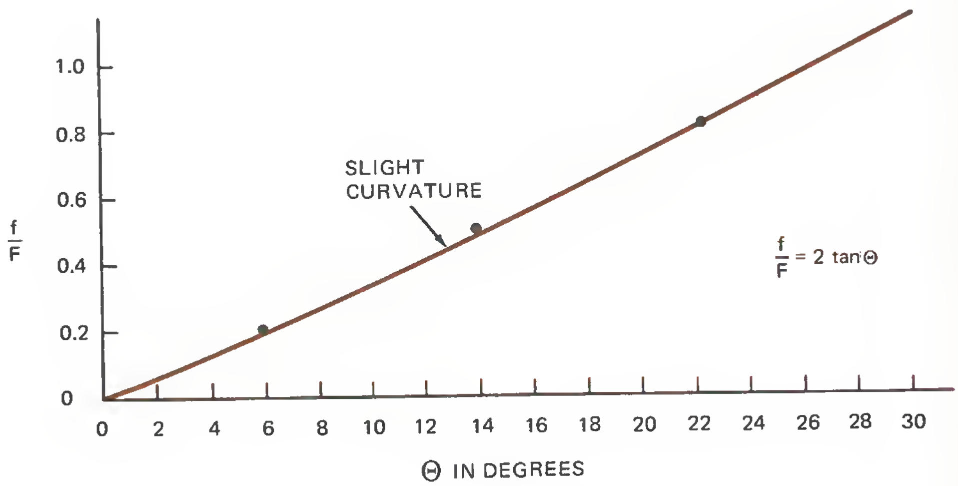


Fig. 29-3 f/F Versus Θ

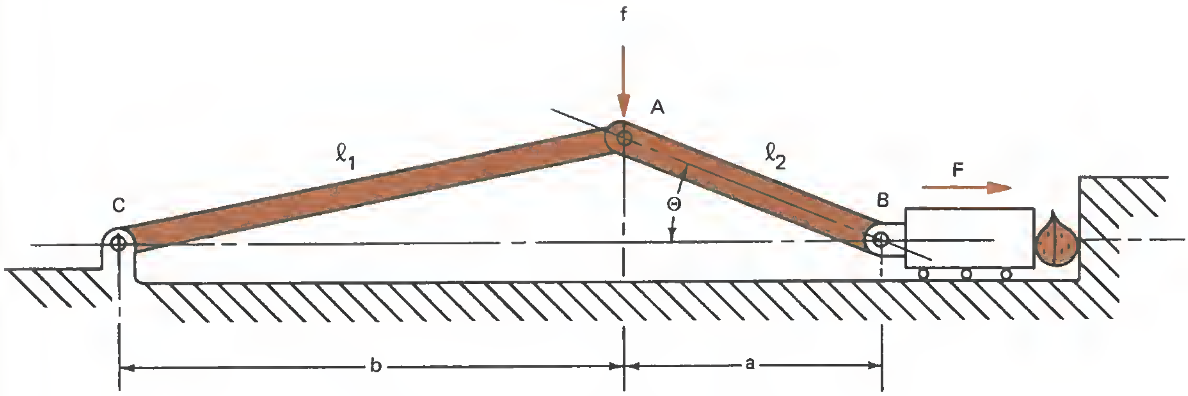


Fig. 29-4 A Toggle With Unequal Arms

$$f_1 = \frac{a}{b} f_2$$

Substituting this into the sum-of-vertical-forces equation will give us

$$\frac{a}{b} f_2 + f_2 = f$$

or

$$f_2 \left(1 + \frac{a}{b}\right) = f$$

which can be solved for f_2 in terms of f

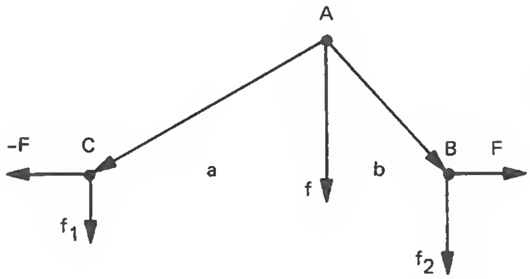
$$f_2 = \frac{f}{1 + a/b}$$

Substituting this into equation 29.3 allows us to solve for the ratio f/F:

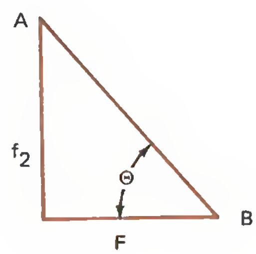
$$\boxed{\frac{f}{F} = \left(1 + \frac{a}{b}\right) \tan \Theta} \qquad (29.4)$$

In this type of mechanism, when Θ changes, the values of a and b also change. Consequently, the relationship between f/F and Θ is different for each position of the mechanism. However, as Θ is quite small, then

$$a \approx \ell_2 \text{ and } b \approx \ell_1$$



(A) FORCES AT POINTS A, B, & C



(B) FORCES AT POINT B

Fig. 29-5 Forces in Unequal Link Toggle

So, we can approximate the ratio f/F using

$$\frac{f}{F} \approx \left(1 + \frac{\ell_2}{\ell_1}\right) \tan \Theta$$

when Θ is a small angle. It is often slightly more convenient to rearrange the coefficient of tan Θ and use the form

$$\boxed{\frac{f}{F} \approx \frac{\ell_1 + \ell_2}{\ell_2} \tan \Theta} \qquad (29.5)$$

in actual practice.

MATERIALS

- 1 Steel rule 6 in. long
- 1 Protractor
- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 1 Shaft hanger with bearing
- 4 Bearing holders with bearings
- 3 Shafts 4″ X 1/4″
- 2 Spring balances with clamps and posts
- 1 Lever arm 2 in. long with 1/4 in. bore hub
- 1 Pulley approx. 3/4 in. OD with 1/4 in. bore hub
- 1 Rigid coupling
- *1 Wire loop link approx. 3 in. long
- 1 Machine screw 6-32 x 1/4
- 1 Spacer No. 6 x 1/8 with 1/32 in. wall thickness
- 5 Collars
- 2 Pieces of string approx. 6 in. long

*See Appendix A for wire link construction details.

PROCEDURE

1. Inspect all of your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 29-6.
3. Measure and record the link lengths used in the toggle mechanism.
4. Check to see that both spring balances read zero when no force is applied.
5. Position the input spring balance completely back against its post.
6. Move the output spring balance forward until it reads zero. Then carefully move it back until the *input* force is 1 ounce.
7. Check to be sure that:

   (a) The string connecting the input force to the toggle joint approaches the joint *vertically.* Adjust the position of the lever arm shaft as necessary to make the input string vertical.

   (b) The point of attachment of the wire link to the slider block must be as shown in figure 29-7. You may have to hold the screw in this position while taking your force and angle readings.

   (c) Both spring balances must be horizontal and they must not touch any part of the mechanism.

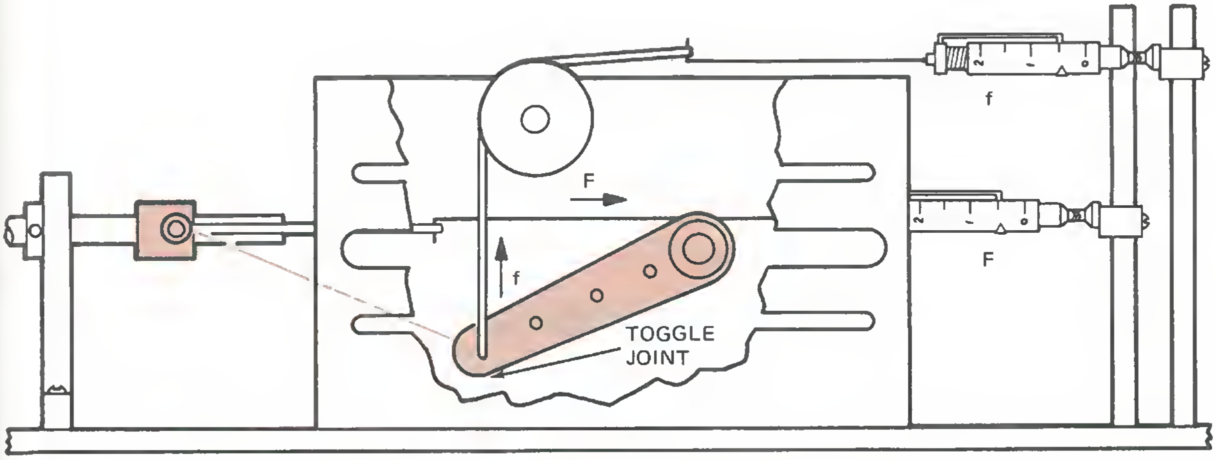


Fig. 29-6a Experimental Setup Side View

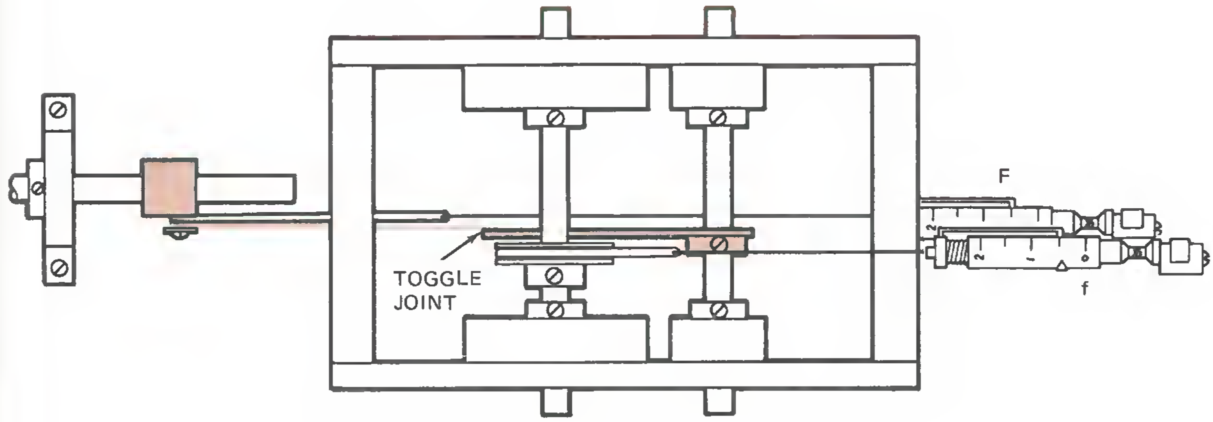


Fig. 29-6b Experimental Setup Top View

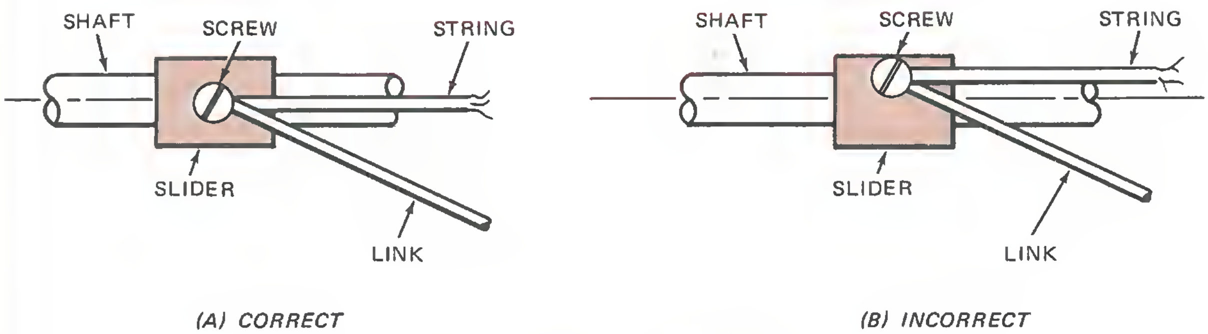


Fig. 29-7 Slider and Link Alignment

8. With the conditions in step 7 satisfied, repeat step 6.

9. Record the input (f) and output (F) forces, then carefully measure and record the angle (θ) between the wire link and the slider shaft center line.

10. Repeat steps 6 through 9 for input forces of 2, 3, 4, 5, 6, and 7 ounces. Considerable care must be exercised in taking these data to insure that they are reliable.

11. For each data set compute the ratio f/F.

12. Plot the angle θ versus the ratio f/F on a sheet of graph paper.

13. On the same set of axes plot equation 29.5.

ℓ_1 =		ℓ_2 =	
f (ounces)	F	f/F	θ
1V			
2			
3			
4			
5			
6			
7			

Fig. 29-8 The Data Table

ANALYSIS GUIDE. In the analysis of these data you should compare the two curves and discuss their differences and similarities. List and discuss some of the possible sources of error in the experiment. Discuss each of the conditions given in step 7 and explain why each is important.

PROBLEMS

1. A certain stone crushing machine employs equal length arms in a toggle mechanism. If the toggle angle never exceeds 2.0 degrees, what input force is required to a minimum load force of 1,000,000 pounds?
2. Make a sketch showing how a nutcracker using a toggle mechanism can be adjusted to accommodate nuts of various sizes.
3. A pair of toggle pliers requires a force of 22 pounds to hold the toggle angle at 3 degrees. If the toggle links are of equal length, what is the load force?
4. A certain toggle mechanism has unequal links of ℓ_1 = 8 in. and ℓ_2 = 5 in. What is the ratio f/F at a toggle angle of: 1.0 degree? 5.0 degrees? 30 degrees?
5. Work out problem 4 using both equations 29.4 and 29.5. What difference, if any, do you observe?
6. List several practical applications of a toggle mechanism.

experiment 30 TOGGLE LATCHING

INTRODUCTION. Mechanisms that latch or hold a load in position are widely used in many practical applications. In this experiment we shall examine a simple example of a latching mechanism.

DISCUSSION. Toggle mechanisms are frequently used to latch a load in position. Figure 30-1 shows a simplified toggle used in this way. In this mechanism ℓ_1 and ℓ_2 are the toggle links with ℓ_3 the load support link.

When the toggle links are to the right of the center line as shown in figure 30-1, the load support is latched. The load force tends to hold the toggle in the position shown.

If we apply an unlatching force from the right, we can push the toggle joint past its center line and release the load. The amount of unlatching force required depends on the geometry of both the toggle and the support arms as well as on the load force. In the case of figure 30-1 we can determine the effective toggle end load (F′) applied through the load support using figure 30-2.

The load support is a class-two lever and the force effective at the end of the toggle can be found by observing that the moments acting on the lever must be equal

$$F(S_1 + S_2) = F'(S_2)$$

or

$$F' = F\frac{S_1 + S_2}{S_2} \qquad (30.1)$$

Then if the toggle angle (Θ) is relatively small, the unlatching force necessary to just hold the system in equilibrium is

$$f \simeq F'\frac{\ell_1 + \ell_2}{\ell_1} \tan\Theta \qquad (30.2)$$

Combining the results of equations 30.1 and 30.2 gives us

$$f \simeq F\frac{(S_1 + S_2)(\ell_1 + \ell_2)}{\ell_1 S_2} \tan\Theta \qquad (30.3)$$

as the relationship between the unlatching force and the load force. In actual practice there will be some friction and slack in the mechanism so that an unlatching force slightly greater than that predicted by equation 30.3 is usually required for unlatching.

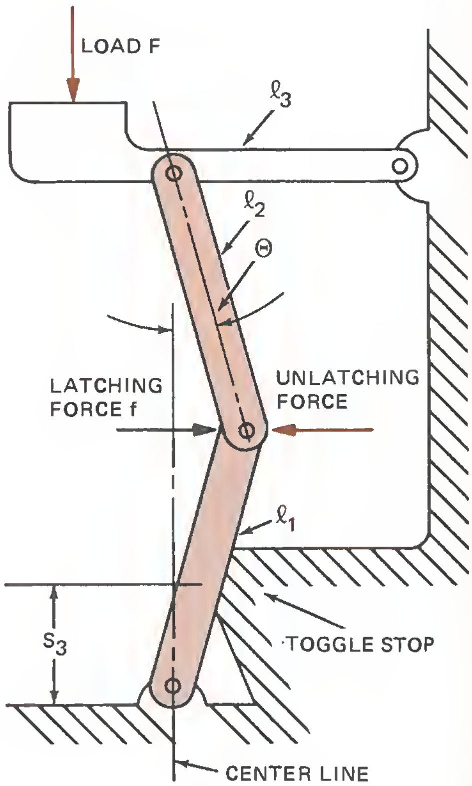


Fig. 30-1 A Latching Toggle

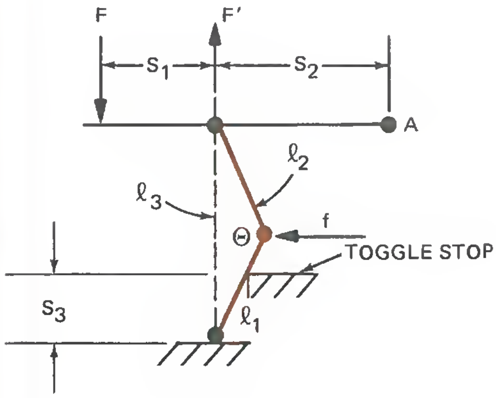


Fig. 30-2 Determining Unlatching Force

It is worth noting that if the toggle stop is located a distance (S_3) above the ℓ_1 pivot, then the force (f′) acting on the stop will be

$$f' \simeq f \frac{\ell_1}{S_3} \qquad (30.4)$$

if the angle between ℓ_1 and the center line is small when the mechanism is resting on the stop.

Toggle mechanisms of this type are used in a variety of practical applications. Figure 30-3 shows a typical example of a toggle latch used in a pair of pliers. In this case the lower jaw is, in effect, a bell crank used to change the direction of the load force.

Notice also that the left end of ℓ_1 is not fixed. It can be adjusted with a positioning screw in the handle. This allows the spacing of the latched jaws to be set to a desired value. Because of this adjustability a force analysis of the type discussed above is only valid at one setting.

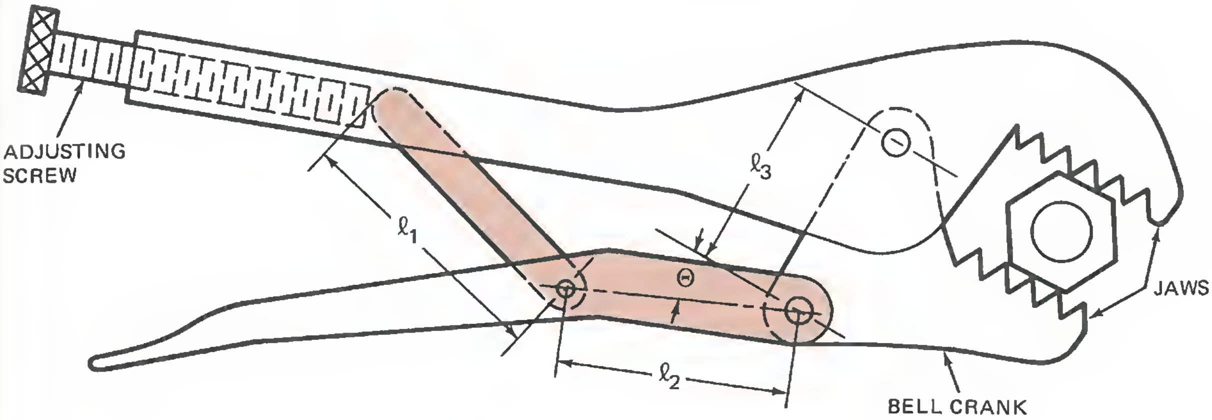


Fig. 30-3 Toggle Latch Pliers

MATERIALS

- 1 Steel rule 6 in. long
- 1 Protractor
- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 1 Shaft hanger with bearing
- 4 Bearing holders with bearings
- 3 Shafts 4″ x 1/4″
- 2 Spring balances with clamps and posts
- 1 Lever arm 2 in. long with 1/4-in. bore hub
- 1 Pulley approx. 3/4 in. OD with 1/4-in. bore hub
- 1 Rigid coupling
- *1 Wire loop link approx. 3 in. long
- 1 Machine screw 6-32 x 1/4
- 1 Spacer No. 6 x 1/8 with 1/32 in. wall thickness
- 5 Collars
- 2 Pieces of string approx. 6 in. long

*See appendix A for wire link construction details.

PROCEDURE

1. Inspect all of your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 30-4.
3. Measure and record the link lengths used in the toggle mechanism.
4. Check to see that both spring balances read zero when no force is applied.
5. Position the input spring balance so that it reads zero with the toggle against the stop.
6. Move the output spring balance forward until it reads zero. Then carefully move it back until it reads 14 ounces.
7. Check to be sure that:
   - (a) The string connecting the input force to the toggle joint approaches the joint *vertically*. Adjust the position of the lever arm shaft as necessary to make the input string vertical.
   - (b) The point of attachment of the wire link to the slider block must be as shown in figure 30-5. You may have to hold the screw in this position while taking your force and angle readings.
   - (c) Both spring balances must be horizontal and they must not touch any part of the mechanism.
8. With the conditions in step 7 satisfied repeat step 6.
9. Record the input (f) and output (F) forces, then carefully measure and record the angle (Θ) between the wire link and the slider shaft center line.
10. Repeat steps 6 through 9 for input forces of 1, 2, 3, 4, etc. oz. until the toggle unlatches. Keep the output force at 14 oz. Considerable care must be exercised in taking these data to insure that they are reliable.
11. Using the link lengths, work out an equation for the approximate unlatching force you would expect for the experimental mechanism. Use an analysis approach similar to the one in the discussion.
12. With your equation and your values of link lengths (also your value of output force), compute the expected unlatching force and record it.

ANALYSIS GUIDE. In the analysis of these data you should compare the experimental and computed values of unlatching forces, then discuss their differences and similarities. List and discuss some of the possible sources of error in the experiment. Discuss each of the conditions given in step 7 and explain why each is important.

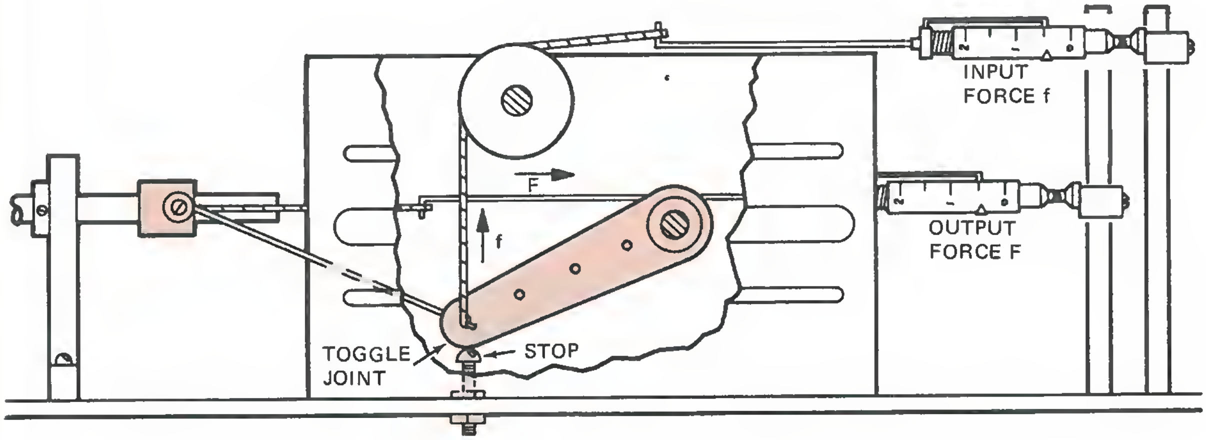


Fig. 30-4a Experimental Setup Side View

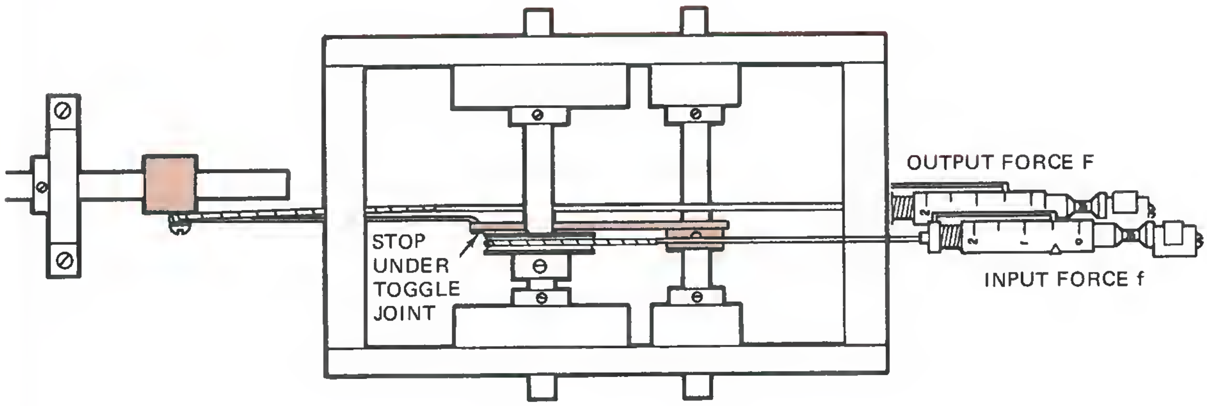


Fig. 30-4b Experimental Setup Top View

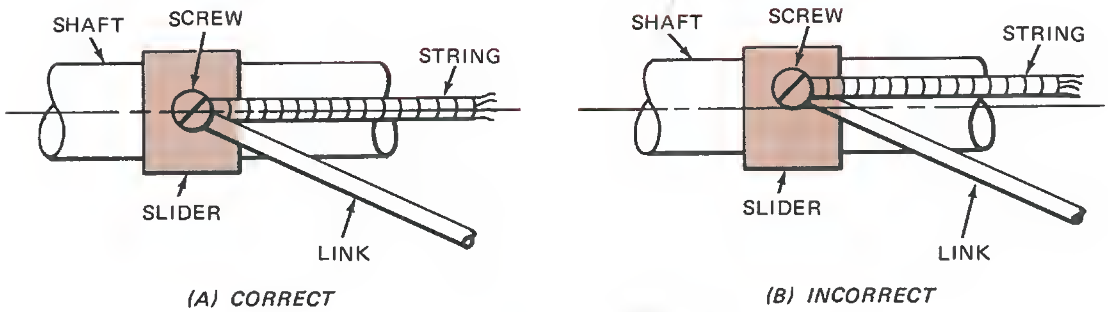


Fig. 30-5 Slider and Link Alignment

ℓ_1 =		ℓ_2 =
f	F	Θ
Unlatching force equation		
Unlatching force value		

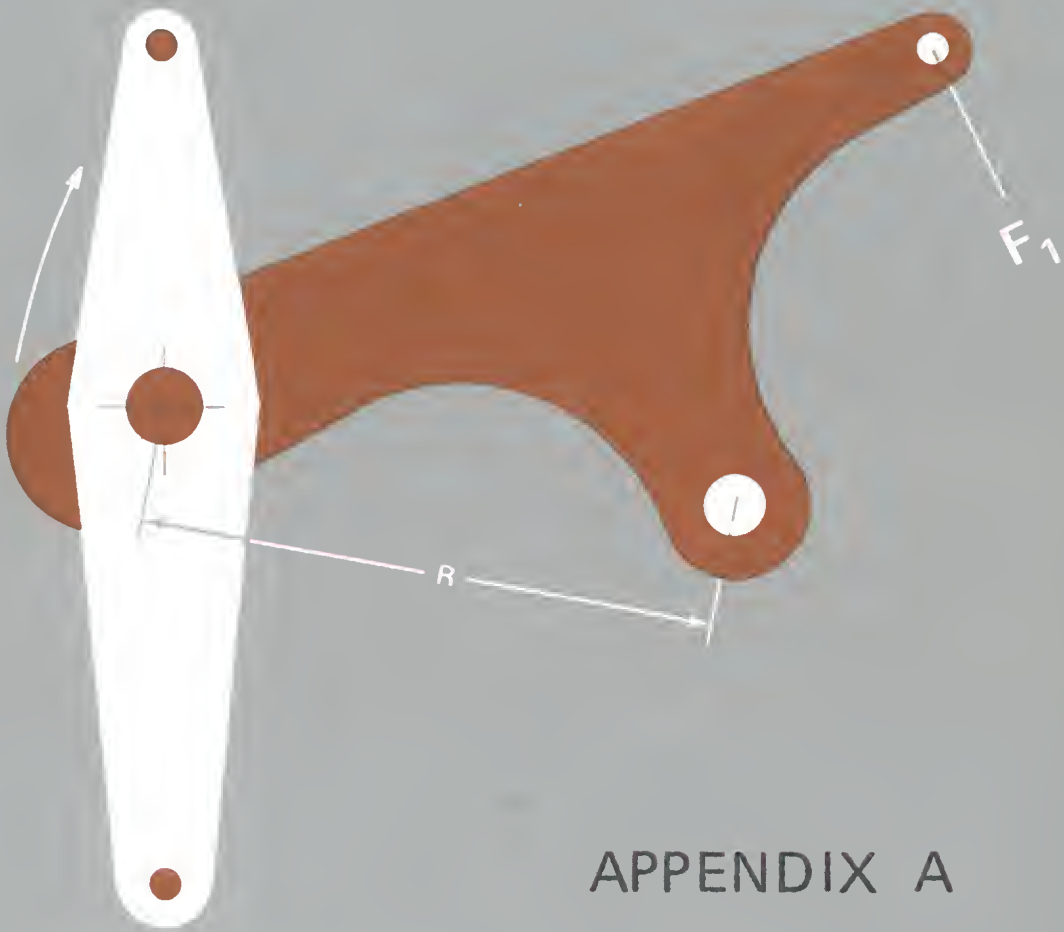
Fig. 30-6 The Data Table

PROBLEMS

1. A pair of toggle latch pliers has equal length toggle links and a toggle angle of two degrees. What is the unlatching force if the load effective at the toggle end is 300 pounds?
2. What type of four-bar mechanism is represented by the pliers in figure 30-3? Assume ℓ_1 = 1-3/8 in., ℓ_2 = 1-3/8 in., ℓ_3 = 1 in. and ℓ_0 = 2-1/2 in.
3. A mechanism like figure 30-1 is used to lock an automobile lift in its "up" position. If the links are ℓ_1 = 4 ft, ℓ_2 = 3-1/2 ft, ℓ_3 = 2-1/2 ft, ℓ_4 = 6 in. and the effective load is 4500 pounds, what would be the unlatching force for a toggle angle of 3.5 degrees?
4. What would be the force on the toggle stop in problem 3 if S = 2-1/2 ft? Assume the mechanism to be fully latched when Θ = 3.5 degrees.
5. List three practical applications of a toggle latch mechanism.

MECHANISMS

LINKAGES



APPENDIX A

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Appendix

WIRE LINK CONSTRUCTION

The wire links used in these experiments should be constructed using steel wire with a diameter of 0.05 in. Figure A-1 shows the construction details for a straight link.

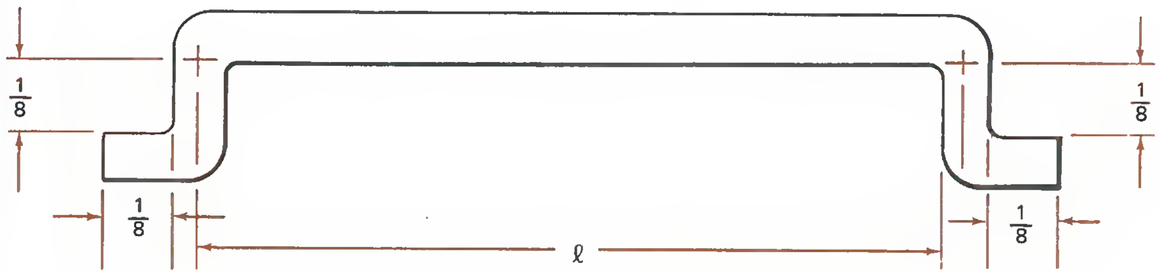


Fig. A-1 Straight Link Construction

The link should be constructed as follows:

1. Cut the wire *slightly* longer than the desired length (ℓ) plus 0.65 in.
2. Smooth off the rough ends with a file and remove any excess length. (The total length should be the desired value plus 0.65 in.)
3. Bend the ends of the wire as shown in figure A-1.
4. Twist the wire if necessary to align the ends.

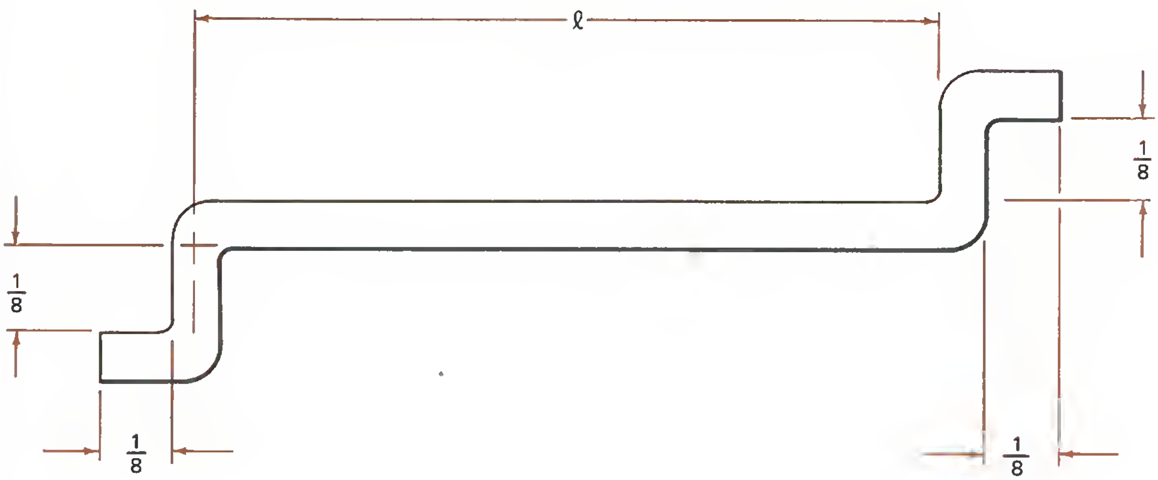


Fig. A-2 Reverse Link Construction

Reverse links are constructed in the same way but with the ends bent in opposite directions as shown in figure A-2.

Loop links are constructed in a similar manner but have a loop bent in one end as shown in figure A-3. The remaining end is constructed like a straight or reverse link.

In most cases the *exact* dimensions of a link are not critical. Don't worry if your link isn't precisely the prescribed dimensions.

When you make one of the links keep it with the other components so that it can be used in later experiments.

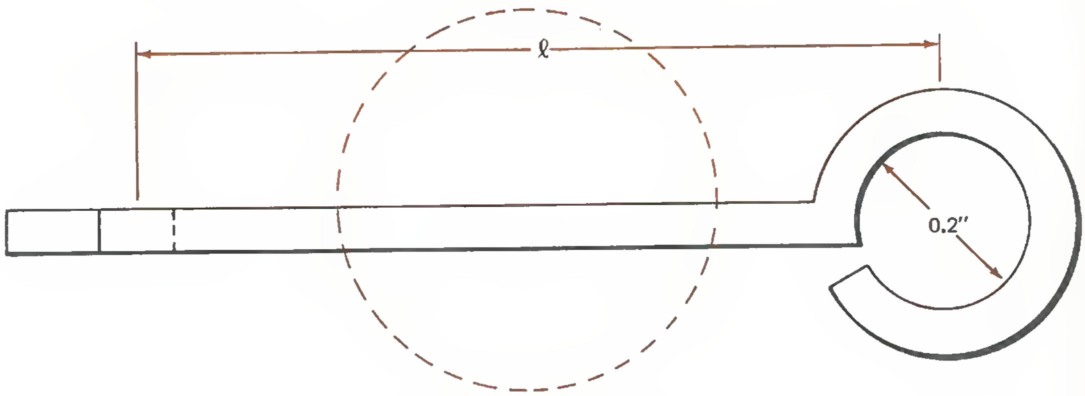


Fig. A-3 Loop Link Construction

APPENDIX B

EXPERIMENT 1 ____________ Name ____________________

Date: ____________ Class ____________ Instructor ____________

F_1	S_1	M_1	F_2	S_2	M_2	% Diff. in M

Fig. 1-5 Data for Coplanar Arms

F_1	S_1	M_1	F_2	S_2	M_2	% Diff. in M

Fig. 1-6 Data for Noncoplanar Arms

EXPERIMENT 2 ______ Name ______

Date: ______ Class ______ Instructor ______

Qty.	First Trial	Second Trial
F_1		
F'_2		
ℓ_1		
ℓ_2		
ℓ'_1		
ℓ'_2		
f		
M_1		
M_2		
M'_1		
M'_2		
MA_1		
MA_2		
$(MA_1)(MA_2)$		
F'_2/F_1		
% Diff.		

Fig. 2-4 The Data Table

EXPERIMENT 3 Name ______________________

Date: ______________ Class ____________ Instructor ______________

Qty.	F_1	F_2	ℓ_1	ℓ'_2	ℓ'_1	ℓ_2	f	f′	% Diff.
First Trial									
Second Trial									
Third Trial									

Fig. 3-6 The Data Table

EXPERIMENT 4 Name ______________________

Date: ______________ Class __________ Instructor ______________

F_1	F_2	ℓ_1	ℓ'_2	ℓ'_1	ℓ_2

f	M_1	M'_2	% Diff.	M'_1	M_2	% Diff.

MA_1	MA_2	MA_T	MA'_T	% Diff.

Fig. 4-5 The Data Tables

EXPERIMENT 5

Name ______________________

Date: ______________ Class __________ Instructor ______________

F_1	F_2	ℓ_1	ℓ_2	Θ_1	Θ_2	f_1	f_2	M_1	M_2	% Diff.

F_1/F_2	f_1/f_2	ℓ_2/ℓ_1

Fig. 5-8 The Data Tables

EXPERIMENT 6 Name ____________________

Date: ____________ Class ________ Instructor ____________

Qty Trial	n	N	F_1	F_2	ℓ_A	ℓ_B	ℓ_C	ℓ_D
1								
2								

Fig. 6-3 Data Table A

Qty Trial	Θ_A	Θ_B	Θ_C	Θ_D	f	f′	% Diff.
1							
2							

Fig. 6-3 Data Table B

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

EXPERIMENT 7 Name ____________________

Date: ____________ Class ________ Instructor ____________

Type of Mechanism	Name of Mechanism			
Dimensions of Mechanism	ℓ_o =	ℓ_1 =	ℓ_c =	ℓ_2 =
Sketch of Mechanism		Description of Motion		
Test of Mechanism's Possibility				

Fig. 7-10 Data for the First Mechanism

Type of Mechanism	Name of Mechanism			
Dimensions of Mechanism	ℓ_o =	ℓ_1 =	ℓ_c =	ℓ_2 =
Sketch of Mechanism		Description of Motion		
Test of Mechanism's Possibility				

Fig. 7-11 Data for the Second Mechanism

<table>
<tr><td>Type of
Mechanism</td><td colspan="4">Name of Mechanism</td></tr>
<tr><td>Dimensions of
Mechanism</td><td>$\ell_o =$</td><td>$\ell_1 =$</td><td>$\ell_c =$</td><td>$\ell_2 =$</td></tr>
<tr><td colspan="2">Sketch of Mechanism</td><td colspan="3" rowspan="2">Description of Motion</td></tr>
<tr><td colspan="2">Test of Mechanism's
Possibility</td></tr>
</table>

Fig. 7-12 Data for the Third Mechanism

<table>
<tr><td>Type of
Mechanism</td><td colspan="4">Name of Mechanism</td></tr>
<tr><td>Dimensions of
Mechanism</td><td>$\ell_o =$</td><td>$\ell_1 =$</td><td>$\ell_c =$</td><td>$\ell_2 =$</td></tr>
<tr><td colspan="2">Sketch of Mechanism</td><td colspan="3" rowspan="2">Description of Motion</td></tr>
<tr><td colspan="2">Test of Mechanism's
Possibility</td></tr>
</table>

Fig. 7-13 Data for the Fourth Mechanism

EXPERIMENT 8 Name ______________________

Date: ______________ Class __________ Instructor ______________

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = __________ ℓ_2 = __________ ℓ_c = __________ ℓ_o = __________

Fig. 8-8 Data Table for the First Trial

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = __________ ℓ_2 = __________ ℓ_c = __________ ℓ_o = __________

Fig. 8-9 Data Table for the Second Trial

EXPERIMENT 9

Name ____________________

Date: ____________________ Class __________ Instructor ____________________

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = __________ ℓ_2 = __________

ℓ_c = __________ ℓ_o = __________

Fig. 9-6 Data Table First Trial

Driver Position	Follower Position	Angular Velocity (Rad/sec.)
0°		
20°		
40°		
60°		
80°		
100°		
120°		
140°		
160°		
180°		
200°		
220°		
240°		
260°		
280°		
300°		
320°		
340°		
360°		

ℓ_1 = __________ ℓ_2 = __________

ℓ_c = __________ ℓ_o = __________

Fig. 9-7 Data Table Second Trial

EXPERIMENT 10 ________ Name ____________________

Date: ____________ Class ________ Instructor ____________

Counterclockwise			Clockwise	
Θ_1	Θ_2		Θ_2	Θ_2
		ℓ_o = ____		
		ℓ_1 = ____		
		ℓ_2 = ____		
		ℓ_c = ____		

Fig. 10-6 Data Table I

Counterclockwise

Θ_1	Θ_2

ℓ_o = ____

ℓ_1 = ____

ℓ_2 = ____

ℓ_c = ____

Clockwise

Θ_1	Θ_2

Fig. 10-7 Data Table II

EXPERIMENT 11

Name ______________________________

Date: ____________________ Class ____________ Instructor ____________________

EXPERIMENT 12 Name ______________________

Date: ______________ Class __________ Instructor ______________

ℓ_o =		ℓ_c =	
First Trial		Second Trial	
θ	Θ	θ	Θ

Fig. 12-6 The Data Table

EXPERIMENT 13

Name ______________________

Date: ______________ Class __________ Instructor ______________

ℓ_1	ℓ_c	K

Θ	X	d (Meas.)	d (Comp.)

Fig. 13-6 The Data Tables

Name ______________________

Date: ______________ Class ____________ Instructor ______________

ℓ_1	ℓ_o	ℓ_c	ℓ_2	ℓ'_1	ℓ'_c	ℓ'_o	S
Clockwise		Counterclockwise		Clockwise		Counterclockwise	
Θ	X	Θ	X	Θ'	X'	Θ'	X'

Fig. 14-9 The Data Table

EXPERIMENT 15 Name ______________________

Date: ______________ Class __________ Instructor ______________

First Pass		Second Pass		Third Pass		Average Values	
x	Θ	x	Θ	x	Θ	x	Θ

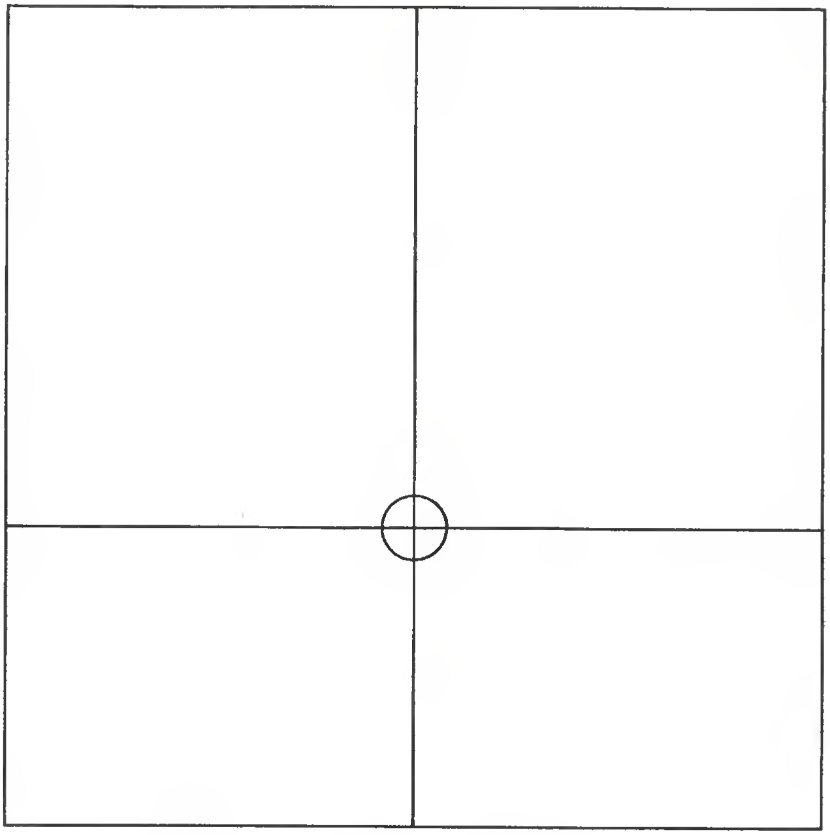
Length of follower lever arm r = ________

Fig. 15-5 The Data Table

EXPERIMENT 16 ____________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

Cam Profile



N_p	N_g	d

Gear & Follower Data

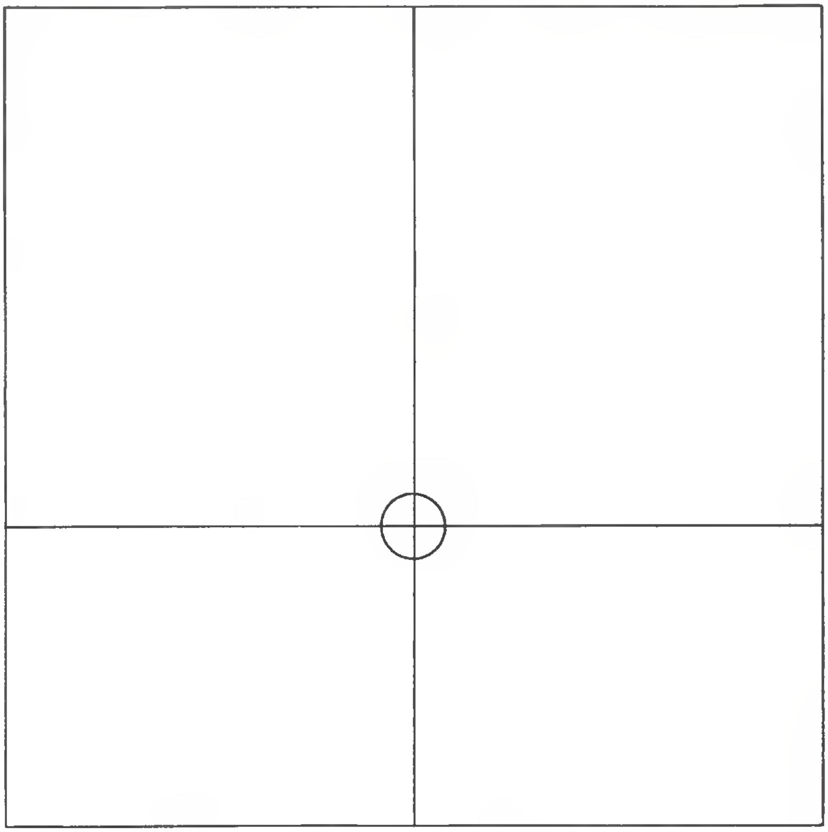
Θ_i	Θ_o	y

Fig. 16-10 The Data Tables

EXPERIMENT 17 __________ Name ______________________

Date: ______________ Class __________ Instructor __________

Cam Profile



N_p	N_g	d

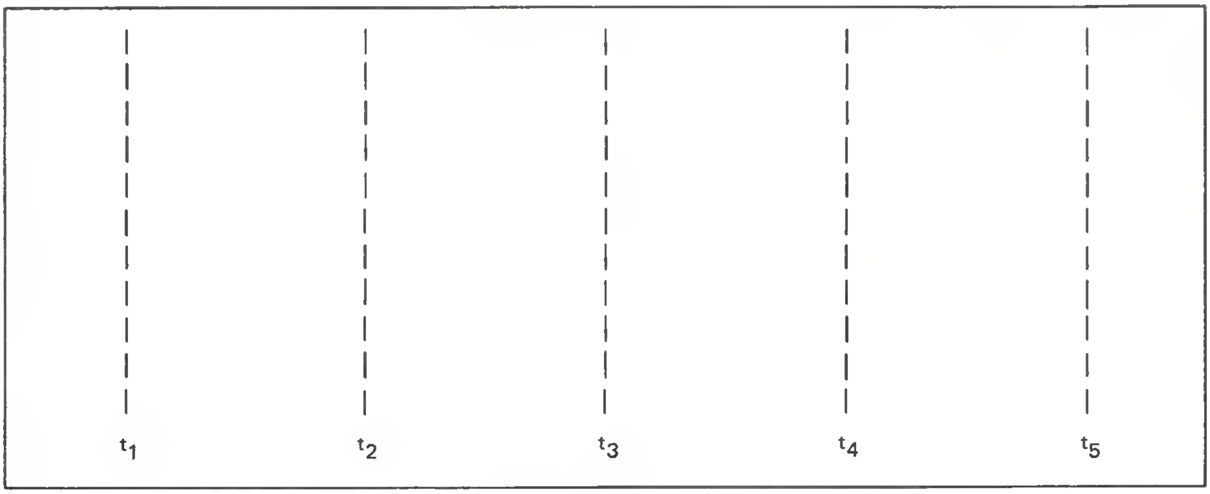
Gear & Follower Data

Θ_i	Θ_o	y

Fig. 17-9 The Data Tables

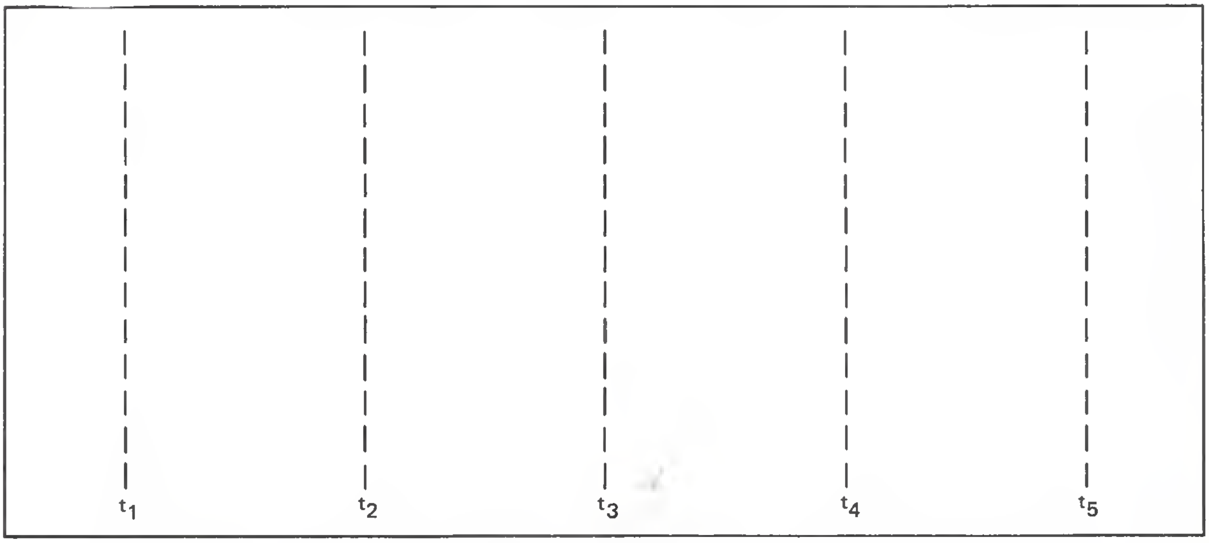
EXPERIMENT 18 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________



RESULTS FROM STEP 9

Fig. 18-13 The Experimental Results



RESULTS FROM STEP 11

Fig. 18-13 The Experimental Results (Cont'd)

EXPERIMENT 19 ______ Name ______

Date: ______ Class ______ Instructor ______

n_1 (3/4 in. OD)	N_1 (2 in. OD	n_2 (3/4 in. OD)	N_2 (2 in. OD)	$\frac{N_1}{n_1}$	$\frac{N_2}{n_2}$

Spur Gears

n_3	N_3	$n_3 - N_3$	Ratio $\frac{n_3}{n_3 - N_3}$

Harmonic Drive

	ω_i	ω_u	ω_o	$\frac{\omega_i}{\omega_o}$ ratio
Calculated				
Measured				

Fig. 19-5 The Data Table

EXPERIMENT 20 ____________ Name ______________________________

Date: ______________________ Class ____________ Instructor ______________

	Crank arm length, ℓ_1	Distance between shafts, ℓ_o
Measured		
Calculated		

Input Angle (degrees)	Output Angle (degrees)
0	
5	
10	
15	
25	
29	
30	
35	
40	
45	
50	
55	
60	

Input Angle (degrees)	Output Angle (degrees)
65	
70	
75	
80	
85	
90	
100	
150	
200	
250	
300	
350	
360	

Fig. 20-3 The Data Tables

EXPERIMENT 21 ____________ Name ______________________________

Date: ____________________ Class __________ Instructor ____________

Angle (degrees)	F_1 (oz)	F_2 (oz)	Ratio
20			
25			
30			
35			
40			
45			
50			
55			
60			
65			
70			
75			

Fig. 21-4 The Data Table

EXPERIMENT 22 ______________ Name ______________________________________

Date: ________________________ Class ___________ Instructor ________________

Input Θ_i (degrees)	Output Θ_o (degrees)

First Setup

Input Θ_i (degrees)	Output Θ_o (degrees)

Second Setup

Fig. 22-7 The Data Tables

EXPERIMENT 23 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

ℓ_1	ℓ_o	ℓ_c	ℓ_2	S	Θ	θ	Θ/θ
Clockwise		Counterclockwise		Clockwise		Counterclockwise	
β	X	β	X	β'	X'	β'	X'

Fig. 23-5 The Data Table

EXPERIMENT 24 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

x	y	output (z)
4	8	
7	12	
8	4	
10	12	
14	16	
11	17	
17	10	
5	13	
12	12	
8	11	

Fig. 24-5 The Data Table

EXPERIMENT 25 ____________ Name ______________________________

Date: ____________________ Class __________ Instructor ______________

Fig. 25-5		Fig. 25-6	
Crank Angle α in degrees	Displacement in inches	Lever Angle α in degrees	Displacement in inches
0			0
20			1/8
40			1/4
60			3/8
80			1/2
100			5/8
120			3/4
140			7/8
160			1
180			1-1/8
200			- 1/8
220			- 1/4
240			- 3/8
260			- 1/2
280			- 5/8
300			- 3/4
320			- 7/8
340			- 1
360			- 1-1/4

Fig. 25-7 The Data Table

EXPERIMENT 26 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

Reading No.	Distance From Reference	Output RPM
1	0 (at Ref point)	
2	6/64 in.	
3	12/64 in.	
4	17/64 in.	
5	22/64 in.	
6	26/64 in.	
7	31/64 in.	
8	38/64 in.	

Fig. 26-8 The Data Table

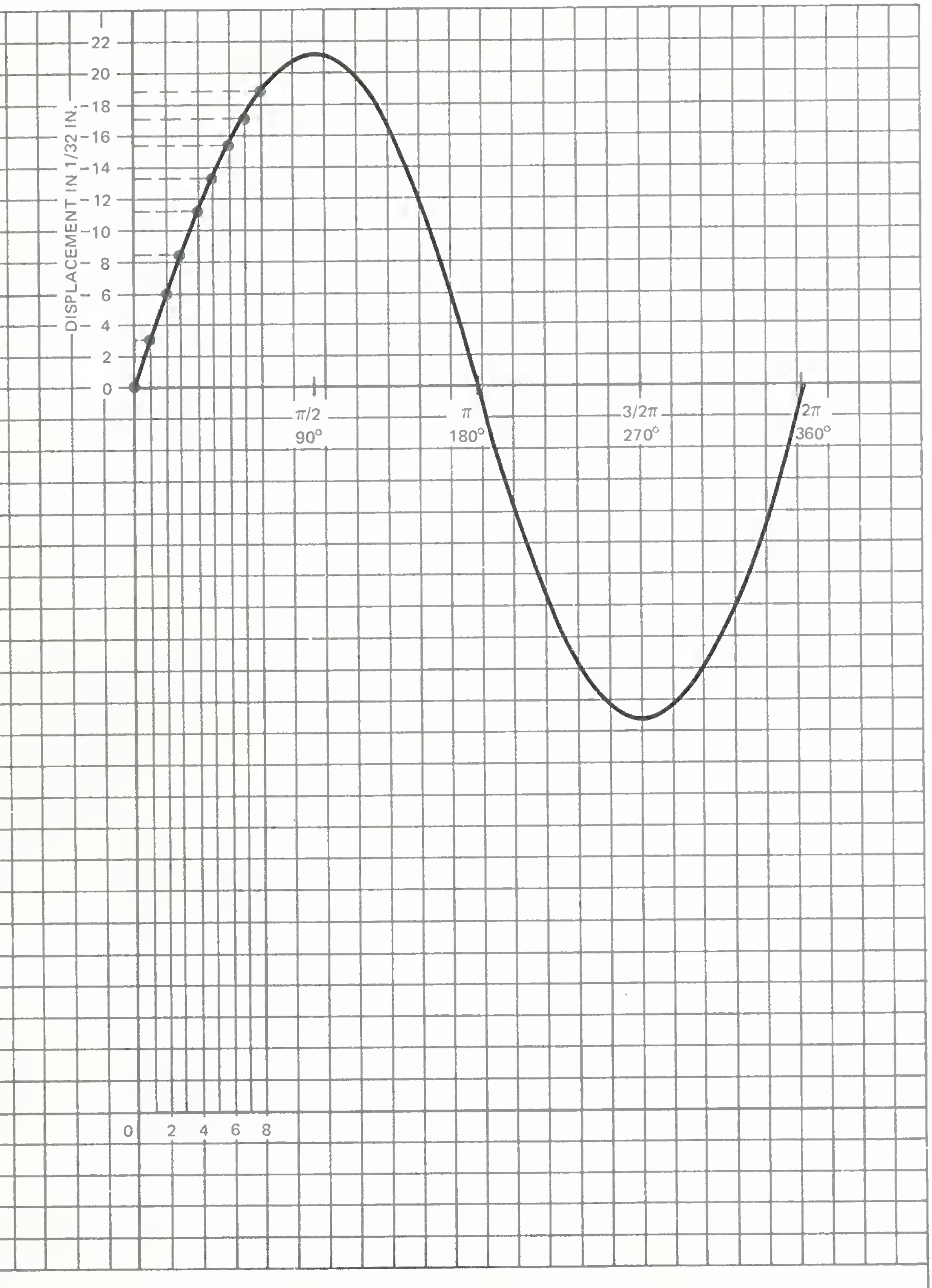


Fig. 26-10 Curves

EXPERIMENT 27 ______ Name ______

Date: ______ Class ______ Instructor ______

Clockwise Rotation		Counterclockwise Rotation	
Input Θ	Output Θ	Input Θ	Output Θ
$\ell_1 =$	$\ell_c =$	$\ell_o =$	$\ell_2 =$

Fig. 27-9 The Data Table

EXPERIMENT 28 ______________ Name ____

Date: ________________________ Class __________ Instructor ________________

Pawl Force	Pulley Force Forward	Pulley Force Backward
4 oz		
8 oz		
12 oz		
16 oz		

ℓ_1	S	Θ	R	r

Fig. 28-6 The Data Table

EXPERIMENT 29 ______________ Name ______________________________________

Date: ________________________ Class __________ Instructor ______________

ℓ_1 =		ℓ_2 =	
f (ounces)	F	f/F	θ
1V			
2			
3			
4			
5			
6			
7			

Fig. 29-8 The Data Table

EXPERIMENT 30 ________ Name ______________________

Date: ______________ Class __________ Instructor ____________

$\ell_1 =$		$\ell_2 =$
f	F	Θ
Unlatching force equation		
Unlatching force value		

Fig. 30-6 The Data Table